

Suboptimal Heuristic Search

Wheeler Ruml



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All actual work done by my fantastic students and collaborators.
Grateful thanks to NSF, BSF, and DARPA.

Heuristic Search Is Fundamental

Why Suboptimal?

■ Search Rocks

- Behavior of A*
- Optimal Isn't
- Problem Settings
- Info Sources
- Classic AI Agent
- Alg as Agent

Greedy Search

Bounded-suboptimal

Bounded Cost

Contract Search

Tunable

Anytime

Situated

Utility

Real-time Search

Conclusion

Search enables planning / action selection

- achieve goal robustly
- optimize resource use (time, energy, pollution, ...)
- autonomy or decision-support
- support retaskability, AGI

Search enables algorithms

- dynamic programming
- discrete optimization
- 'intractable' → possible



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Point 1/3: Suboptimal search is the most important kind!

Dijkstra vs A* for Pathfinding

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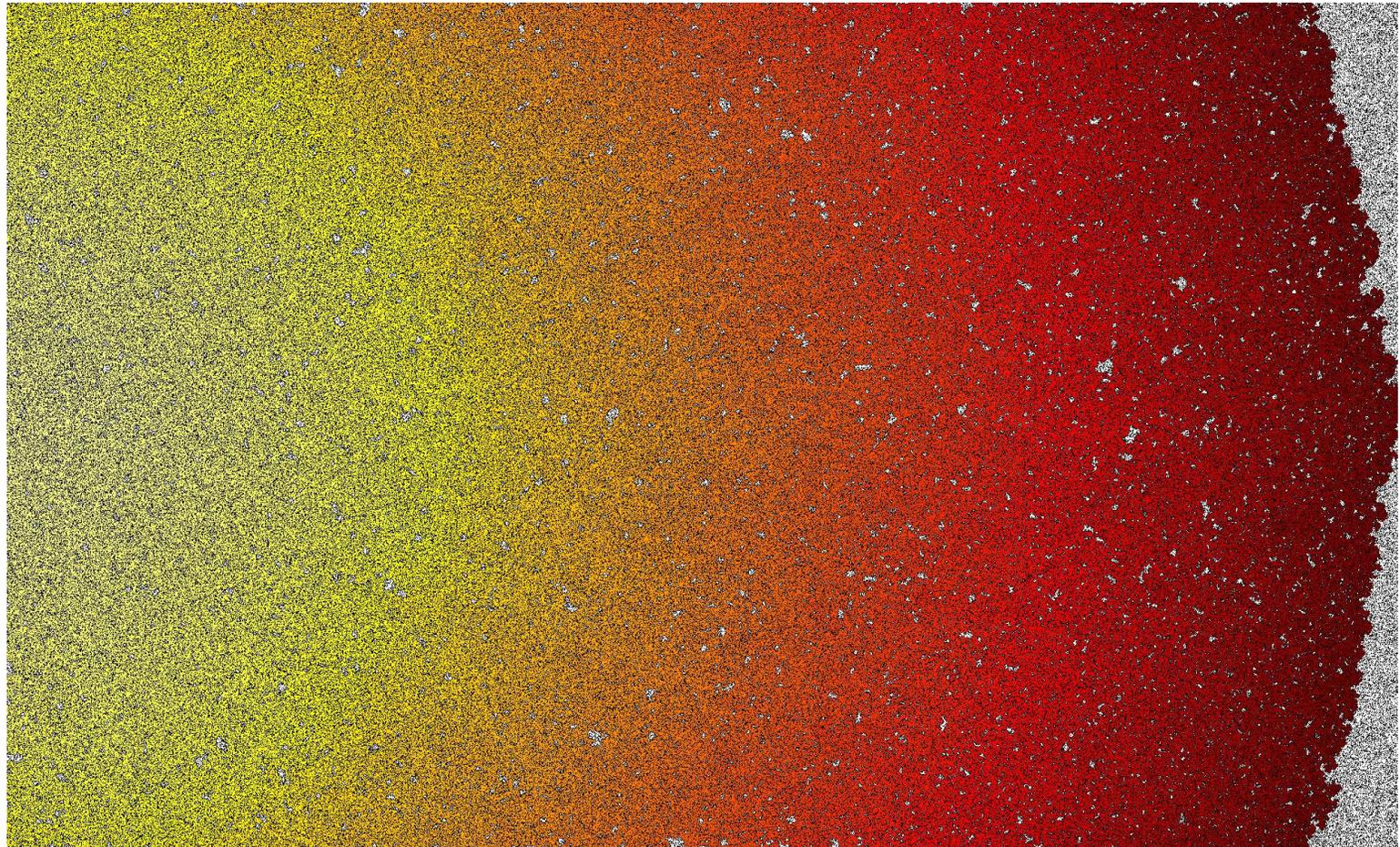
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uniform-cost search: best-first on g

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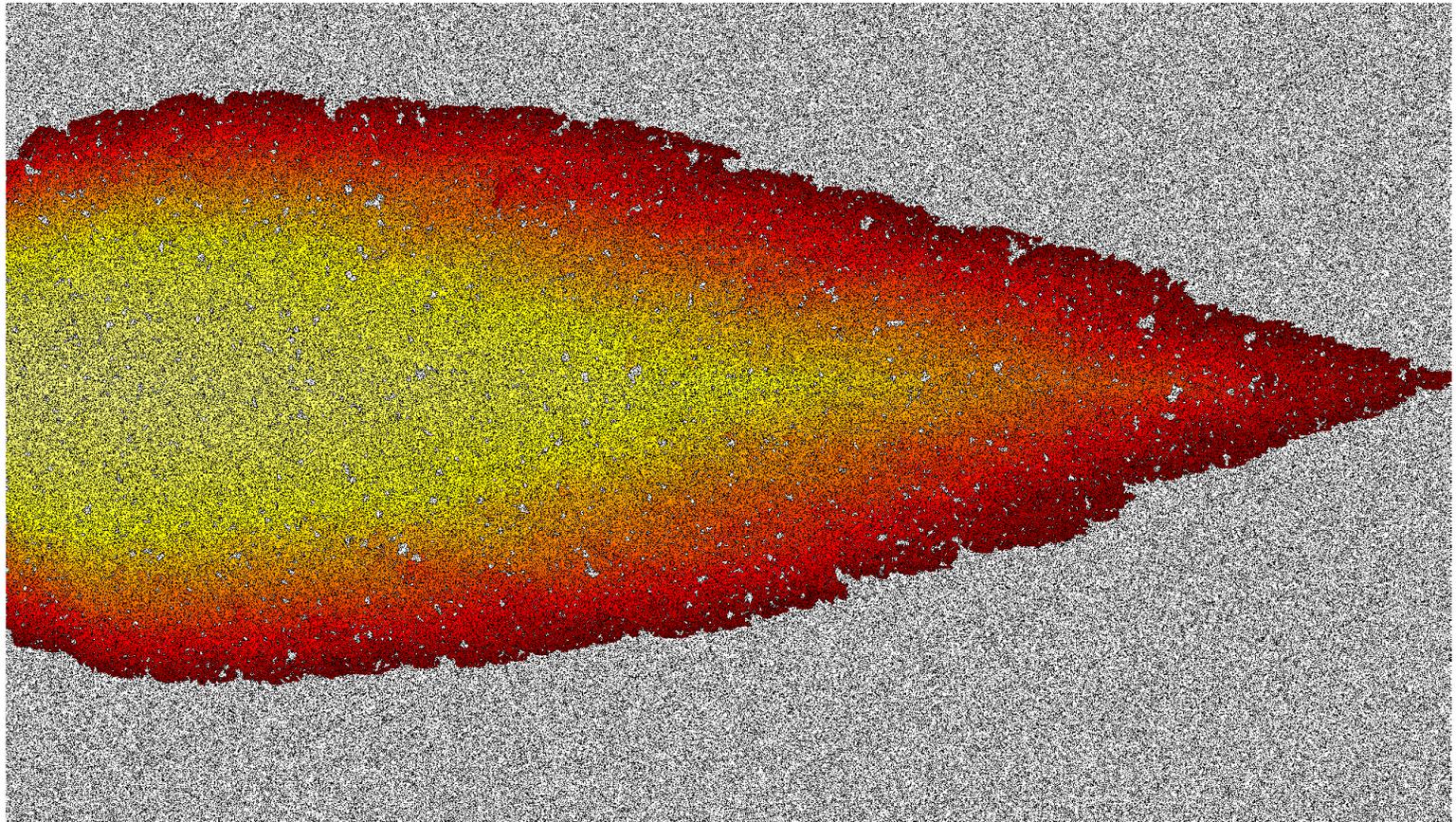
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A*: best-first on $f = g + h$

A* (Hart, Nilsson, and Raphael, 1968)

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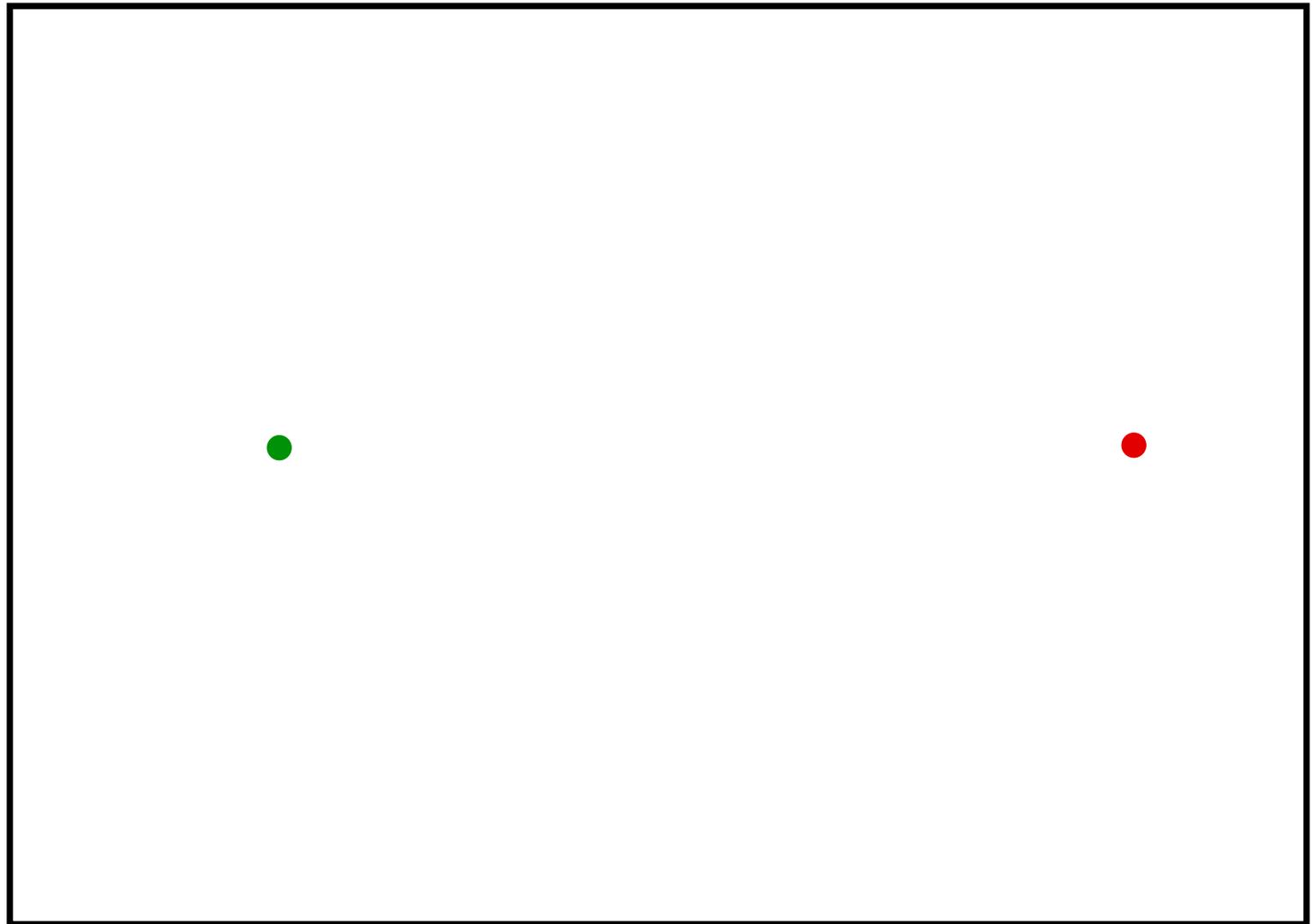
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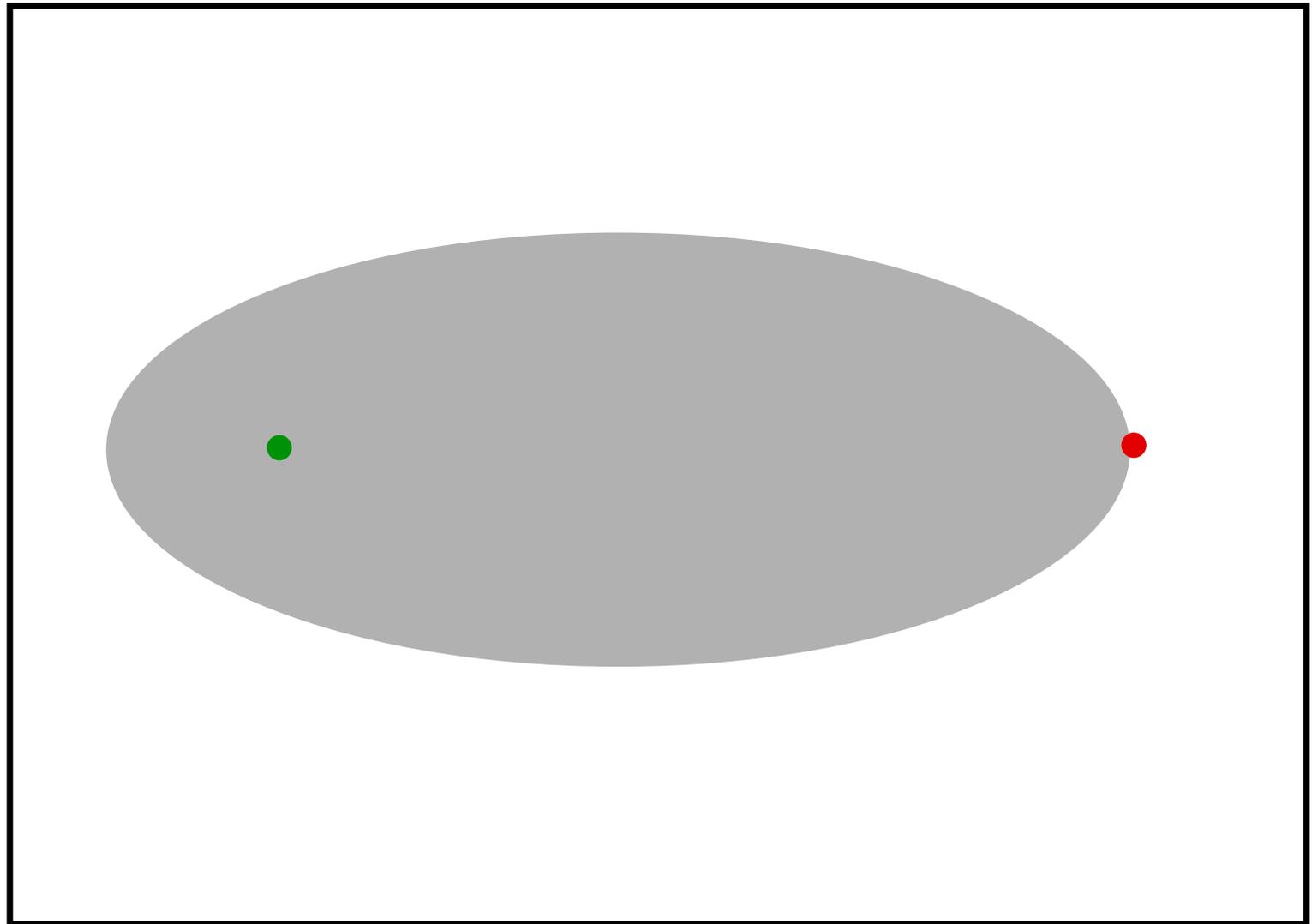
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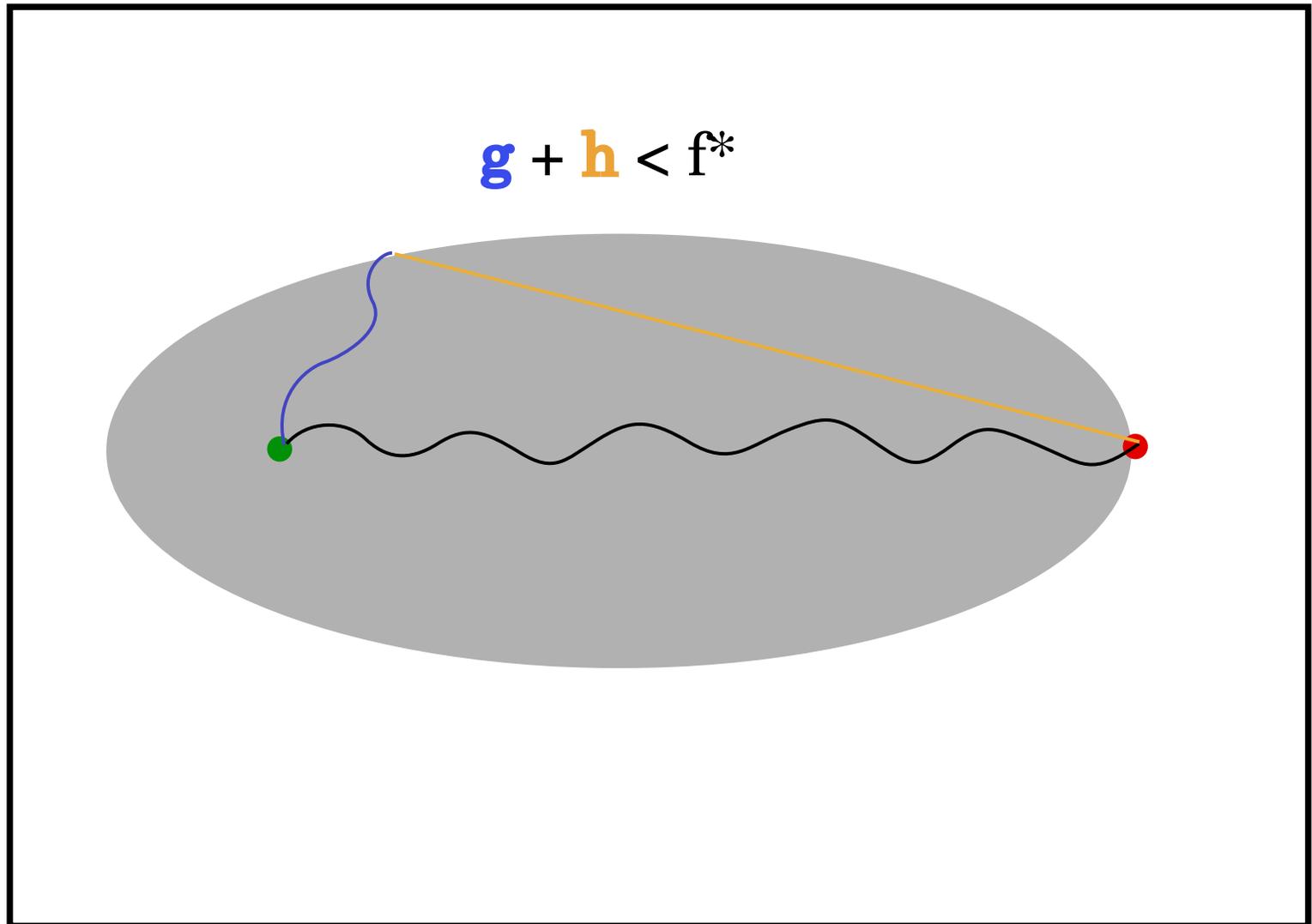
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heuristic is more about procrastination or pruning than guidance

Optimal Search is Provably Impractical

all nodes with $g(n) + h(n) < f^*$

Helmert and Röger, “How Good is Almost Perfect?”, AAI-08 Best Paper Award:

In many cases, such as the GRIPPER domain and a family of MICONIC tasks, there is no significant difference in node expansions between A* with an almost perfect heuristic and breadth-first search.

We suggest that, beyond a certain point, trying to improve a heuristic search algorithm by refining its heuristic estimates is basically fruitless.

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detecting symmetries and partial orders only fixes modeling errors
suboptimal search is the practical answer!

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Suboptimal Search Problem Settings

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optimal: minimize solution cost

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optimal: minimize solution cost

greedy: minimize solving time

bounded suboptimal: minimize time subject to relative cost bound (factor of optimal)

bounded cost: minimize time subject to absolute cost bound

contract: minimize cost subject to absolute time bound

tunable: trade cost for time, no guarantees

anytime: incrementally converge to optimal

situated: minimize goal achievement time

utility: maximize function of cost and time

real-time: return next action within absolute time bound

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My personal very biased view!

Taking Suboptimal Search Seriously: More Information

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information that becomes available during problem-solving

Point 2/3: many sources of information beyond h !

$\hat{h}(n)$: unbiased heuristics (possibly learned on-line from h)

$\hat{d}(n)$: distance-to-go estimates (eg, unit-cost h)

experience so far: eg, how misleading are estimates? how many paths look promising?

beliefs: distributions over values, quantify uncertainty

not today: preferred actions / policies

Classic AI Agent

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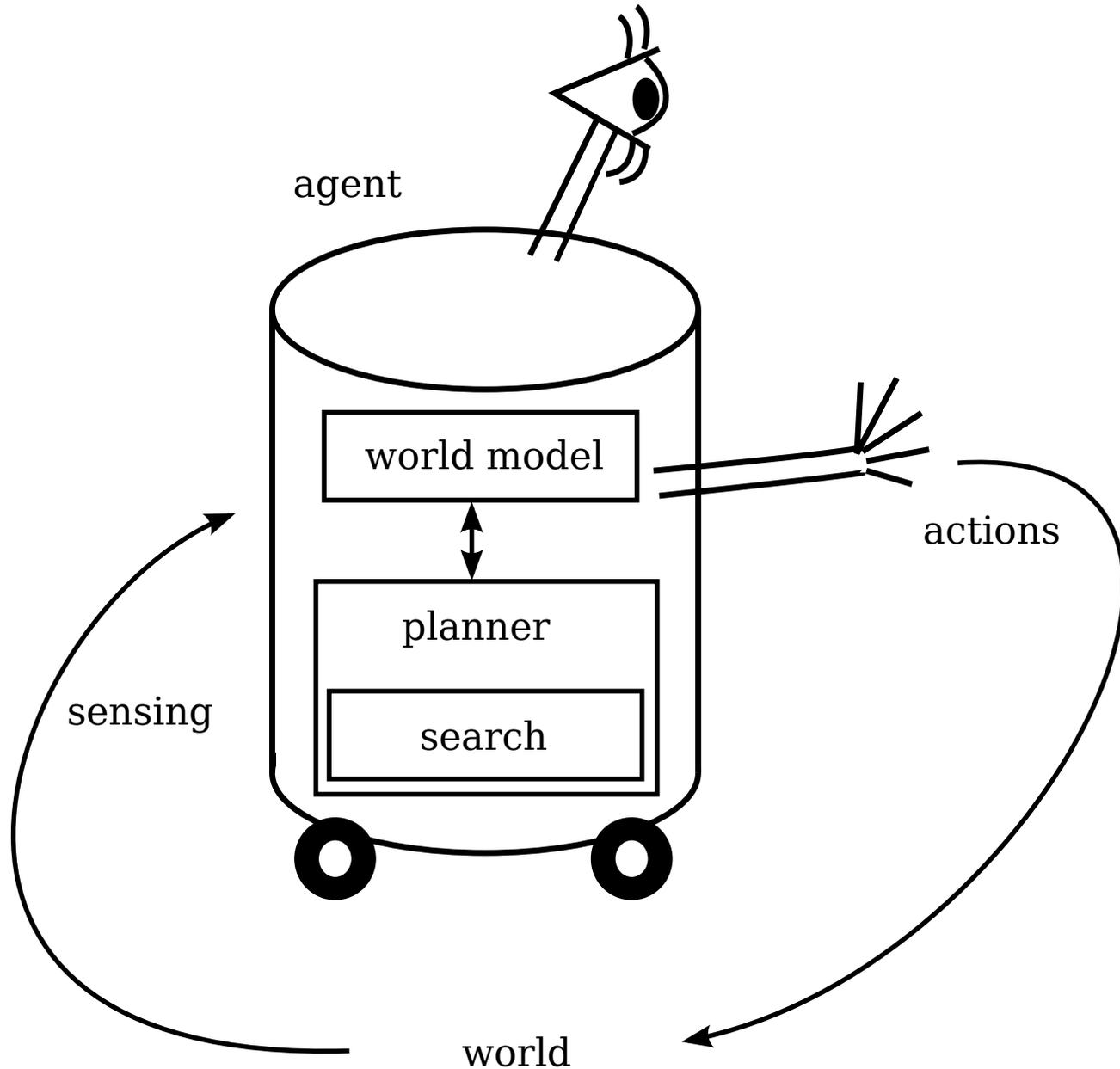
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Point 3/3: Search Algorithm as an Agent

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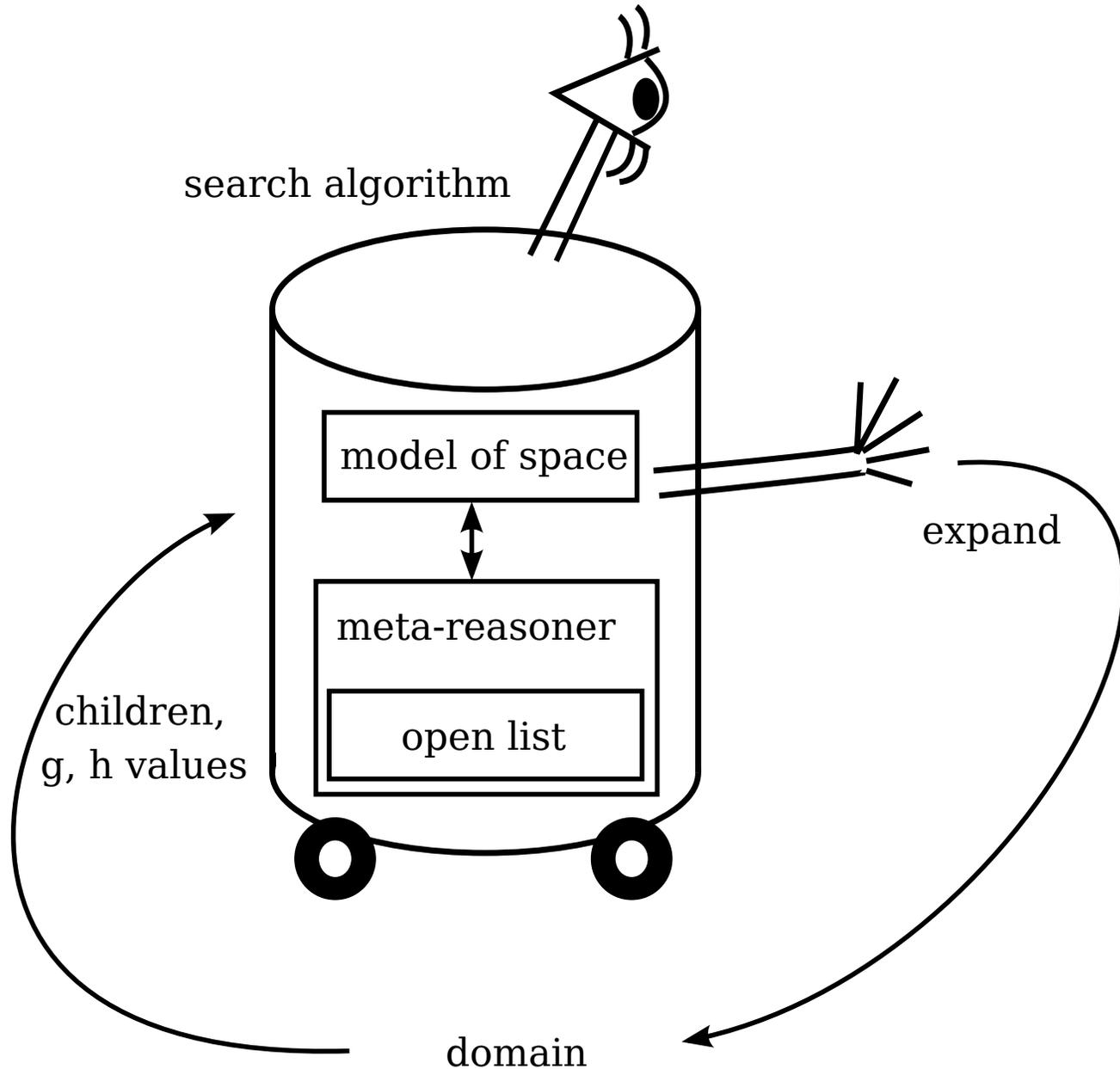
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Why Suboptimal?

Greedy Search

- Inadmissible h
- Distance-to-go
- \hat{d} Performance
- Why?
- GBFS Behavior
- Others

Bounded-suboptimal

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Greedy Search

Inadmissible Heuristics \hat{h}

finding solutions as quickly as possible

Greedy best-first search (GBFS): best-first search on h
inadmissible \hat{h} can be more informed

Why Suboptimal?

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\hat{d} Performance

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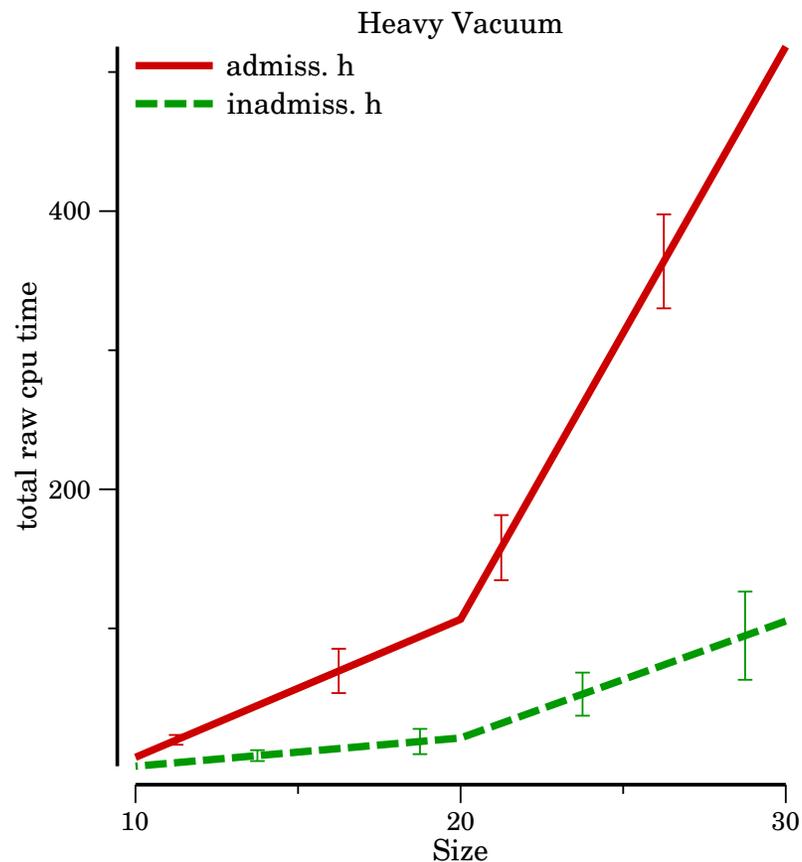
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searching on \hat{h} is faster than h

Distance-to-go

minimize solving time = minimize number of expansions to goal
for domains with costs, this is **not** $h(n)$

Why Suboptimal?

Greedy Search

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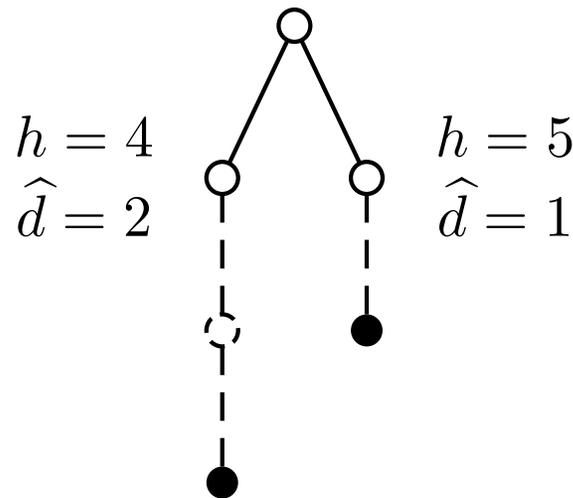
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Distance-to-go

minimize solving time = minimize number of expansions to goal

for domains with costs, this is **not** $h(n)$

$\hat{d}(n)$ distance-to-go, remaining solution path length, arcs-to-go, hops-to-go



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Performance of Distance-to-go

Why Suboptimal?

Greedy Search

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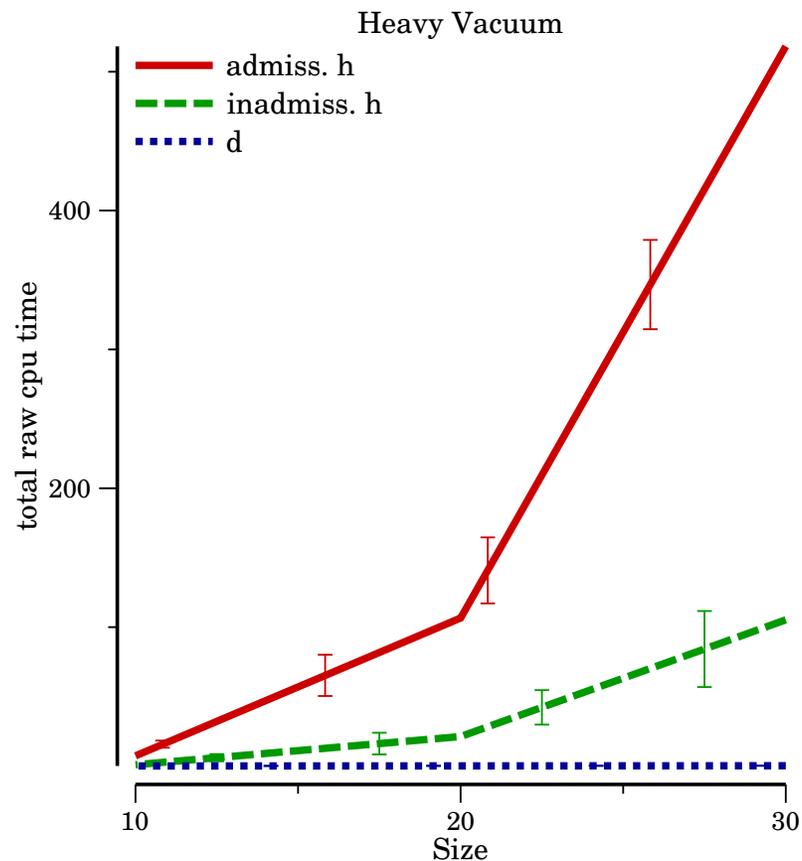
Utility

Real-time Search

Conclusion

Greedy: best-first search on h

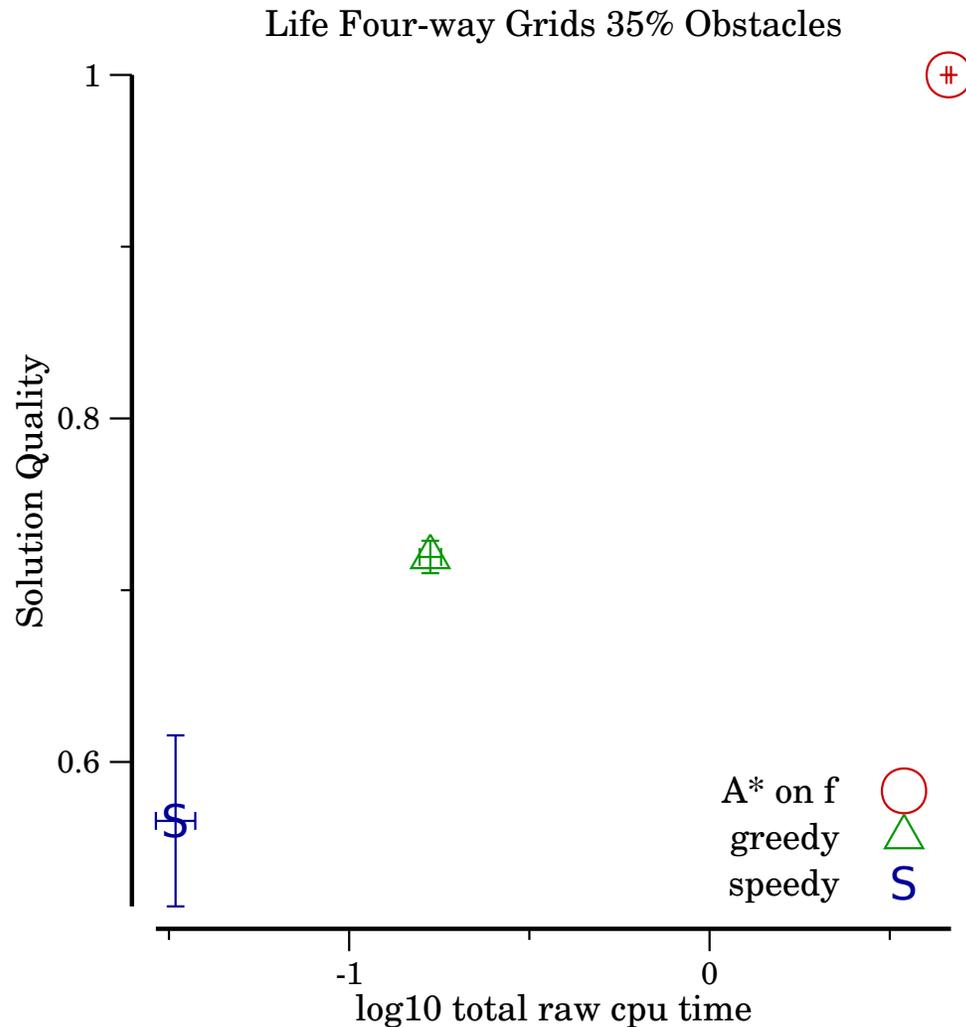
Speedy: best-first search on \hat{d} (Thayer, Ruml, and Kreis, SoCS-09)



searching on \hat{d} is faster than \hat{h}

Performance of Speedy Search

- Why Suboptimal?
- Greedy Search
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 - \hat{d} Performance
 - Why?
 - GBFS Behavior
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Speedy cost often reasonable (sometimes better than Greedy!)

Why Is Speedy Faster Than Greedy?

why faster than h ? (Wilt and Ruml, SoCS-14)

- not: predicts search effort
- **local minima are smaller**

Domain	Cost	Max Local Min Size	Expected Min Size	Exp
Tiles	unit	392	2	801
	inverse	51,532	87	93,010
	rev inv	2091	2	855
Hanoi	unit	7,587	1,892	36,023
	rev sq	35,874	4,416	559,250
	square	2,034	201	4,663
TopSpin	unit	296	250	933
	sum	922	3	749
	stripe	240	3	441

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Minimum Size Controls GBFS

intuition: for high cost ratios, many annoyingly cheap paths required to compensate for one unforeseen expensive action

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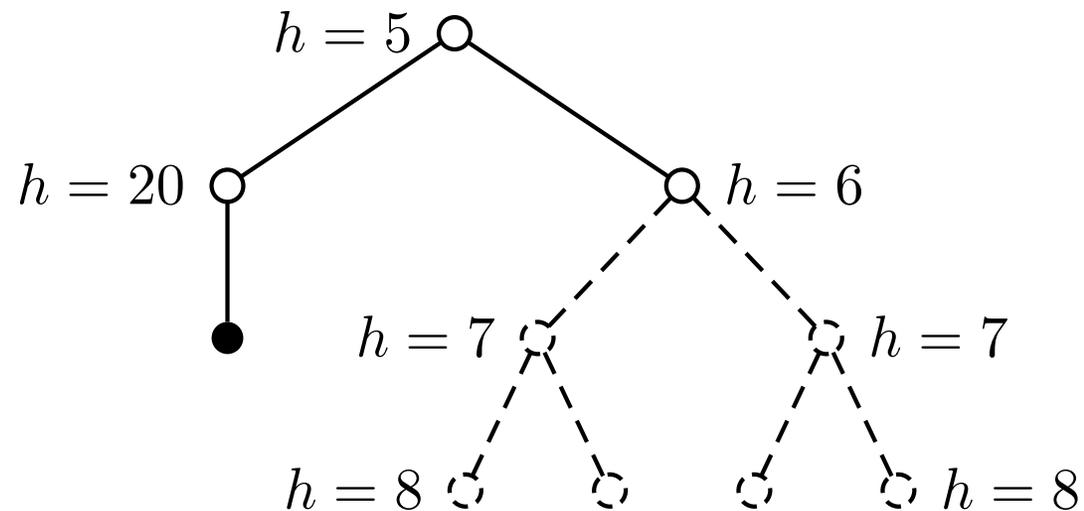
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Other Greedy Search Algorithms

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- Why?
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- **Others**

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robust greedy search is a wide open area!

exploration/diversity: ϵ -greedy, type-wA*

Why Suboptimal?

Greedy Search

Bounded-suboptimal

- Weighted A*
- wA* Bound
- RR- d
- Planning
- Search

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Bounded-suboptimal Search

Bounded-suboptimal Search: Weighted A*

quickly find a solution within factor b of optimal

simple hack (Pohl, AIJ 1970): $f'(n) = g(n) + b \cdot h(n)$

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Bounded-suboptimal Search: Weighted A*

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quickly find a solution within factor b of optimal

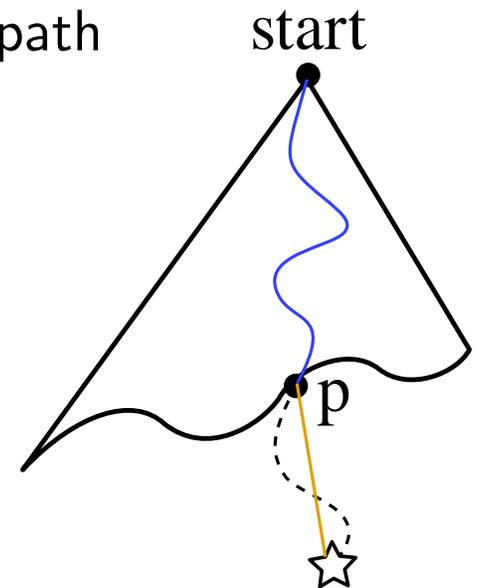
simple hack (Pohl, AIJ 1970): $f'(n) = g(n) + b \cdot h(n)$

$f_{min} = \text{lowest } f(n) \text{ on open}$

the key lemma: f_{min} is a global lower bound

1. any optimal path must pass through the frontier
2. let p be an open node along an optimal path
3. $f_{min} \leq f(p) = g(p) + h(p) \leq f^*$

can expand **any node** with $f(n) \leq b \cdot f_{min}$!



Weighted A*'s Bounded Suboptimality

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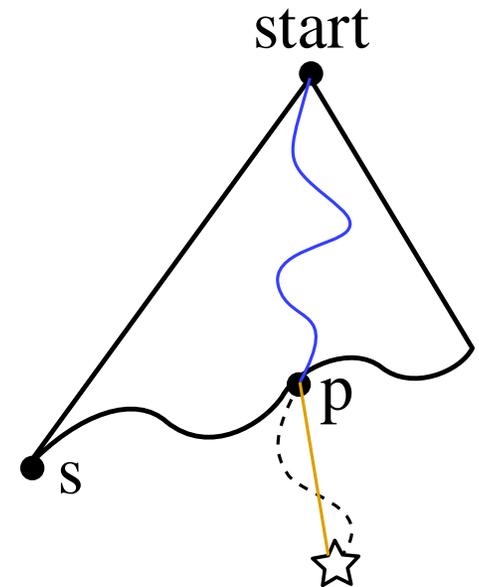
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wA*'s bounded suboptimality:

$$f'(s) \leq f'(p)$$

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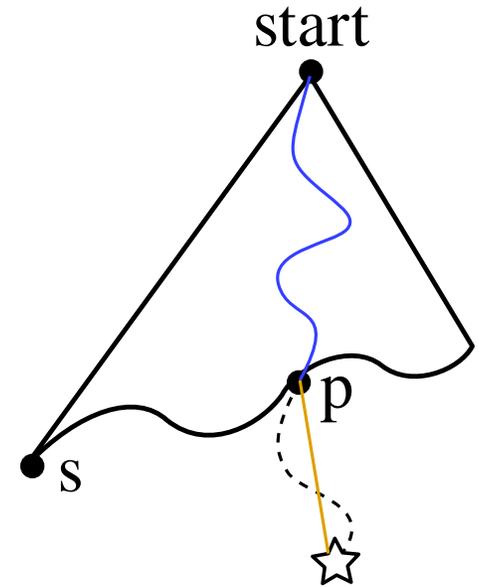
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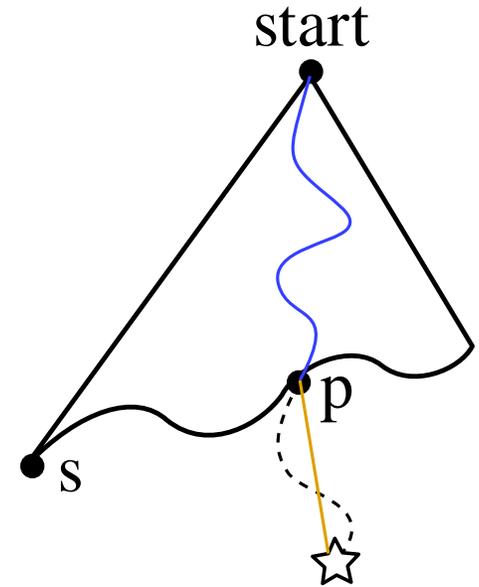
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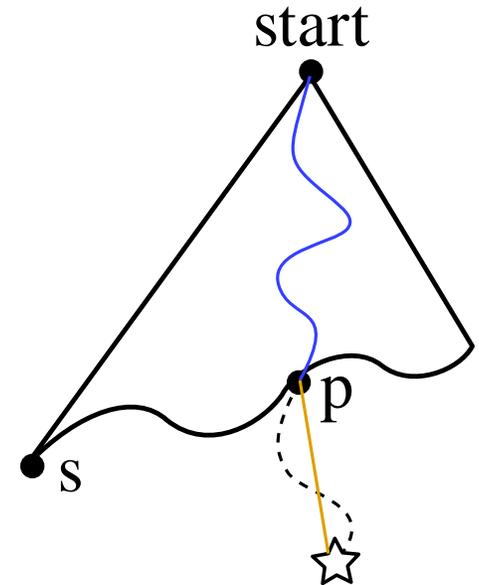
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again: can expand **any node** with $f(n) \leq b \cdot f_{min}$!

Bounded-suboptimal Search: RR- d

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quickly find a solution within factor b of optimal

two tasks: find $\text{sol} \leq b \cdot f_{\min}$, raise f_{\min}

RR- d (Fickert, Gu, and Ruml, AAI-22):

multi-queue alternation (Röger and Helmert, ICAPS-10)

open: sorted on \hat{f} , explore

focal: sorted on \hat{d} , exploit

cleanup: sorted on f , raise bound

filter for open and focal: $f(n) \leq b \cdot f_{\min}$

obvious ablations/substitutions are worse

IPC Coverage ($b = 1.5$)

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Coverage	WA*	EES	DPS	DXES	RR-DPS	RR-d
Sum (1652)	995	967	1012	894	982	1025
Normalized(%)	58.7	57.0	60.0	51.5	57.9	60.7
Expansions	569	558	472	734	665	383

Search Domains

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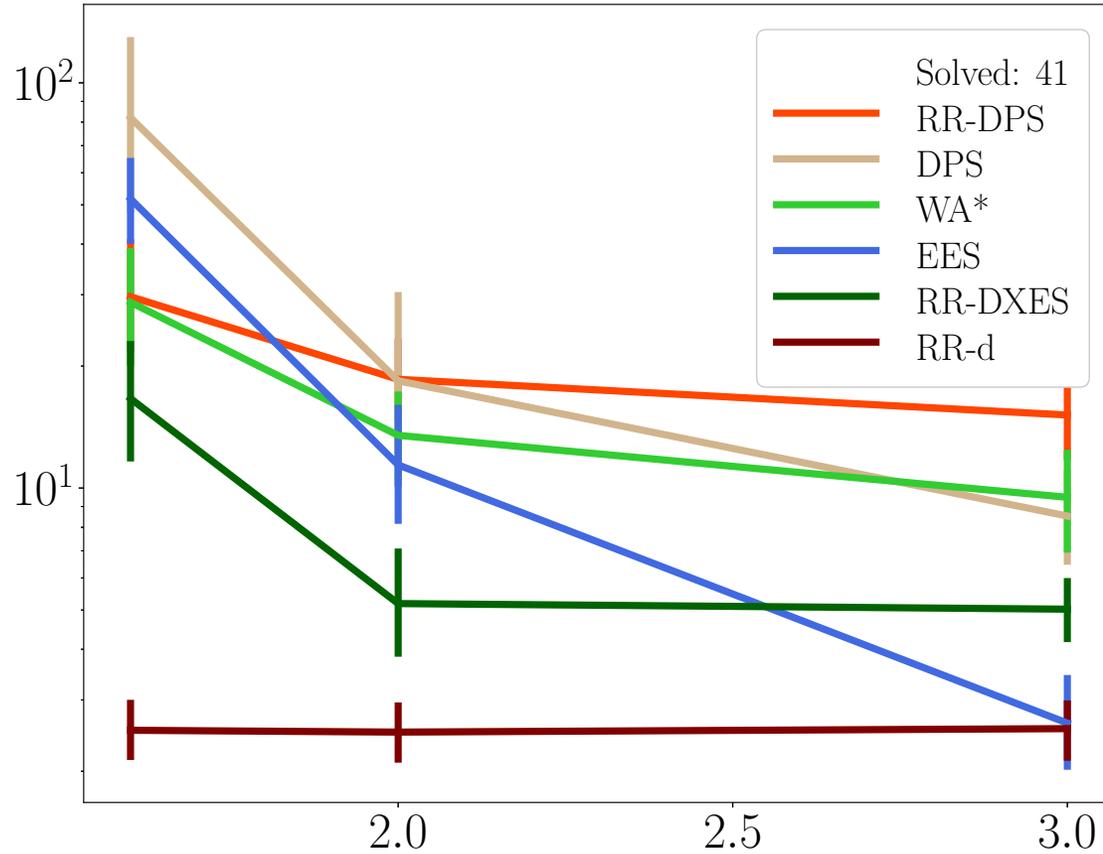
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Uniform Vacuum World



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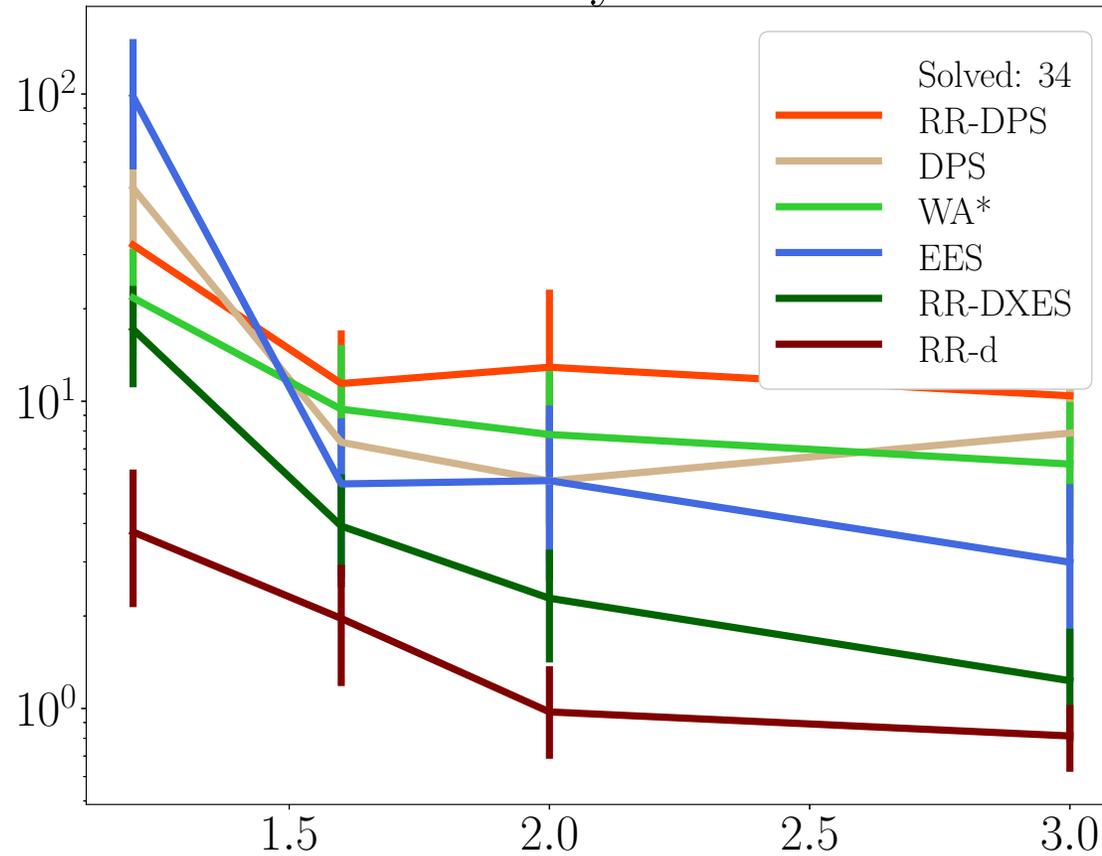
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Heavy Tile



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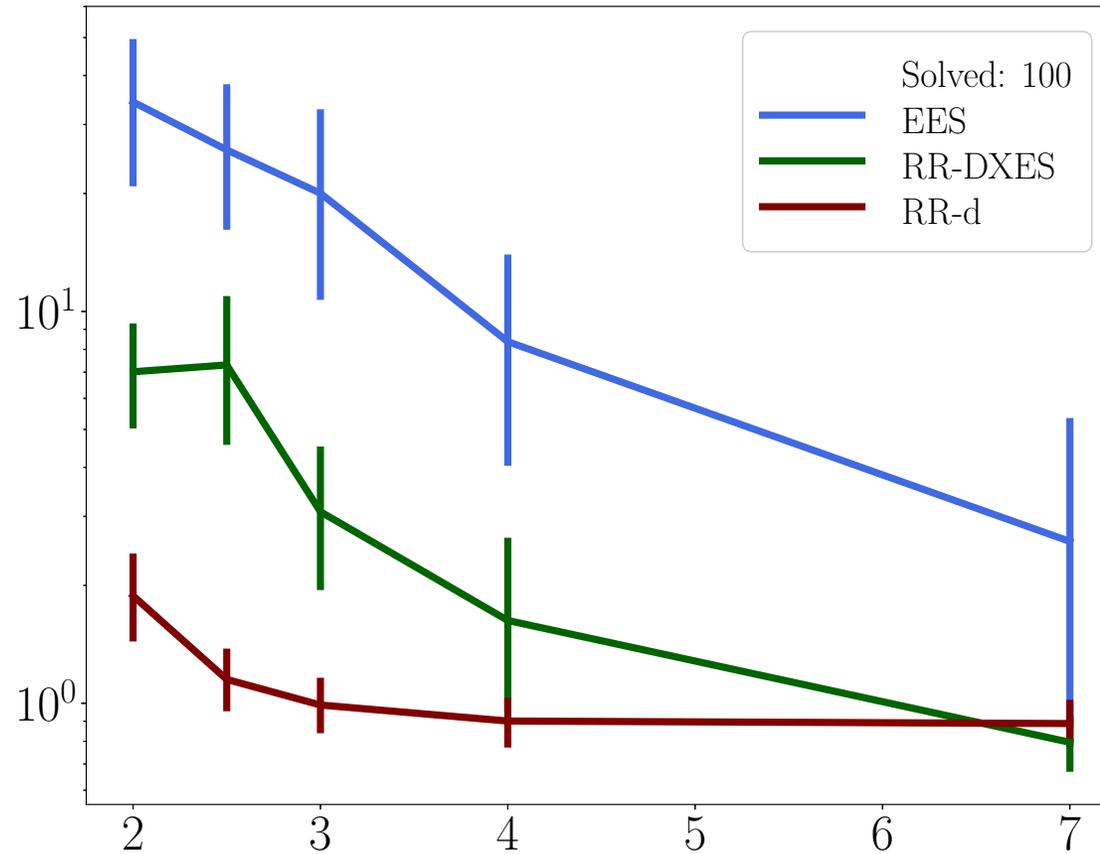
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Inverse Tile



many duplicates: Φ_{pwXD} (Chen and Sturtevant, AAAI-21)

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Expected Effort Search (XES, Fickert et al, IJCAI-21)

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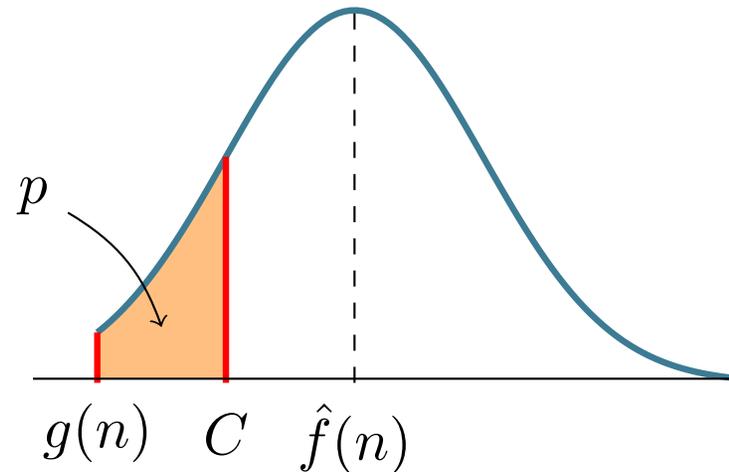
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minimize time subject to absolute cost bound C

best-first on expected effort: $T(n)/p(n)$
(Dobson and Haslum, HSDIP-17)

$T(n)$: expected effort to find goal under n , eg $d(n)$

$p(n)$: probability of cost $< C$ under n
approximate from $g(n) + \hat{h}(n)$ belief distribution



XES on IPC'18 Bounded-Cost Track

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- XES**
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Coverage	GBFS	PTS	BEES	BEES95	XES
Agricola (20)	1	0	0	0	0
Caldera (20)	8	10	10	12	13
Caldera-split (20)	4	2	2	2	2
DataNetwork (20)	2	0	3	3	4
Nurikabe (20)	4	10	10	11	9
Settlers (20)	4	5	10	11	11
Snake (20)	4	5	4	4	5
Spider (20)	7	11	10	10	9
Termes (20)	11	9	11	11	13
Sum (180)	45	52	60	64	66
Expansions (*10 ³)	1.93	3.93	2.10	2.25	1.77

bounded-cost algs dominate GBFS, XES best overall

Why Suboptimal?

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■ Ingredients

■ DAS

■ DAS Results

■ Summary So Far

■ Problem Settings

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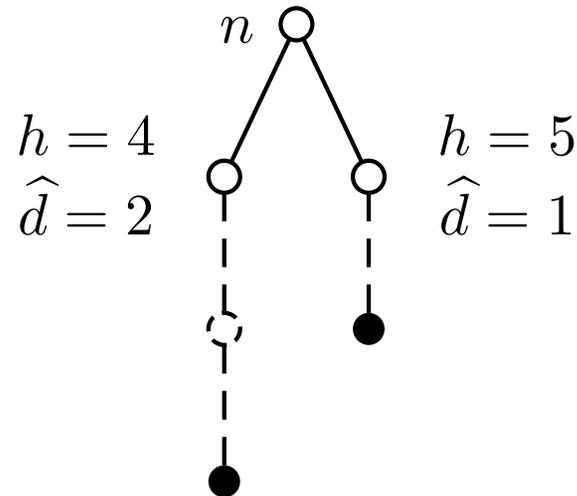
Conclusion

Contract Search

Ingredients for Contract Search

find cheapest solution within deadline

note: anytime algorithms (should) optimize for unknown deadline



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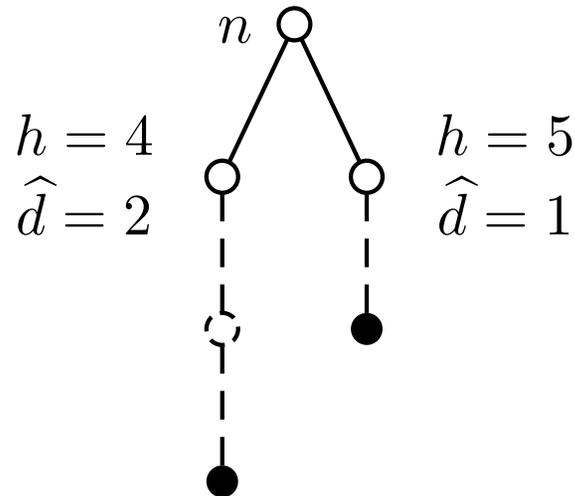
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find cheapest solution within deadline

note: anytime algorithms (should) optimize for unknown deadline



- $\hat{d}(n)$ distance-to-go
- expansion delay: number of expansions when a node is generated and expanded
- $\hat{d}(n) \cdot \text{delay}$ estimates expansions to goal
- time per expansion

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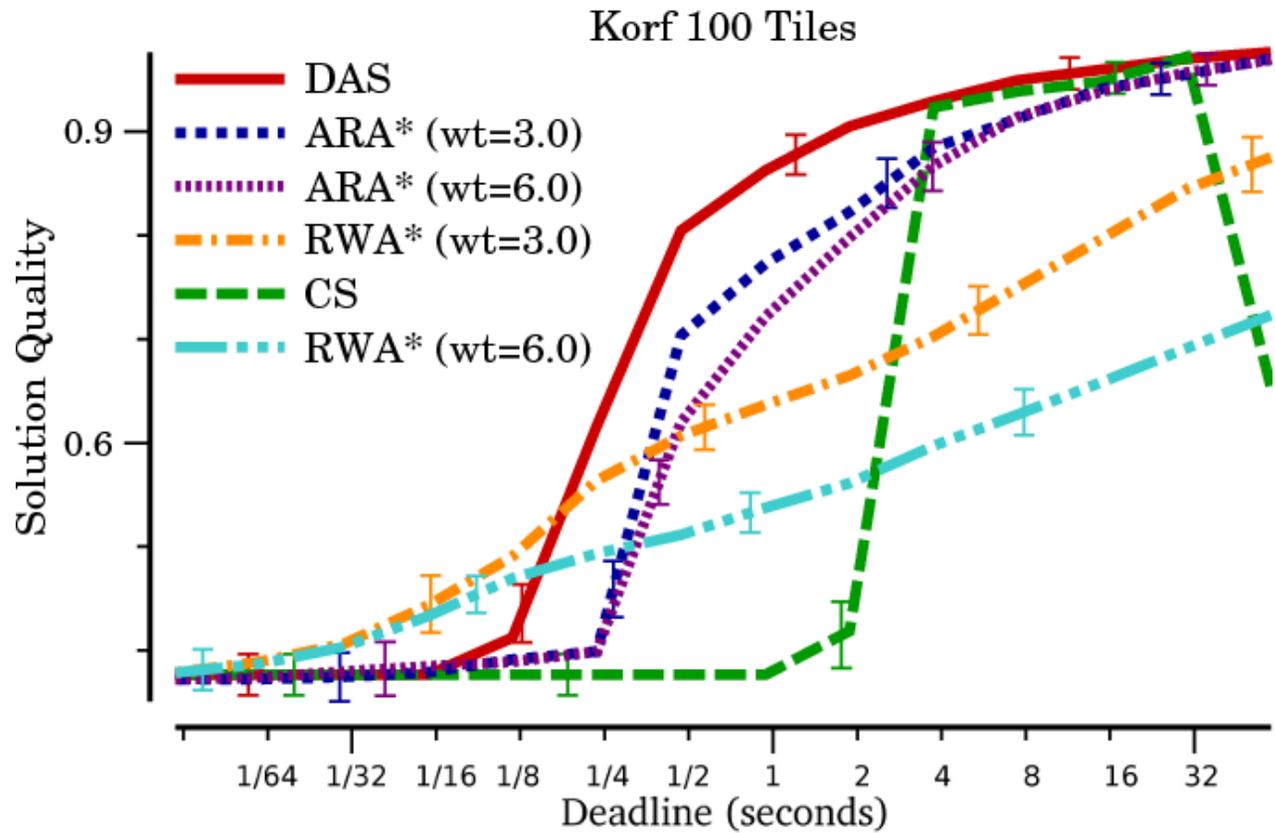
Deadline Aware Search

1. while ($time$) < ($deadline$) and $open$ is non-empty
2. $d_{max} \leftarrow$ calculate d bound
3. $s \leftarrow$ pop lowest f state from $open$
4. if s is a goal and is better than $incumbent$
5. $incumbent \leftarrow s$
6. else if $\hat{d}(s) < d_{max}$, expand s
7. else prune s
8. if $open$ empties, recover some pruned states
9. return $incumbent$

ripe for improvement!

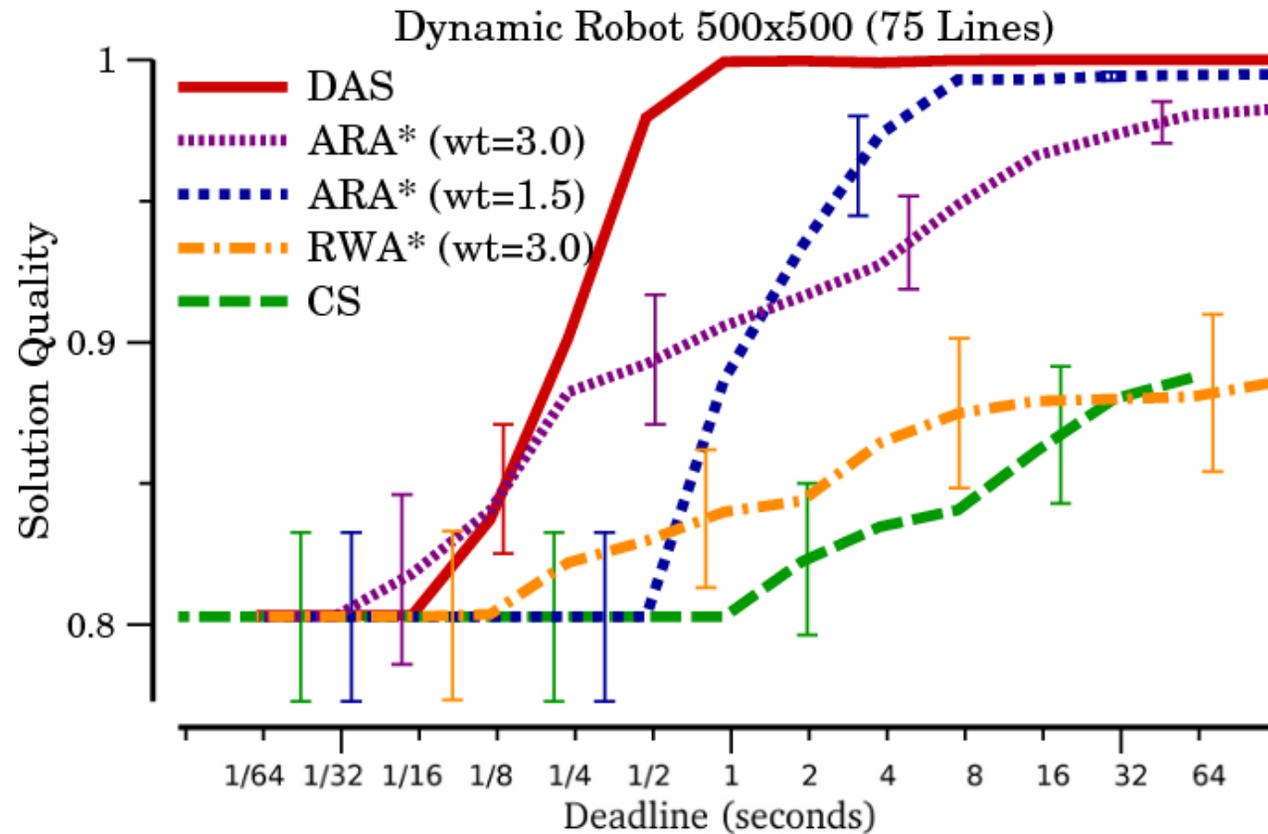
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- Optimal search is impractical
- Lots of room for creativity in suboptimal search

Going beyond lower bounds on cost-to-go:

- Inadmissible cost-to-go \hat{f}
- Inadmissible distance-to-go \hat{d} : Speedy
- RR- d uses f , \hat{f} , and \hat{d}
- DAS uses expansion delay

Exploit estimates of uncertainty in beliefs

break?

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optimal: minimize solution cost

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optimal: minimize solution cost

greedy: minimize solving time

bounded suboptimal: minimize time subject to relative cost bound (factor of optimal)

bounded cost: minimize time subject to absolute cost bound

contract: minimize cost subject to absolute time bound

tunable: trade cost for time, no guarantees

anytime: incrementally converge to optimal

situated: minimize goal achievement time

utility: maximize function of cost and time

real-time: return next action within absolute time bound

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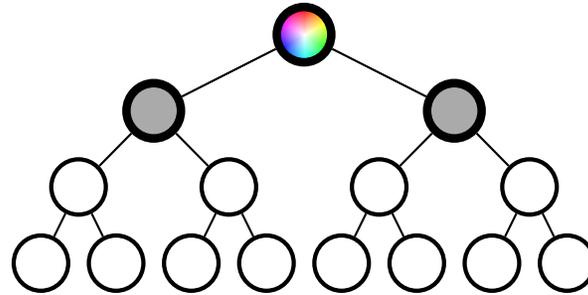
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restricted **breadth-first** search (not best-first!)



Beam search with width 2.

optimal with width = ∞ , usually faster with smaller k
but no guarantees!

compare comparable nodes, enforce diversity
not forced to fill minima

beam uses d

(see also Steve Wissow's SoCS talk)

Beam Search

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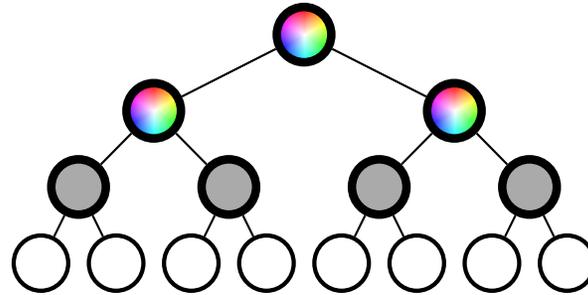
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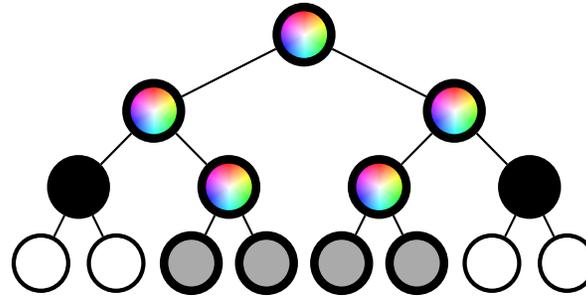
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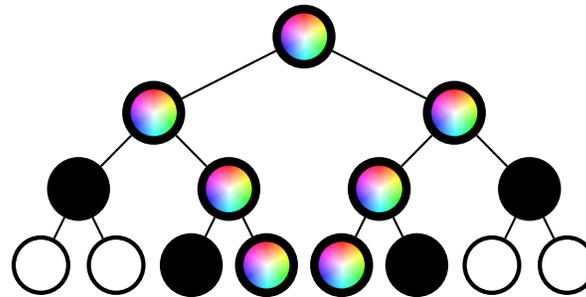
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Bead on Tiles

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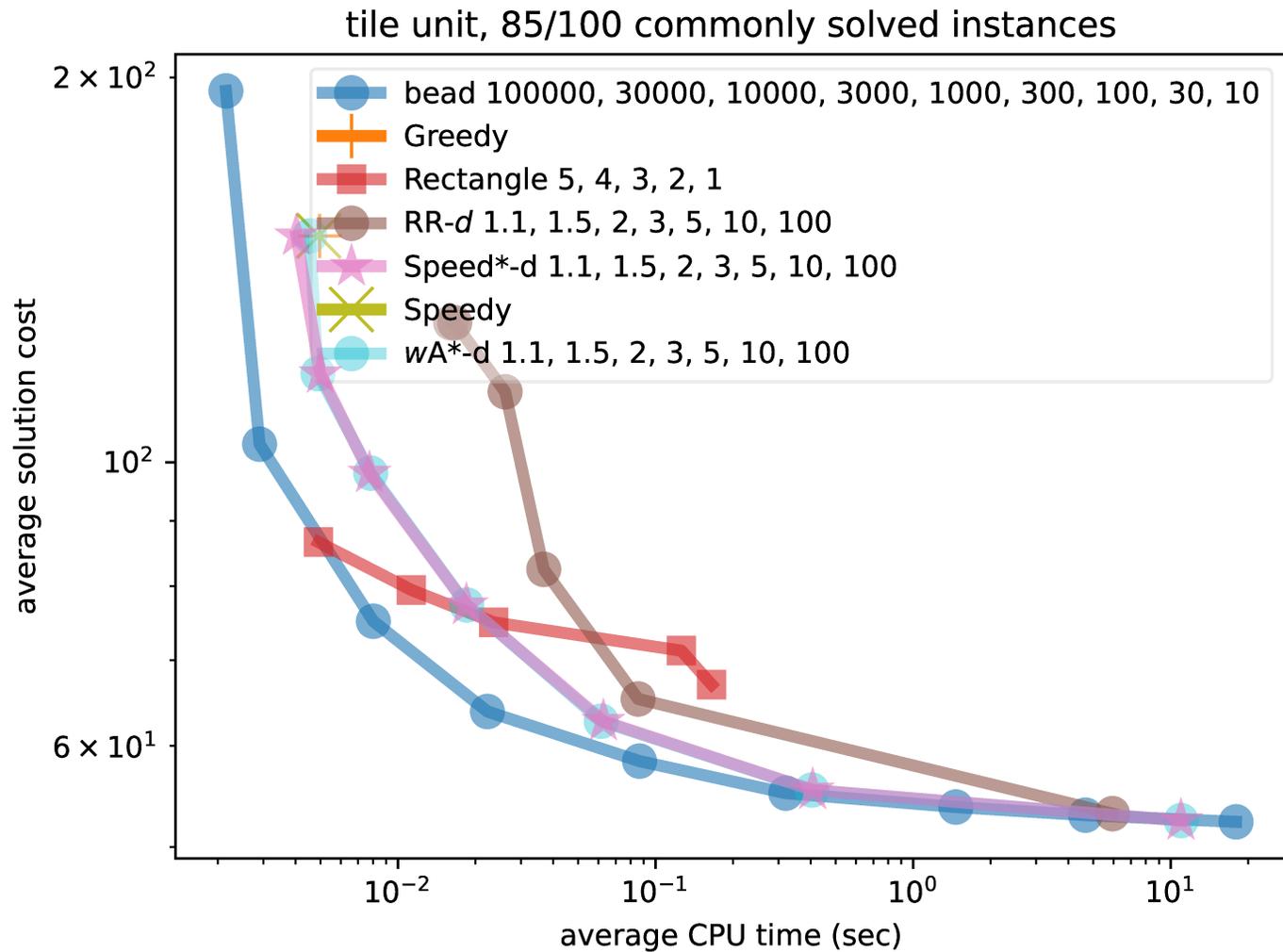
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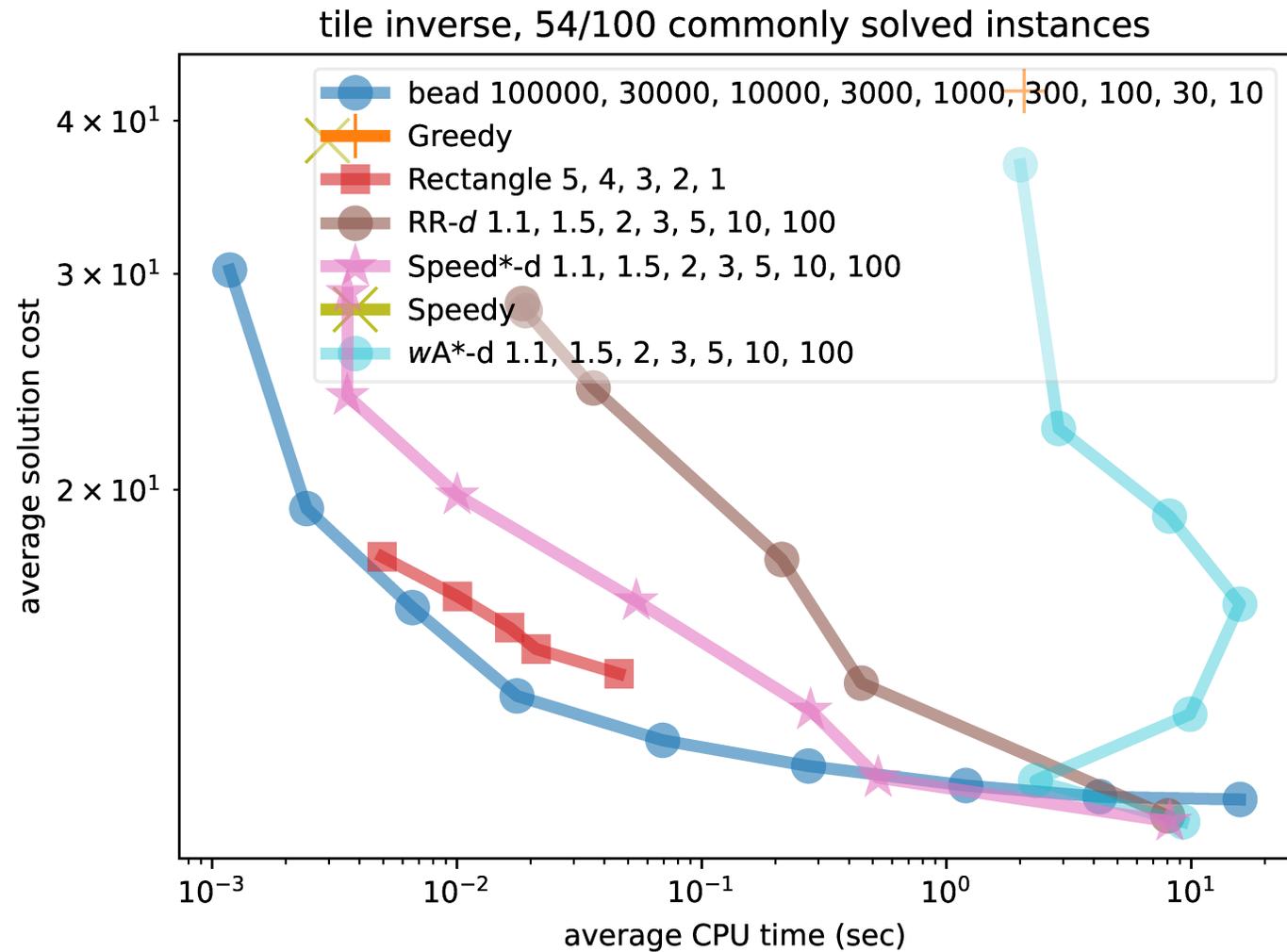
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iteratively widening and deepening beam

great for large minima!

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Situated Planning

Achieve goal as quickly as possible!

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- Optimal search: slow planning, fast execution



- Greedy search: fast planning, slow execution



- To achieve goal quickly: minimize the sum!



Situated Planning

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minimize goal achievement time

special case of optimizing a utility function of planning time and plan makespan

with concurrent planning and execution: very realistic

see Coles et al, “Planning and Acting While the Clocks Ticks” at ICAPS!

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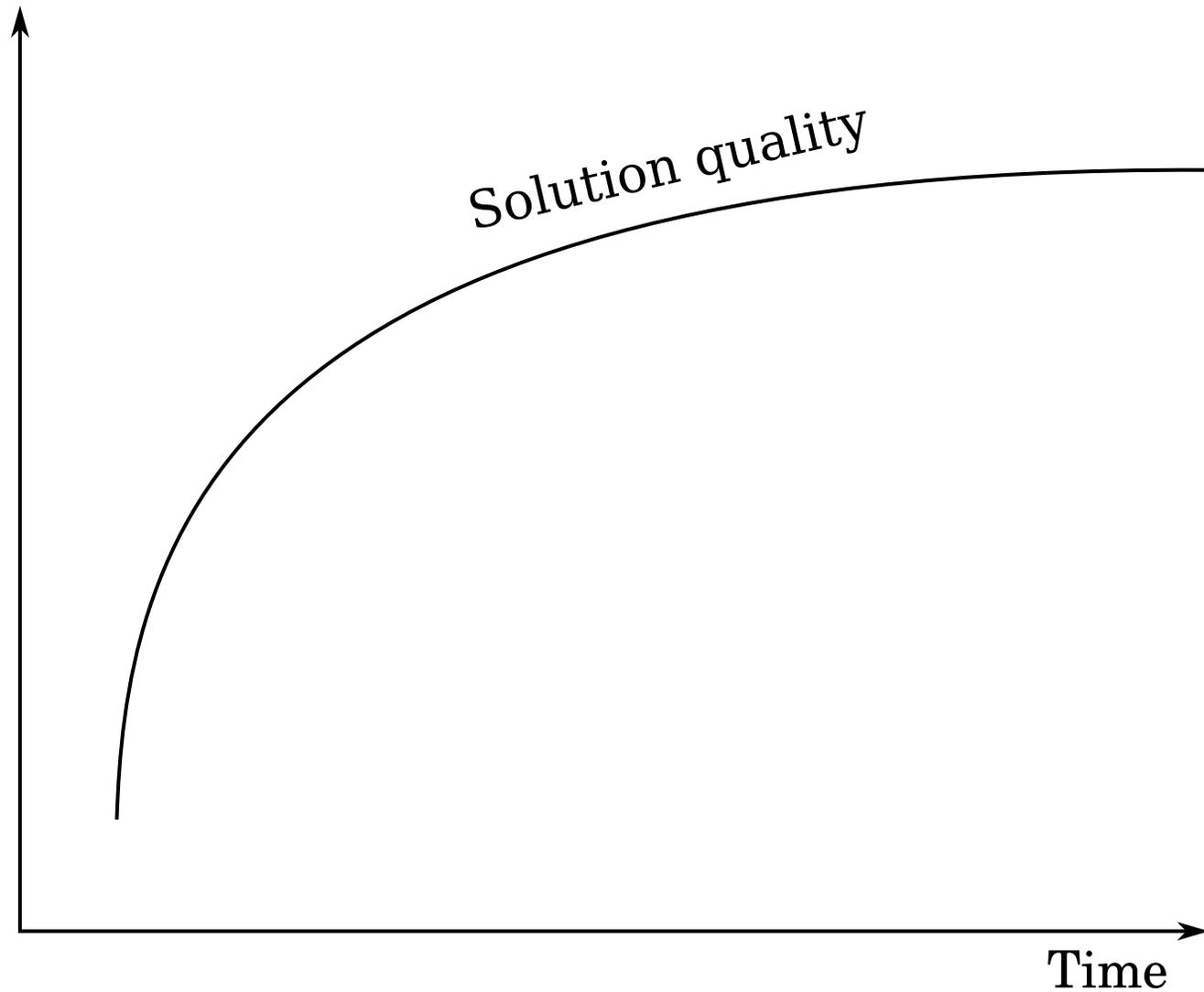
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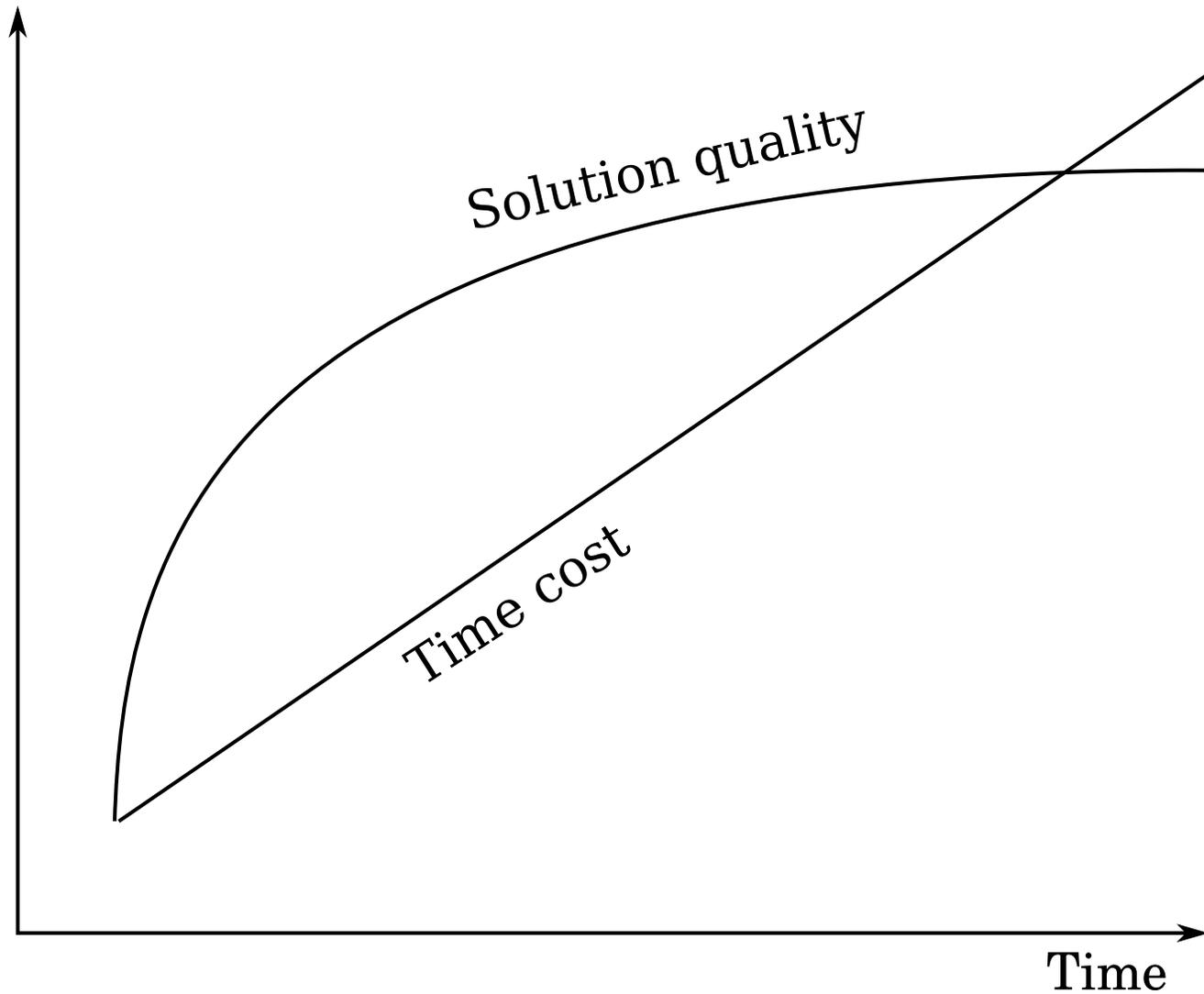
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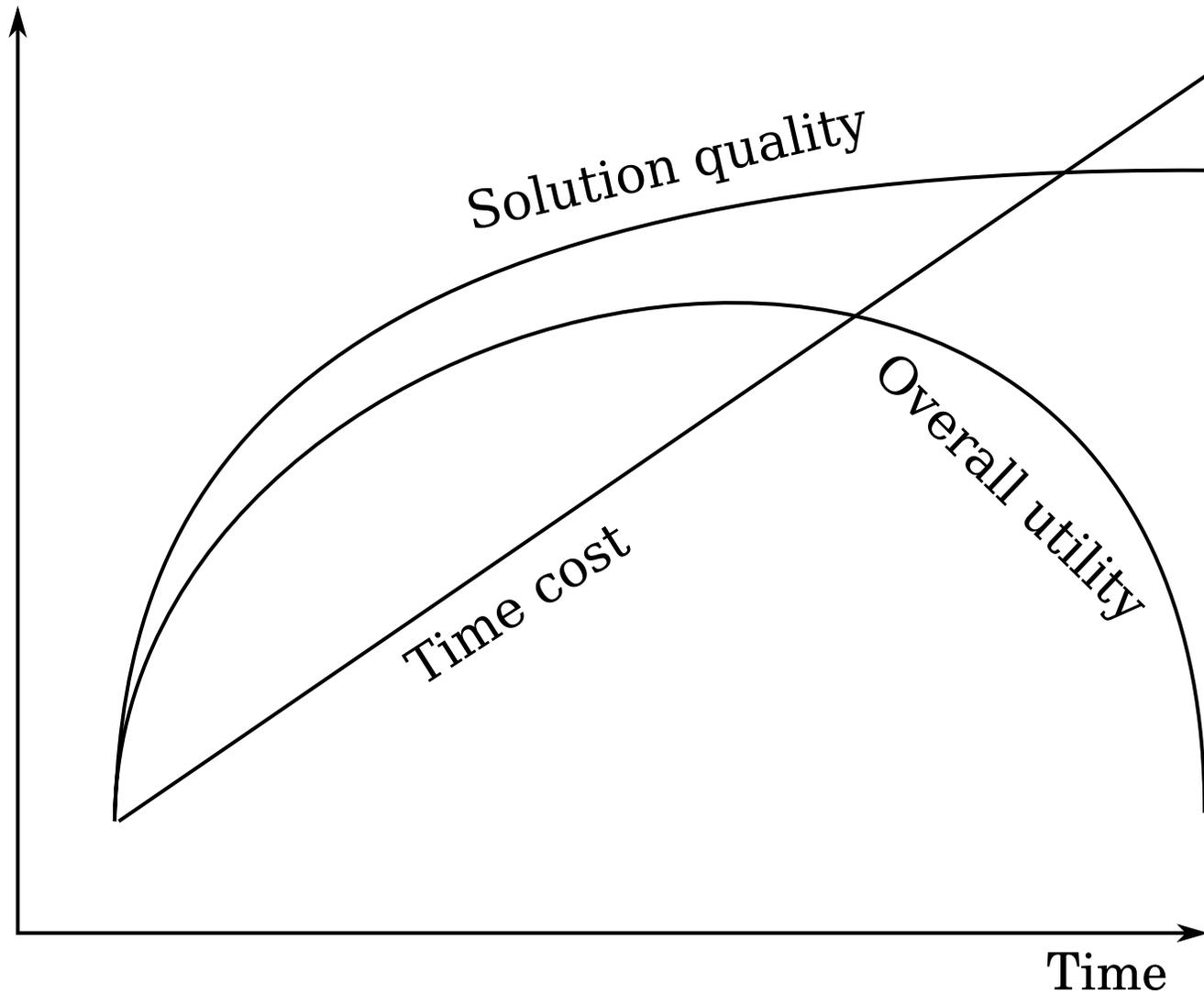
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Utility vs Anytime

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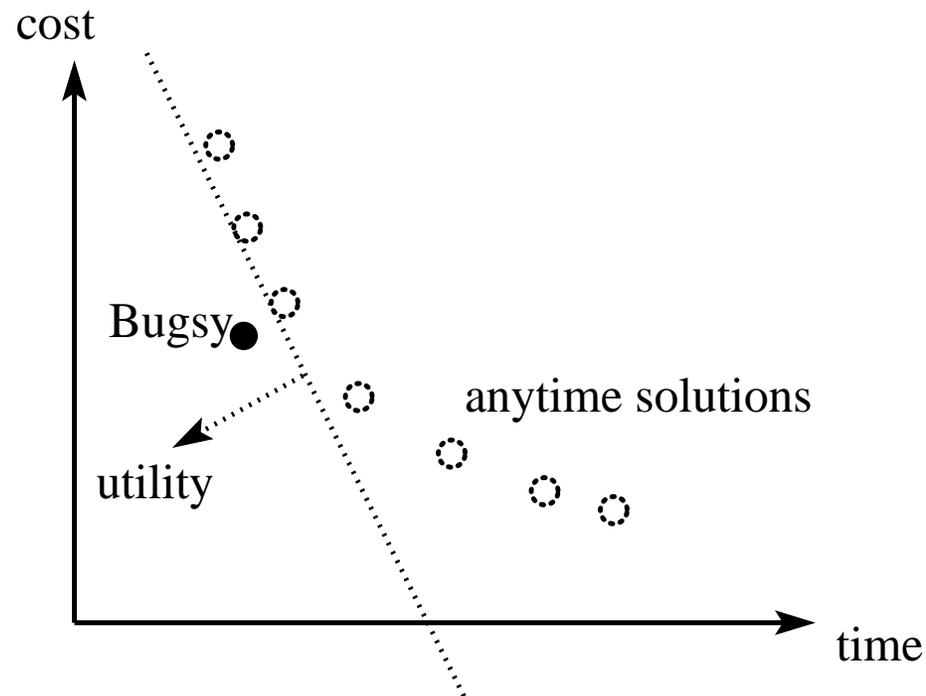
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Different from anytime algorithms

- no need for termination policy, training data
- can spend all effort pursuing one solution
- no fixed trade-off



Utility vs Anytime

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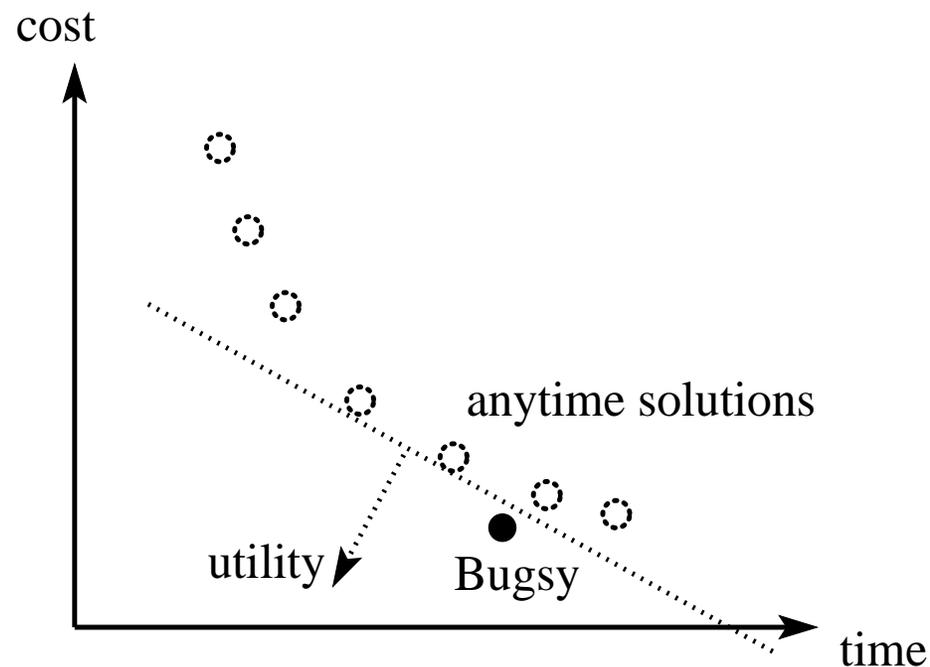
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Different from anytime algorithms

- no need for termination policy, training data
- can spend all effort pursuing one solution
- no fixed trade-off



Best-first Utility-guided Search, Yes!

Act to obtain the highest utility outcome.

Possible actions:

terminate: U_{def} = utility of immediately returning null solution

expand(n): estimate utility of outcome enabled by expansion:

$$\hat{u}(n) = -(w_f \cdot \widehat{cost}(n) + w_t \cdot \widehat{time}(n))$$

Expand node with highest $\hat{u}(n)$, unless $< U_{def}$.

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$$\hat{u}(n) = -(w_f \cdot \widehat{cost}(n) + w_t \cdot \widehat{time}(n))$$

$$\widehat{cost}(n) = f(n)$$

Estimate the amount of vacillation and account for it:

$\hat{d}(n)$ number of search steps

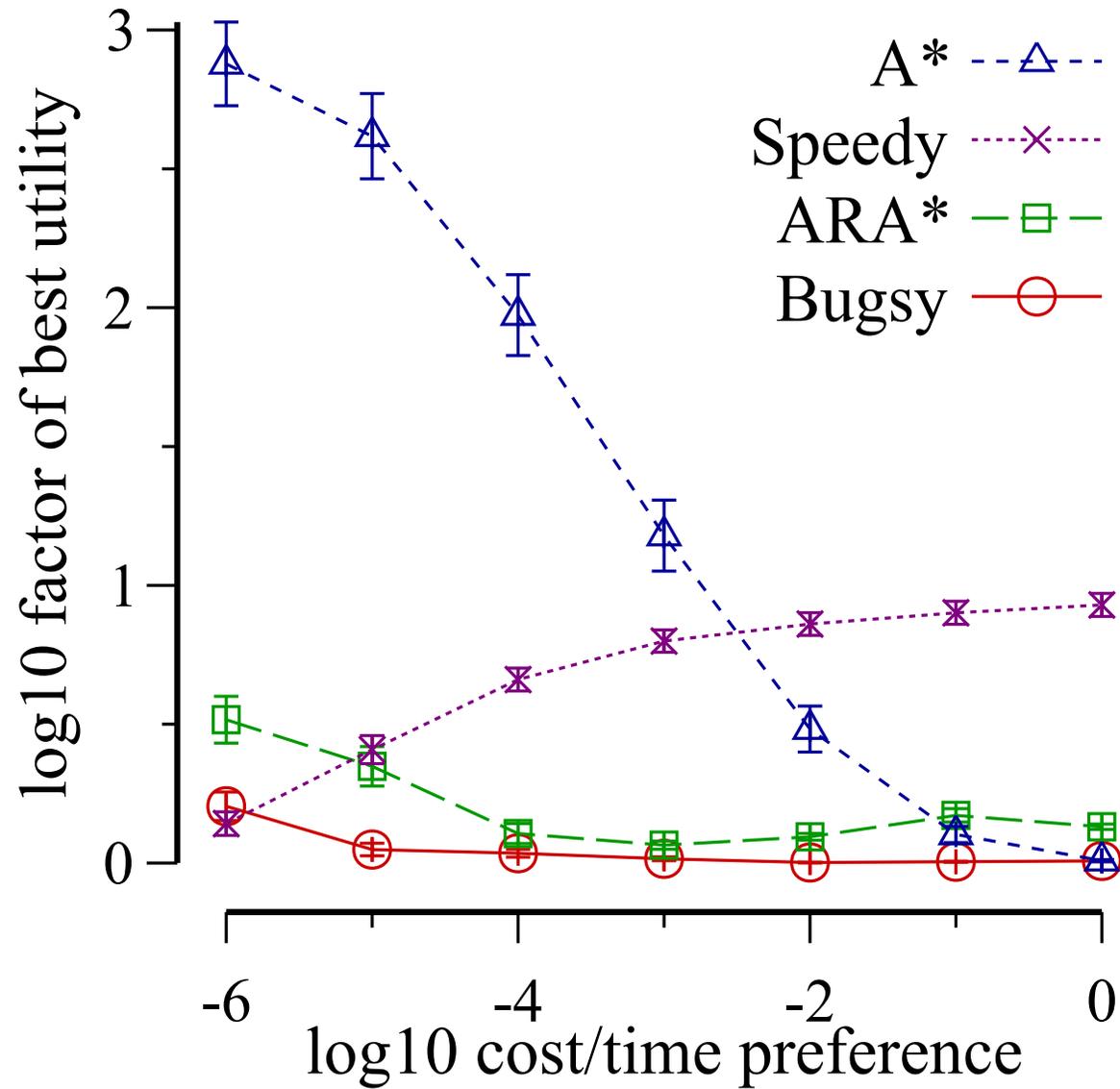
$delay$ number of expansions between steps

t_{expand} time per expansion

$$\widehat{time}(n) = \hat{d}(n) \cdot delay \cdot t_{expand}$$

Bugsy on the 15-puzzle

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Multiple Sequence Alignment

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$U()$	Bugsy	ARA*	AA*	Gr	A*
time only	100	100	100	100	54
0.1 sec	99	97	98	96	54
0.5 sec	92	83	88	76	52
1 sec	80	68	79	54	51
5 sec	75	68	71	25	73
10 secs	78	75	74	25	78
cost only	82	82	82	24	82

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Real-time Heuristic Search

return next action within prespecified time bound



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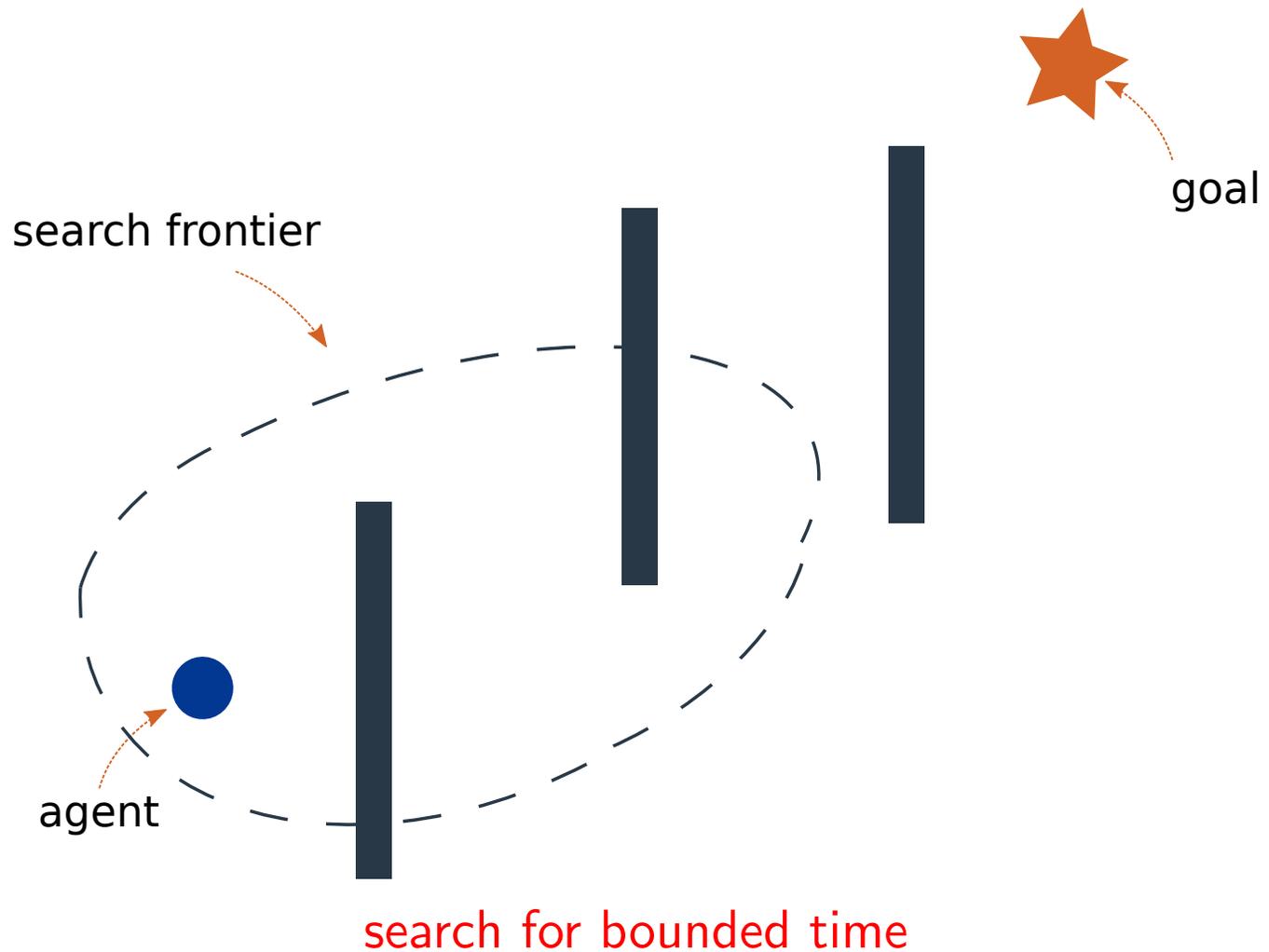
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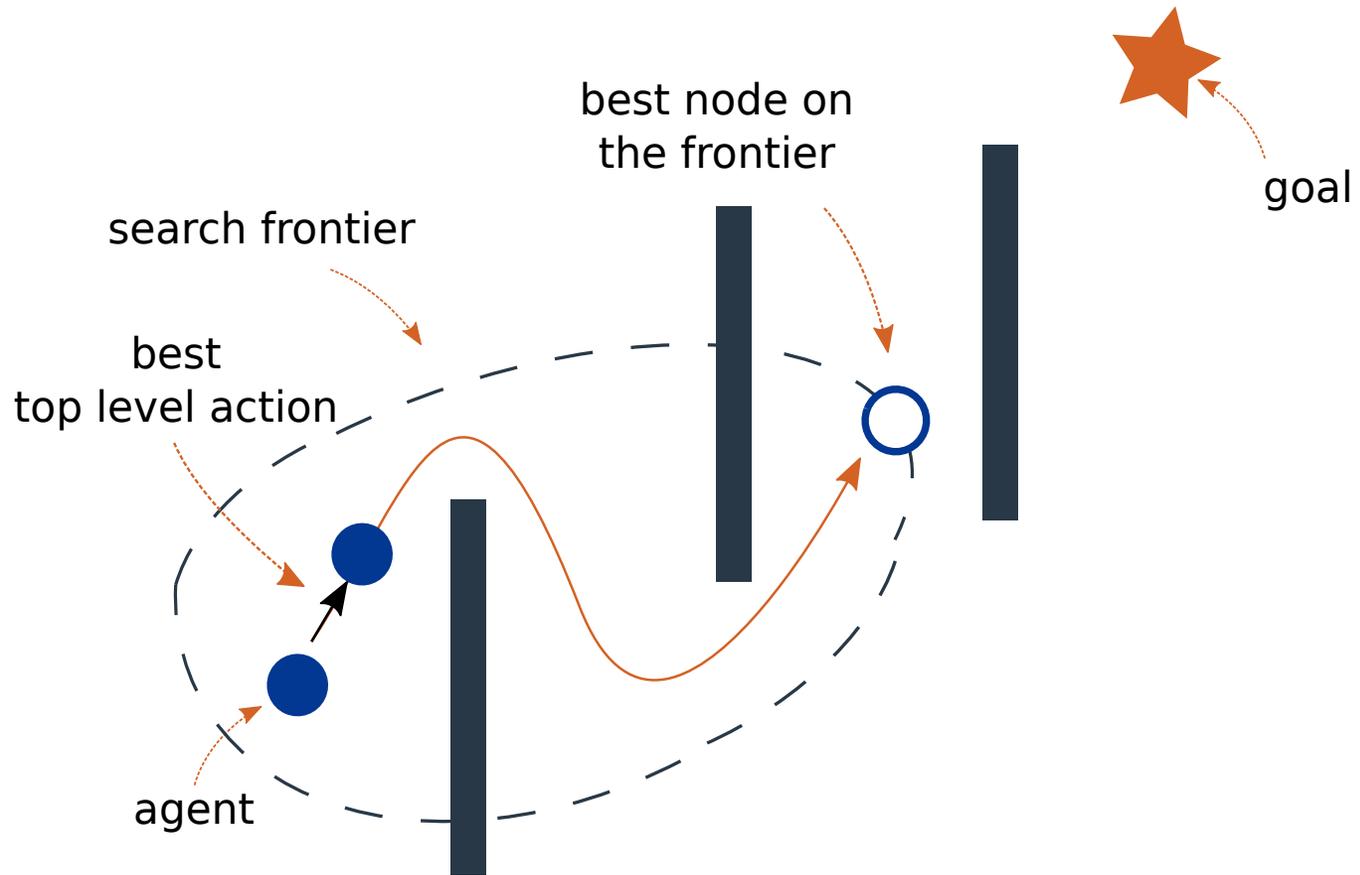
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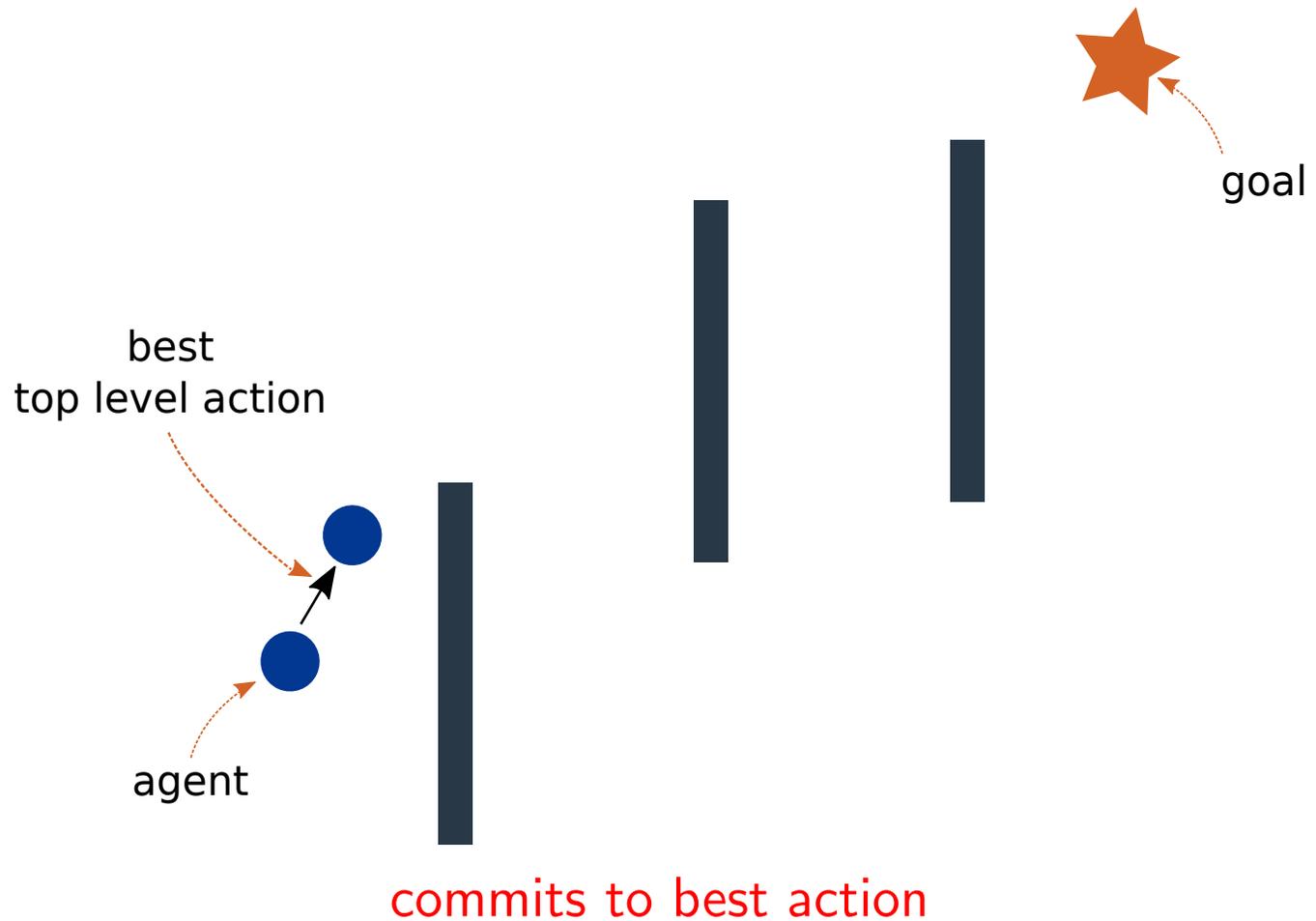
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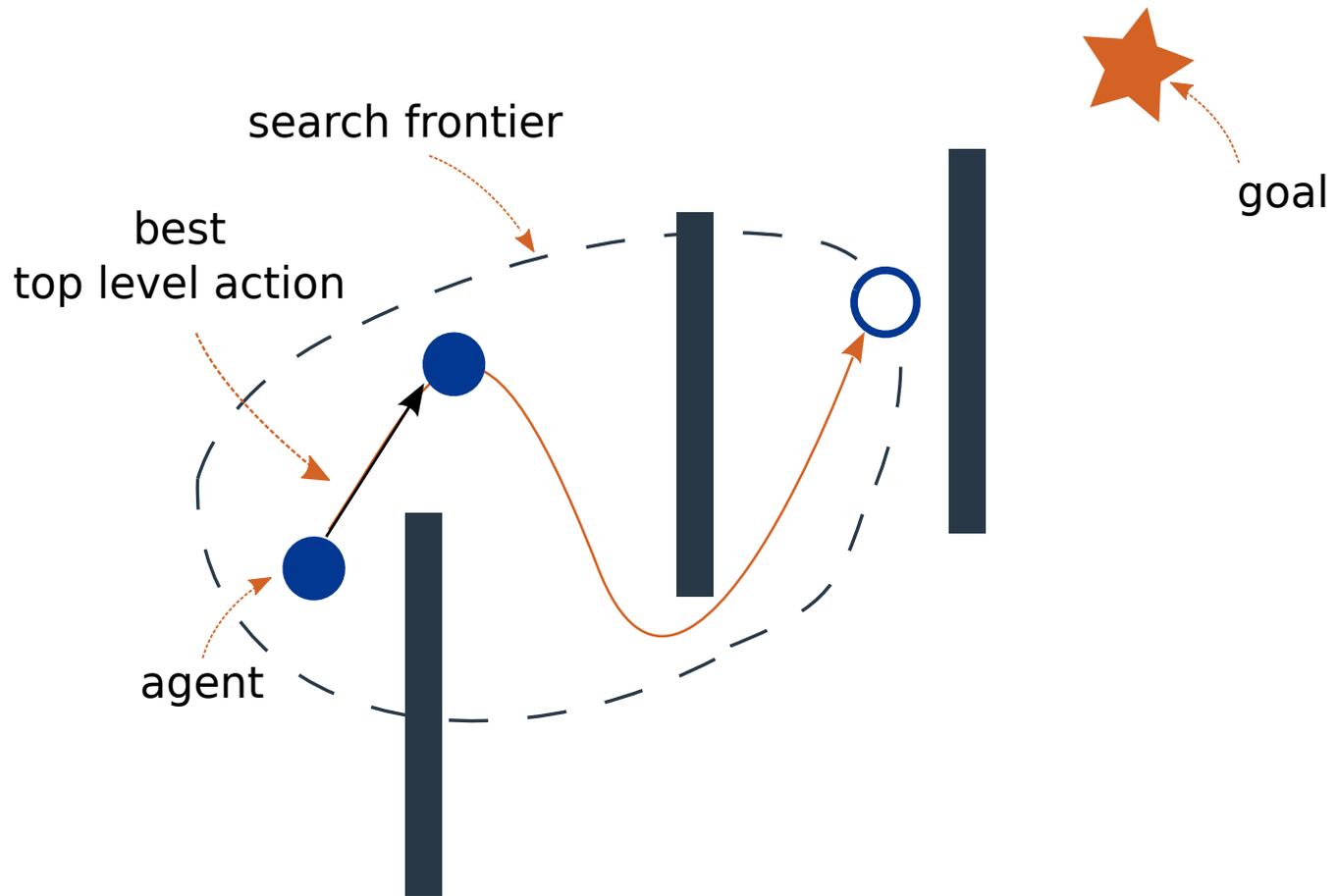
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concurrent search and execution, online planning,
'receding horizon control'

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Classic Real-time Search: LSS-LRTA* (Koenig&Sun 2008)

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three phases:

1. Lookahead:
expand minimum f node
2. Decision-making:
backup minimum f from frontier ('minimin')
select top-level action with minimum f
3. Learning:
update heuristic values
(avoid loops, escape local minima, ensure completeness)

repeat until goal achieved

Taking Real-time Search Seriously

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three phases:

1. Lookahead:

Which nodes to expand?

minimum f optimal for A^* (offline optimal)

what about online?

2. Decision-making:

Which action to pick?

lowest f optimal for A^* (offline optimal)

what about online?

3. Learning:

How to backup from frontier?

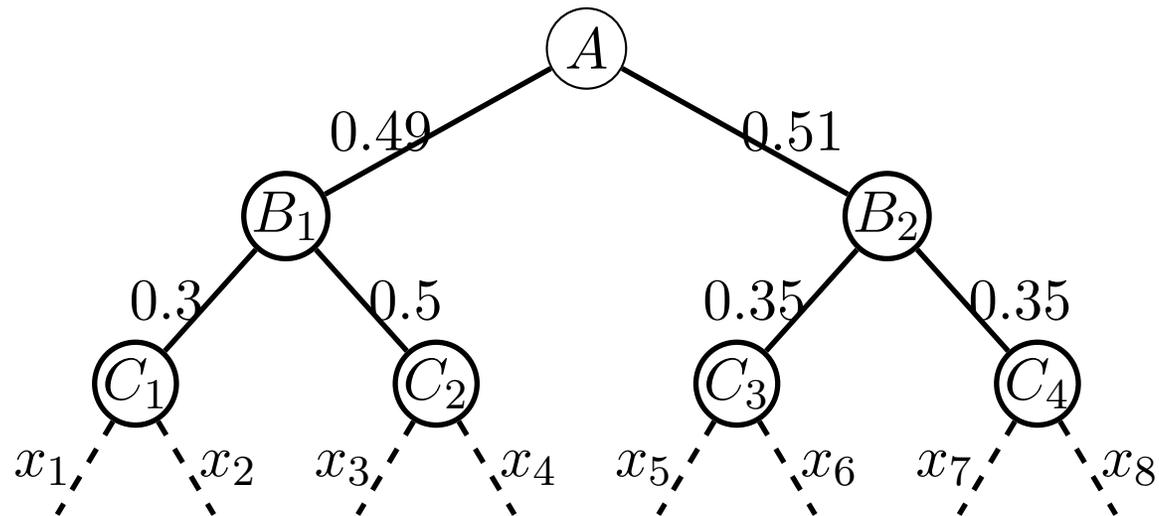
minimin optimal for A^* (offline optimal deterministic)

Bellman optimal for VI (offline optimal stochastic)

what about online?

Decision-making: An Example (Pemberton & Korf 1995)

Should an agent at A move to B_1 or B_2 ?
(x_i are unknown but i.i.d. uniform 0-1)



lower bound on cost-to-go $h = 0$, so $f = g$

some x_i will be revealed at the next step

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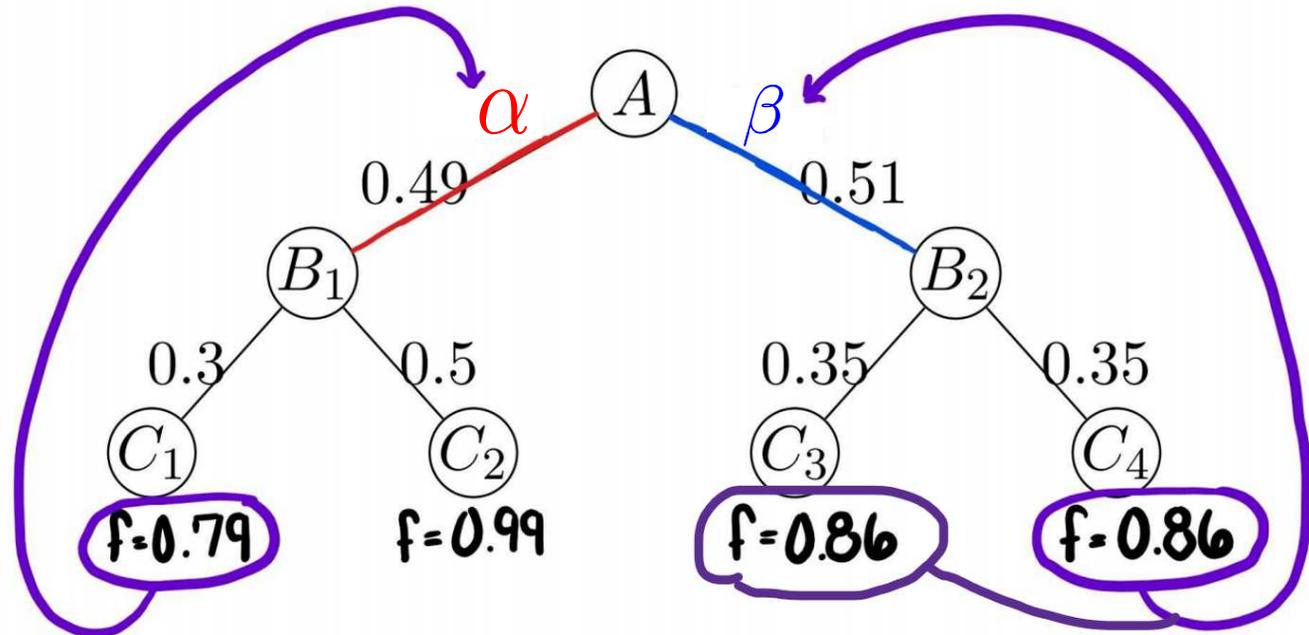
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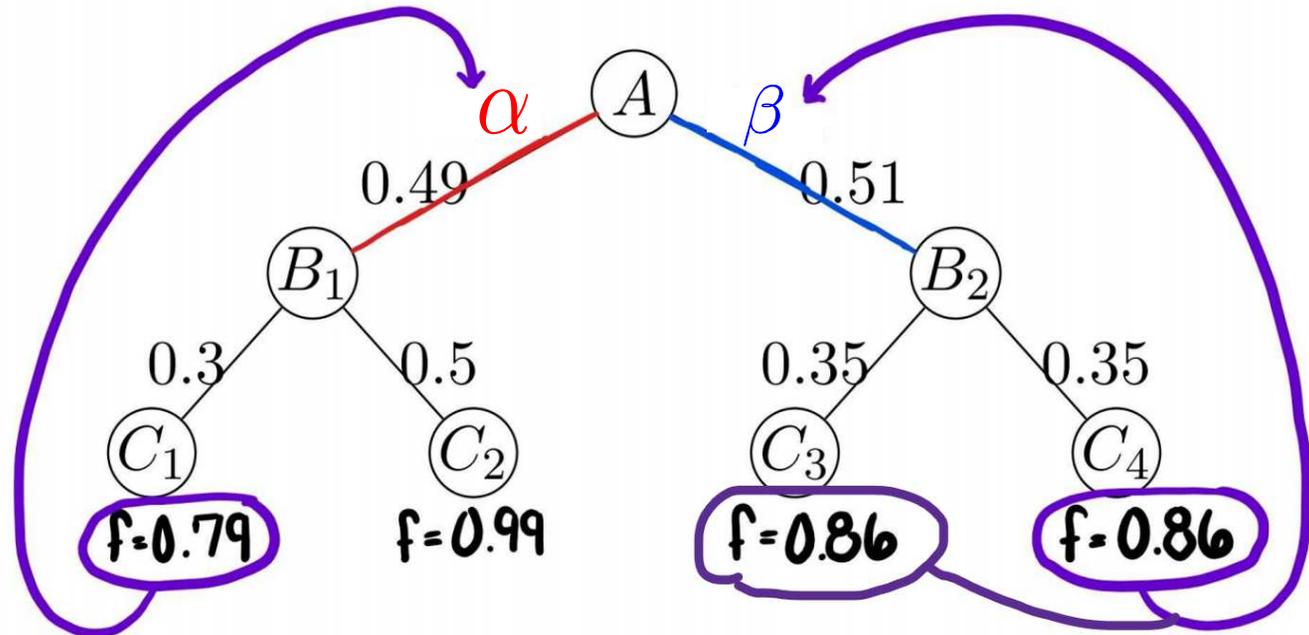
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Should an agent at A move to B_1 or B_2 ?
(x_i are unknown but i.i.d. uniform 0-1)



but decision theory says minimize expected value

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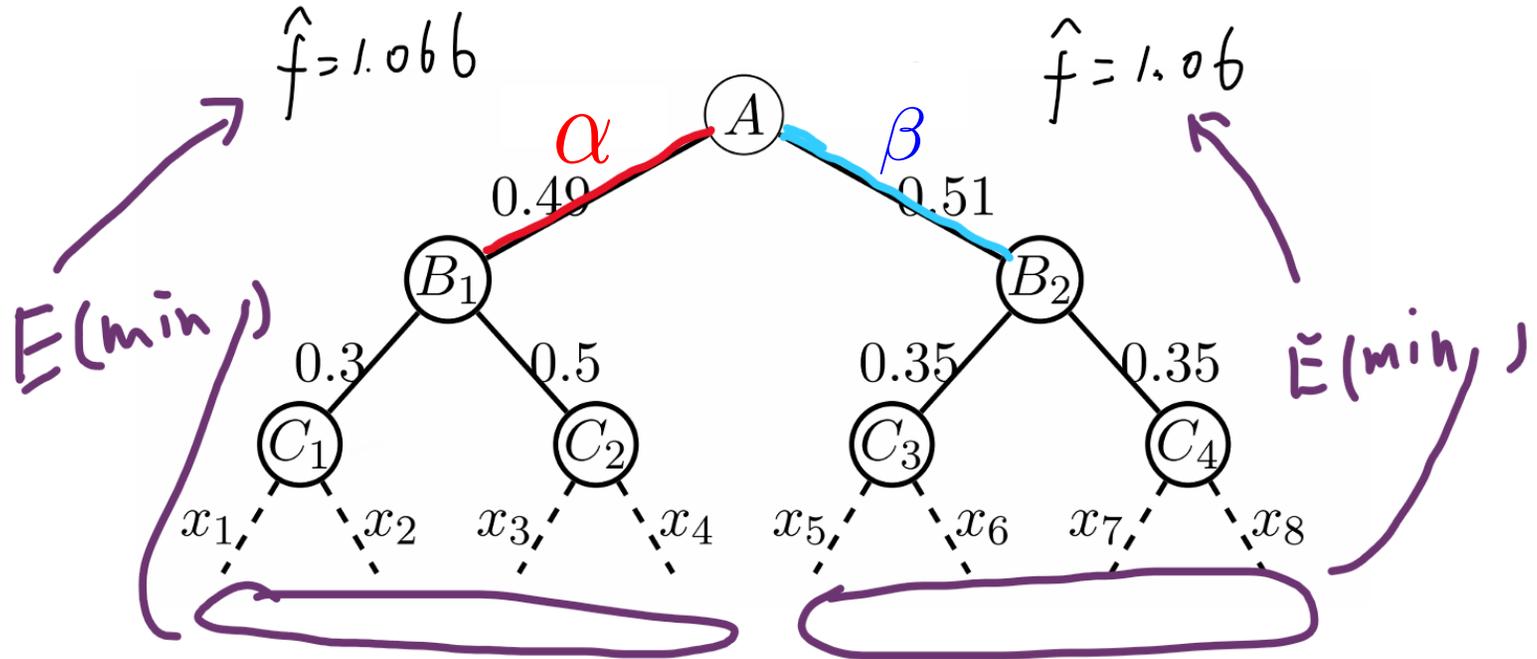
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Should an agent at A move to B_1 or B_2 ?
 (x_i are unknown but i.i.d. uniform 0-1)



\hat{f} is expected total plan cost

four x_i will be revealed at the next step

Why Suboptimal?

Greedy Search

Bounded-suboptimal

Bounded Cost

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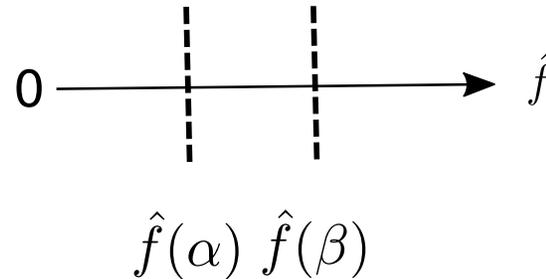
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\hat{f} is expected value

Should an agent expand nodes under α or β ?

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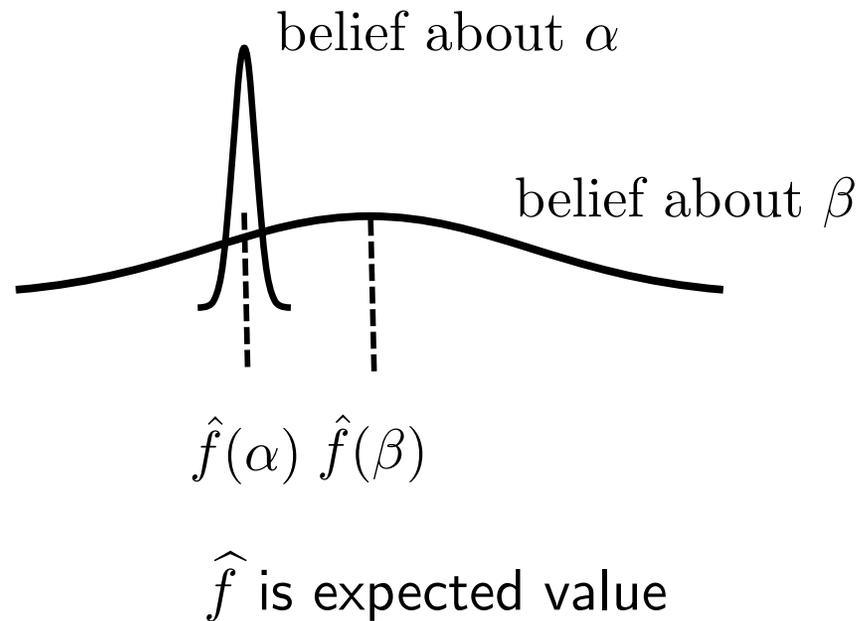
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Should an agent expand nodes under α or β ?

\hat{f} is not the answer: what to do?
want to maximize value of information
need to consider uncertainty of estimates

Nancy: Risk-based Lookahead

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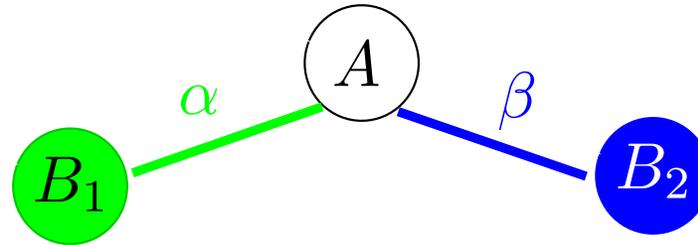
Conclusion

Nancy (Mitchell et al, AAAI-19; Fickert et al, AAAI-20)

- want to maximize value of information
- expand nodes which minimize expected regret
- relies on belief over values
- choose expansions that decrease uncertainty about best

Risk-based Lookahead Example

expand under α or β ?



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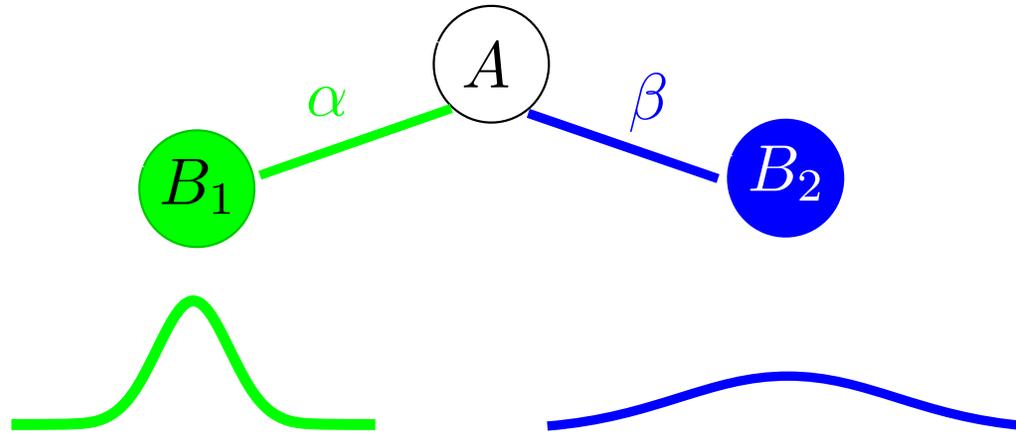
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Risk-based Lookahead Example

expand under α or β ?



need 2 things:

- 1) current beliefs
- 2) estimate of how beliefs might change with search

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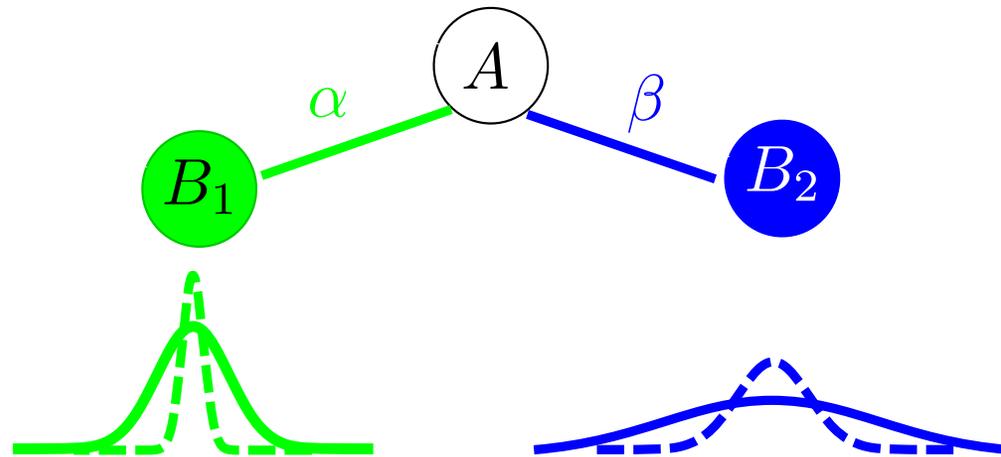
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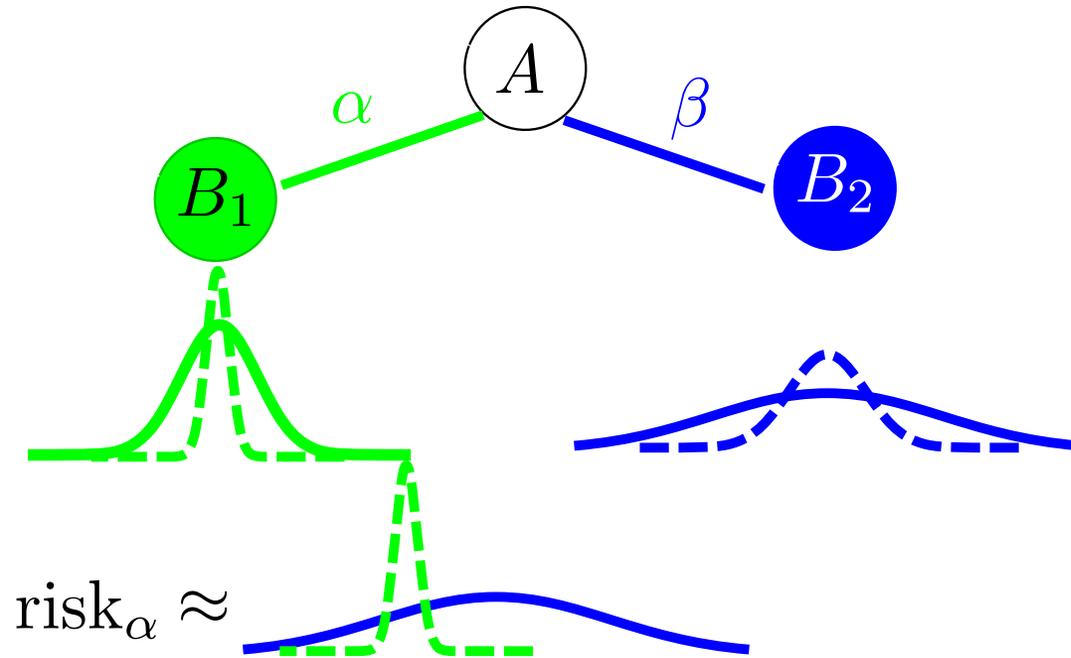
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Risk-based Lookahead Example

expand under α or β ?



Risk: expected regret if a suboptimal action is selected
 α is TLA with lowest expected value, other is β

$$\mathbb{E} \left[\underbrace{f^*(\alpha) - f^*(\beta)}_{\text{our regret}} \mid \underbrace{f^*(\beta) < f^*(\alpha)}_{\text{when } \alpha \text{ not best}} \right]$$

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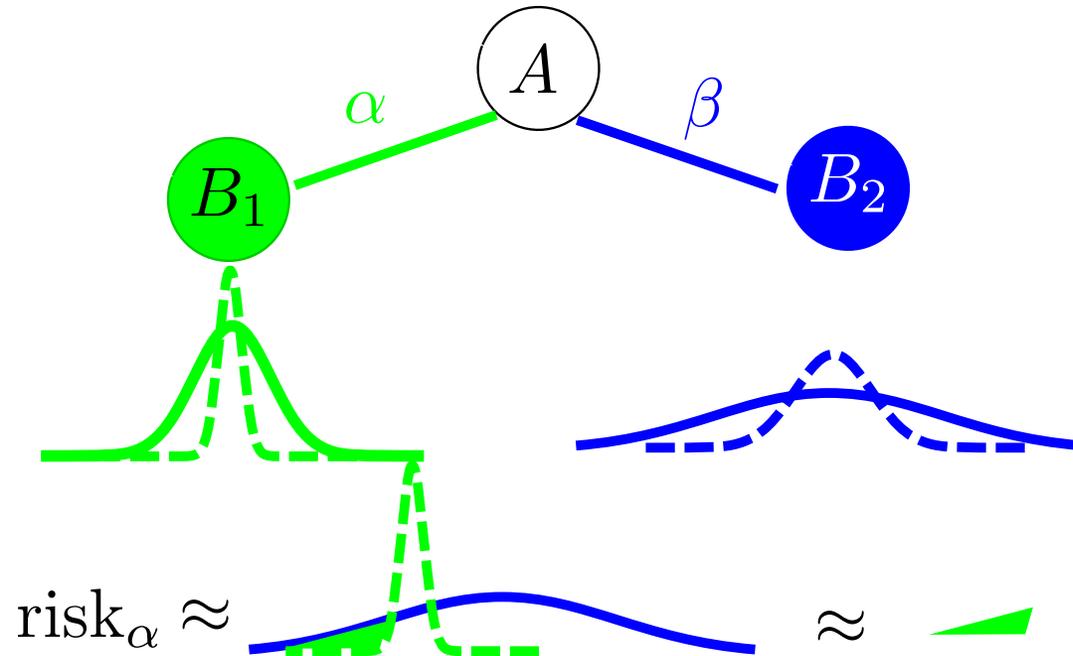
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Risk-based Lookahead Example

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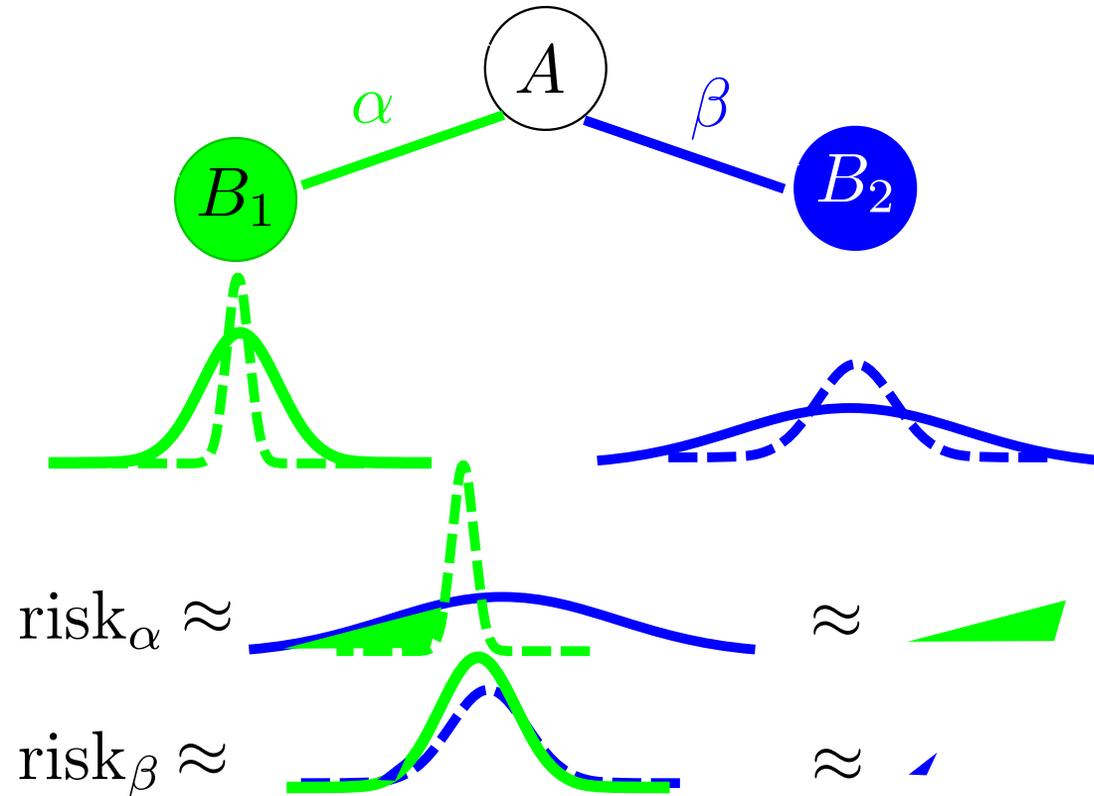
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Risk-based Lookahead Example

expand under α or β ?



expand under the TLA that minimizes risk!
expand under β !

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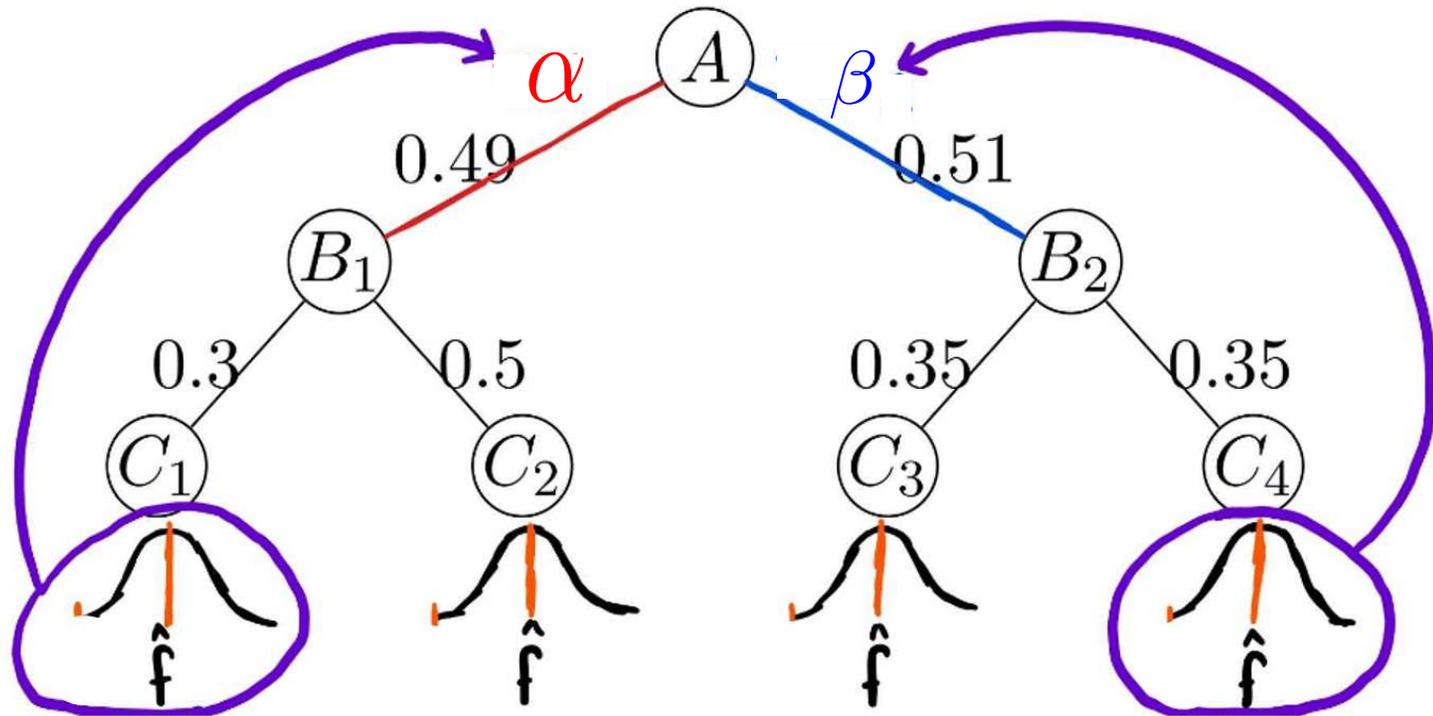
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Nancy Backups

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Nancy:

parent \leftarrow belief with minimum \hat{f} among successors

conveys an entire belief distribution

Open Problem: How to Back-up Frontier Values

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minimin: parent gets best child's f
assumes no more information will become available

Bellman: parent gets expected-best child's \hat{f}
assumes no more information will become available

Nancy: parent gets expected-best child's belief
assumes no more information will become available

Cserna: parent gets expected min over all children's beliefs
assumes we will know optimal choices

something intermediate would seem appropriate!

Nancy's Response to the Central Issues

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1. Lookahead:
Which nodes to expand?
those that minimize risk
2. Decision-making:
Which action to pick?
minimum \hat{f} (rationality)
3. Learning:
How to backup from frontier?
backup beliefs ('Nancy backups')

minimizing uncertainty drives the search

see also XES (bounded-cost search, IJCAI-21)

How to Form Beliefs?

Nancy: Heuristic values: scalar \rightarrow probability distribution (belief)

How to form beliefs?

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Nancy: Heuristic values: scalar \rightarrow probability distribution (belief)

How to form beliefs?

assumptions:

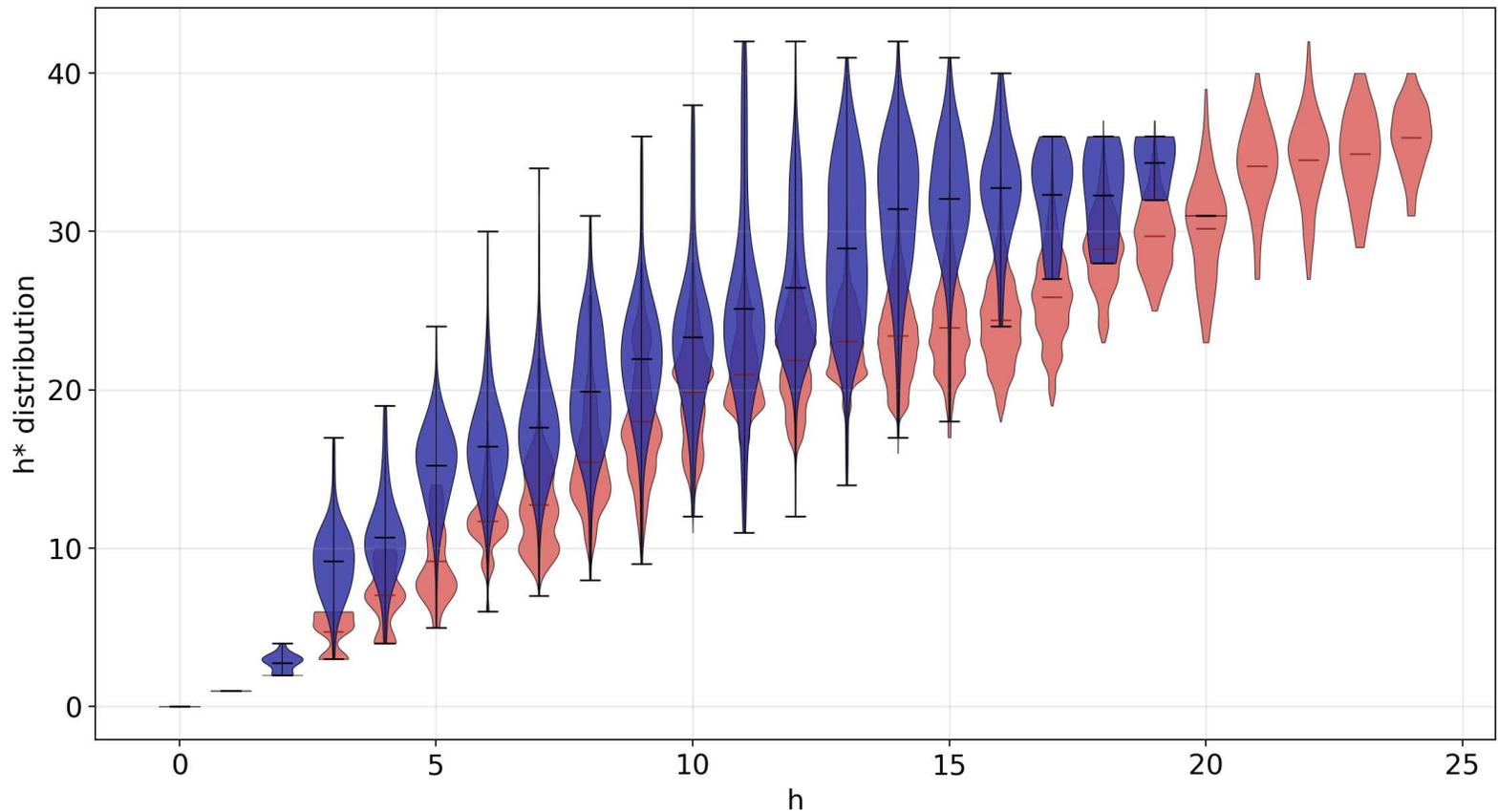
Gaussian at \hat{f} with width $\propto \hat{d}$, truncated at f
online learning with few parameters

training data:

histogram of previous h^* given h
offline learning with many parameters

Example h^* Distribution: Transport vs Blocks World

What do the distributions look like?



Beliefs differ by domain. Often not Gaussian!

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A New General Completeness Proof

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conditions on problem:

1. initial beliefs have finite expected value
2. positive action costs
3. finite state space
4. no dead-ends

conditions on algorithm:

1. goal-aware
2. learning creates local consistency (eg, DP)
3. selects actions via \hat{f}

This proof applies to any LSS-LRTA*-style algorithm

Example Results: Racetrack

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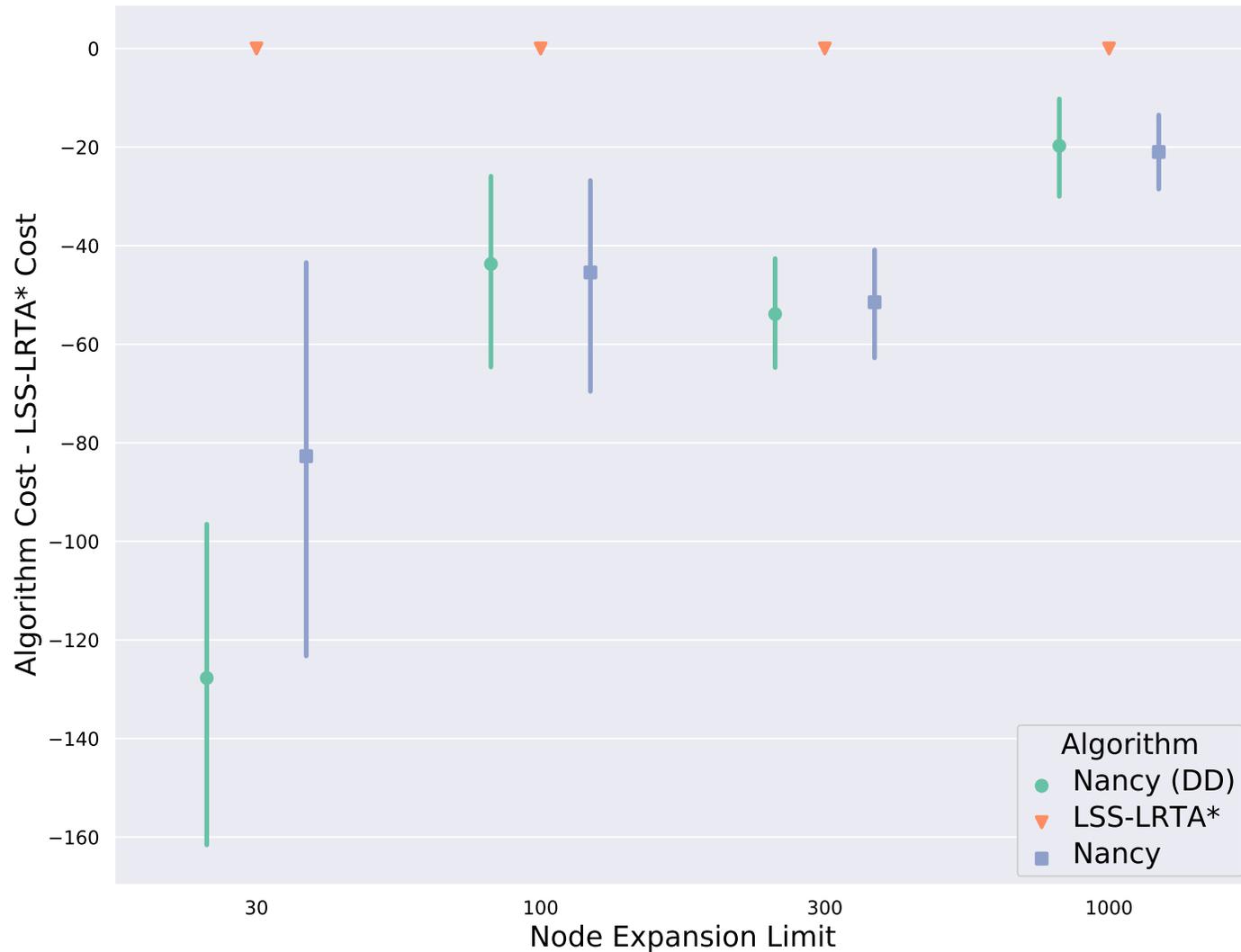
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Even assumptions work well!

IPC Planning: Mean Solution Cost

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	LSS- LRTA*	Nancy (Gauss)	Nancy (data)
Barman	559	702	415
Blocksworld	35	39	34
Elevators-unit	34	27	26
Parking	62	27	31
Rovers	31	29	33
Satellite	15	17	16
Termes	662	129	238
Tidybot	30	30	29
Transport	499	567	422
Transport-unit	35	29	27
VisitAll	52	50	52

Data works when assumptions don't!

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■ Recap

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Recap: Suboptimal Heuristic Search

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■ Recap

■ Questions?

Suboptimal search is the most important kind!

- practical instead of provably intractable
- distinct settings: bounded-suboptimal, contract, utility...

many sources of information beyond h !

- unbiased estimates (can be learned online)
- distance-to-go, not just cost
- beliefs can model uncertainty

search algorithm as agents

- entire AI agent toolbox applies
 - what to represent, how to estimate
 - how to exploit experience
- search highlights issues more clearly than RL

Suboptimal heuristic search needs YOU!

Questions?

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Debiasing h Via Temporal Difference Learning

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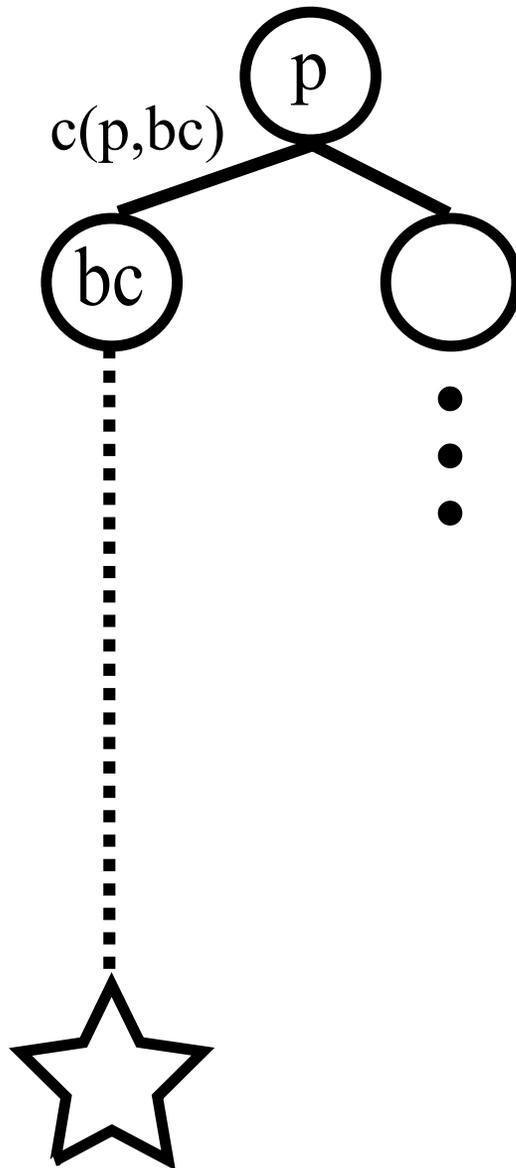
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■ Learning \hat{f}

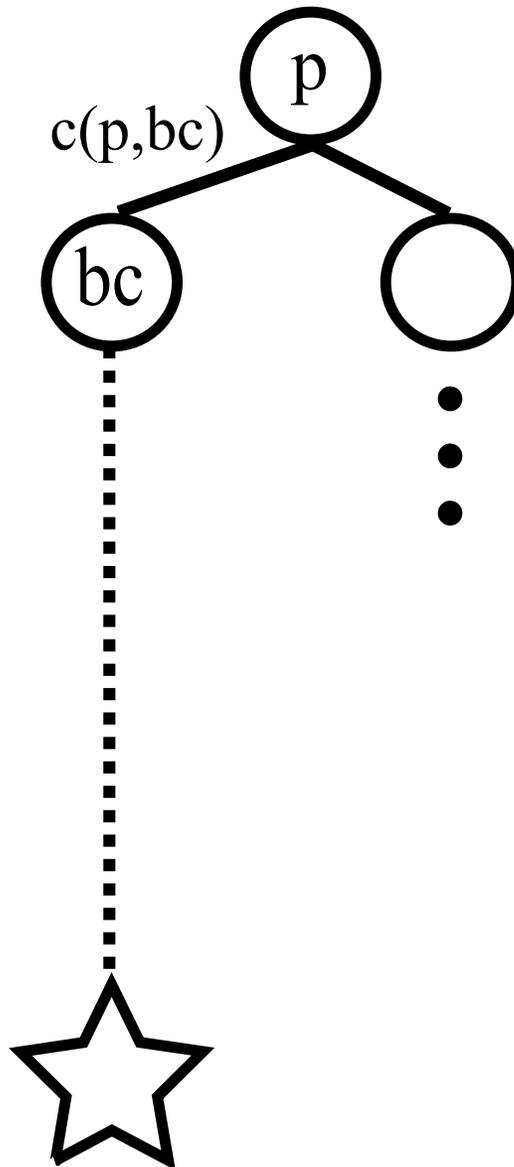
■ EES Bound

■ Subopt. Search



Debiasing h Via Temporal Difference Learning

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$f(p)$ should equal $f(bc)$

Debiasing h Via Temporal Difference Learning

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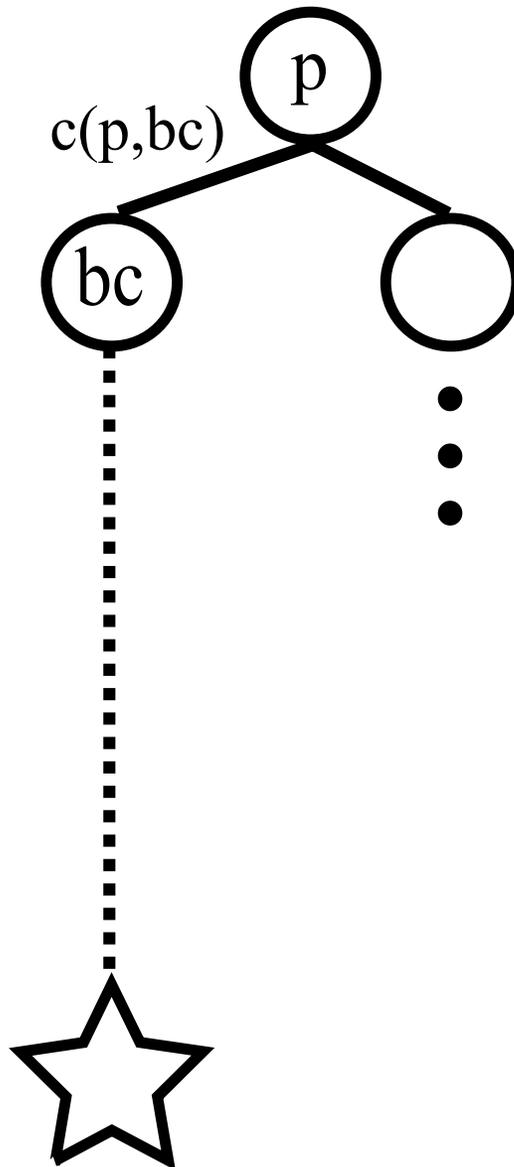
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$f(p)$ should equal $f(bc)$

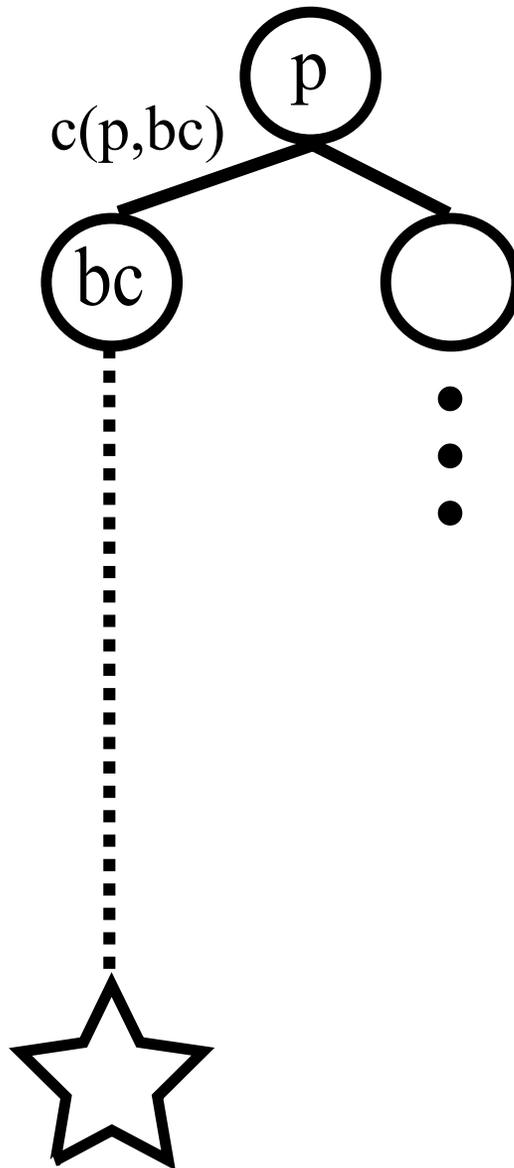
$$f^*(p) = f^*(bc)$$

$$g(p) + h^*(p) = g(bc) + h^*(bc)$$

$$h^*(p) = c(p, bc) + h^*(bc)$$

Debiasing h Via Temporal Difference Learning

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$f(p)$ should equal $f(bc)$

$$f^*(p) = f^*(bc)$$

$$g(p) + h^*(p) = g(bc) + h^*(bc)$$

$$h^*(p) = c(p, bc) + h^*(bc)$$

$$h(p) = h(bc) + c(p, bc) - \epsilon_h$$

$$\epsilon_h = h(bc) + c(p, bc) - h(p)$$

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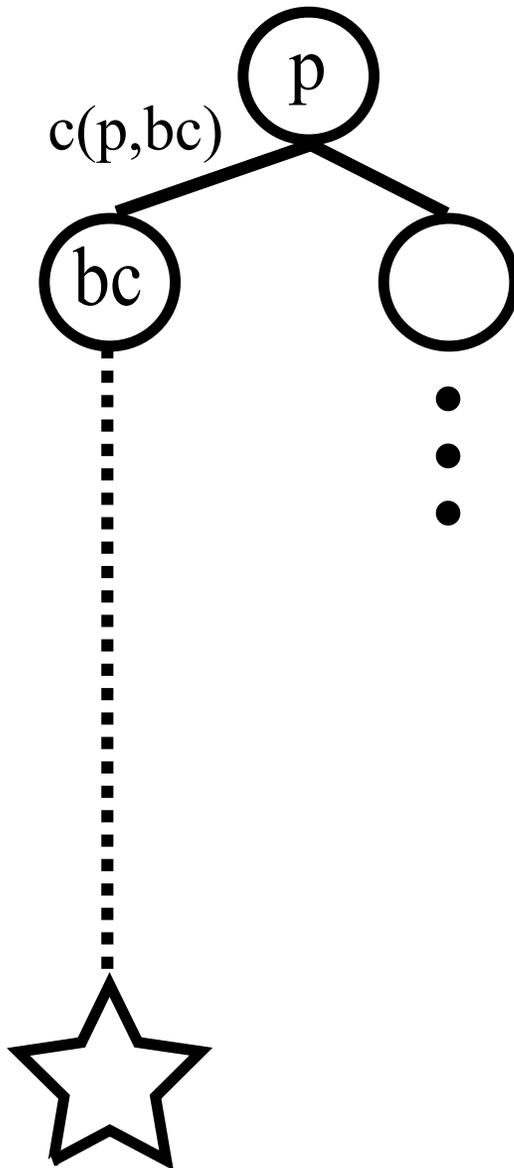
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$$f^*(p) = f^*(bc)$$

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$$h^*(p) = c(p, bc) + h^*(bc)$$

$$h(p) = h(bc) + c(p, bc) - \epsilon_h$$

$$\epsilon_h = h(bc) + c(p, bc) - h(p)$$

$$\hat{h}(n) = h(n) + \bar{\epsilon}_h \cdot \hat{d}(n)$$

EES' Bounded Suboptimality

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■ Learning \hat{f}

■ **EES Bound**

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$$g(n) = \begin{matrix} f(n) \leq w \cdot f_{min} \\ \leq w \cdot f^* \end{matrix}$$

A Science of Suboptimal Search

it's time to take suboptimality seriously!

- estimates, not lower bounds
- belief distributions to quantify uncertainty

acting under uncertainty to maximize utility

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= all of AI

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