Scale-Adaptive Balancing of Exploration and Exploitation in Classical Planning

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MIT-IBM Watson AI Lab

This work was supported through DTIC contract FA8075-18-D-0008, Task Order FA807520F0060, Task 4 - Autonomous Defensive Cyber Operations (DCO) Research & Development (R&D).

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  - first principled adaptation of Monte Carlo Tree Search (MCTS) for forward state space search
  - can readily extend to non-unit cost
  - empirically better than GBFS without ad hoc tweaks

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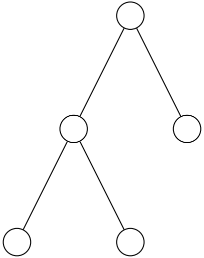

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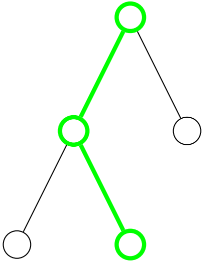

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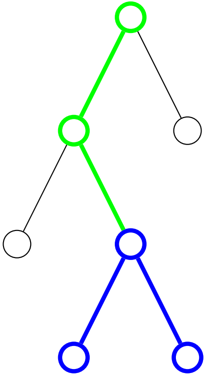

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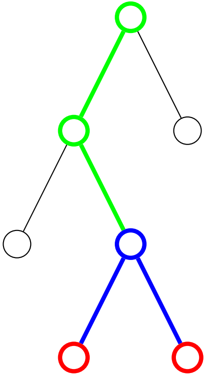

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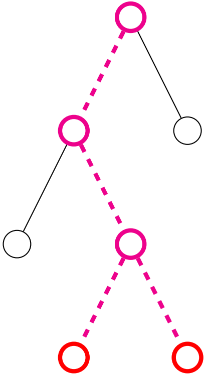

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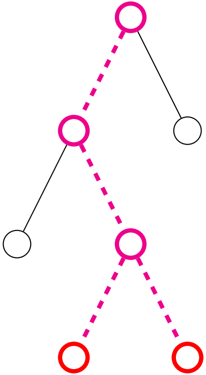

nodes store things like:

h heuristic value

$\hat{\mu}$ sample mean

t visitation counter

...modified via backup

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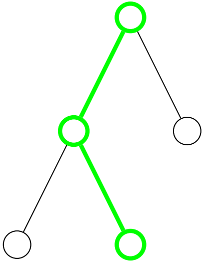

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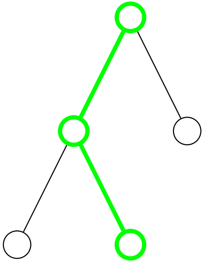

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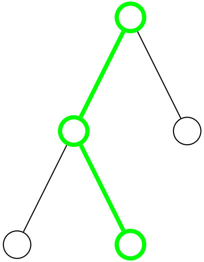

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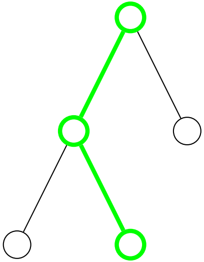

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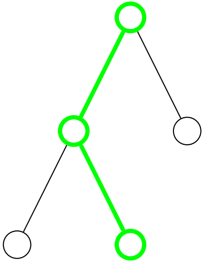

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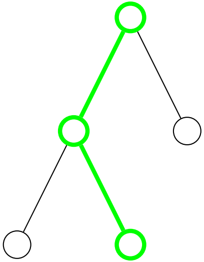

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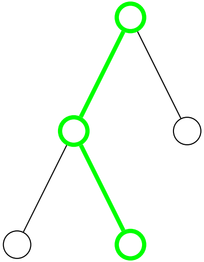

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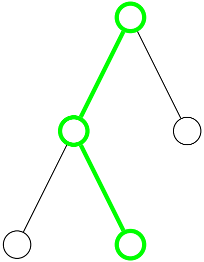

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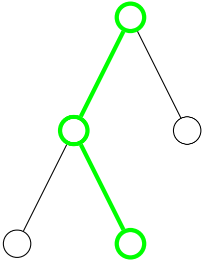

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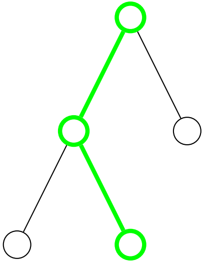

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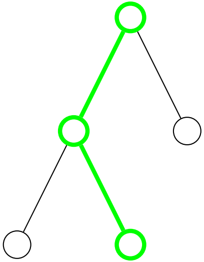

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- some existing MABs use μ, σ^2, but with drawbacks

GUCT-01[1] (Schulte & Keller SoCS-14)

- GUCT-01 adapts UCB1 by normalizing siblings to $[0, 1]$

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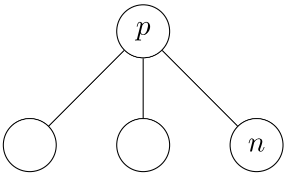

$$M = \max_{n' \in S(p)} h(n')$$

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$$f_{\mathrm{GUCT-01}}(n) = \overset{\textbf{exploitation}}{\frac{h(n) - m}{M - m}} - \overset{\textbf{exploration}}{{\color{red}c}\sqrt{(2\log T)/t}}$$

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$\boldsymbol{S(p)}$ successors of p

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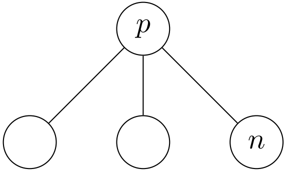

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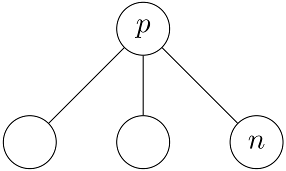

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- but h range varies per subtree!

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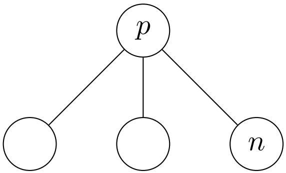
$M - m$

Factor out normalization:

- preserve node ordering
- demonstrate outsize impact on exploration term: will explore too much



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GUCT-Normal2: theoretically sound for classical planning

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(differences in red)

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  - adapts exploration to each child's subtree

Better IPC coverage under 10k node evaluations

h	GBFS	GUCT-01 (c)	**GUCT-Normal2**
h^{FF}			
h^{add}			
$h^{\max}$			
h^{GC} (for Atari)			

Better IPC coverage under 10k node evaluations

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h^{max}	221.4	242.2 (0.5) 227.6 (1.0)	
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h^{FF}	522.4	369.6 (0.5) 354.8 (1.0)	**563.8**
h^{add}	501.6	345.2 (0.5) 312.8 (1.0)	**519.2**
$h^{\max}$	221.4	242.2 (0.5) 227.6 (1.0)	**301.0**
h^{GC} (for Atari)	351.2	307.0 (0.5) 295.2 (1.0)	**374.6**
h^{FF}+PO	-	403.2 (0.5) 387.0 (1.0)	
h^{FF}+DE	474.0	355.6 (0.5) 344.8 (1.0)	
h^{FF}+DE+PO	-	406.4 (0.5) 404.4 (1.0)	

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- non-unit cost setting

More on S&K

- empirical: only improved over GBFS with preferred operators (PO), deferred heuristic evaluation (DE)
- regret bounds: GUCT-Normal2's polynomial regret bound expected to be better than GUCT-01's logarithmic regret bound in practice in context of deterministic, discrete, finite-state space search like planning, **where estimated variance can approach or equal true variance**.

Full Coverage Results

$h =$	h^{FF}		h^{add}		$h^{\max}$		h^{GC}		h^{FF}+PO		h^{FF}+DE		h^{FF}+DE+PO	
$c =$	0.5	1	0.5	1	0.5	1	0.5	1	0.5	1	0.5	1	0.5	1
GUCT	413.2	396.4	405.8	373.8	224.8	222.2	296	278	439.2	411.8	418.6	354.6	450	393.2
*	508.8	440.8	496.2	453.8	239.4	234.2	306.2	303	542.4	448	441.8	386.8	477	422
-01	369.6	354.8	345.2	312.8	242.2	227.6	307	295.2	403.2	387	355.6	344.8	406.4	404.4
*-01	393.6	372	373	343.6	236.2	226.4	306.2	289.8	430.2	401.2	377.6	363	426.2	421.2
-V	329.8	307.2	325	297.6	215	200	264.8	243.8	383.8	348.4	334.4	310	384.4	377.4
-Normal	-	278	-	261.4	-	209.2	-	231.8	-	331.6	-	269.2	-	342.6
*-Normal	-	311.6	-	294.8	-	212.2	-	244	-	338.2	-	285.2	-	343.8
-Normal2	-	**563.8**	-	**519.2**	-	**301**	-	**374.6**	-	**596.4**	-	**496.8**	-	**550.8**
*-Normal2	-	551.2	-	516.2	-	258.2	-	338.6	-	593.8	-	490.6	-	543.4
GBFS	-	522.4	-	501.6	-	221.4	-	351.2	-	-	-	474	-	-

10k node evaluation limit, average of 5 runs, no results for configurations unsupported by Pyperplan, subset of IPC benchmarks compatible with PDDL extensions supported by Pyperplan.
Terminated experiments when grounding exceeded 5 min., removing 751 problem instances across 24 domains.

Weaknesses of previous MABs

- UCB1-Normal (Auer et al. ML-02)
  - scale-adaptive, but relies on conjectures that are not guaranteed to hold
- UCB1-Tuned (Auer et al. ML-02):
  - assumes bounded reward distribution
  - lacks regret bound
- UCB-V (Audibert et al. 2009):
  - proven regret bound, but assumes bounded reward distribution
  - needs initialization pulls
- Bayes-UCT2 (Tesauro et al. 2010):
  - lacks regret bound
  - convergence proved only for bounded reward distributions
  - only tested on synthetic trees of fixed depth, width, and rewards