

Heuristic Search for Large Problems with Real Costs

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- Problem: A* runs out of memory in ~ 10 minutes

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- Problem: A* runs out of memory in ~ 10 minutes
- Problem: IDA* fails with many duplicates and real costs

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- Problem: A* runs out of memory in ~ 10 minutes
- Problem: IDA* fails with many duplicates and real costs
- Solution: A* on disk (“external memory”)

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- Problem: IDA* fails with many duplicates and real costs
- Solution: A* on disk (“external memory”)
- Problem: Previous methods assume integer costs

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- Problem: Previous methods assume integer costs
- Problem: Most previous methods use breadth-first search

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- Problem: IDA* fails with many duplicates and real costs
- Solution: A* on disk (“external memory”)

- Problem: Previous methods assume integer costs
- Problem: Most previous methods use breadth-first search
- Solution: PEDAL (this paper)
 - ◆ Best-first search order
 - ◆ Real costs
 - ◆ Provably I/O efficient
 - ◆ Exploits parallelism

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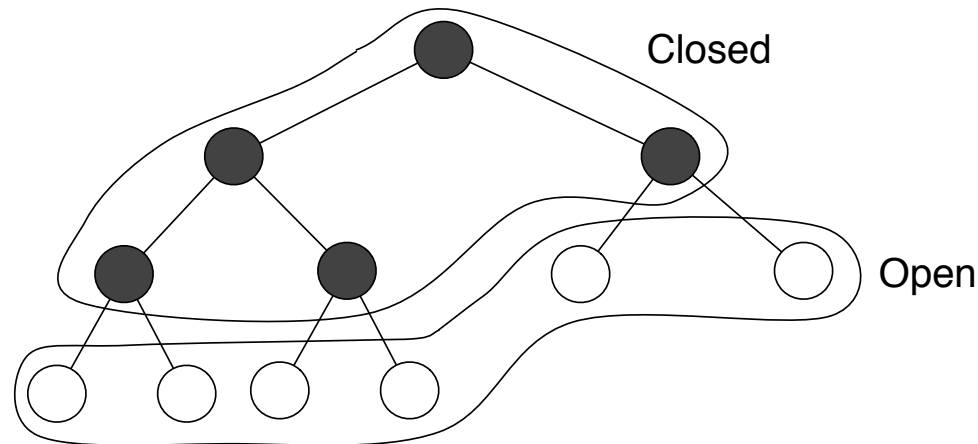
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- Open List: ordered set $f(n) = g(n) + h(n)$
- Closed List: random access!
- How to put this on disk?



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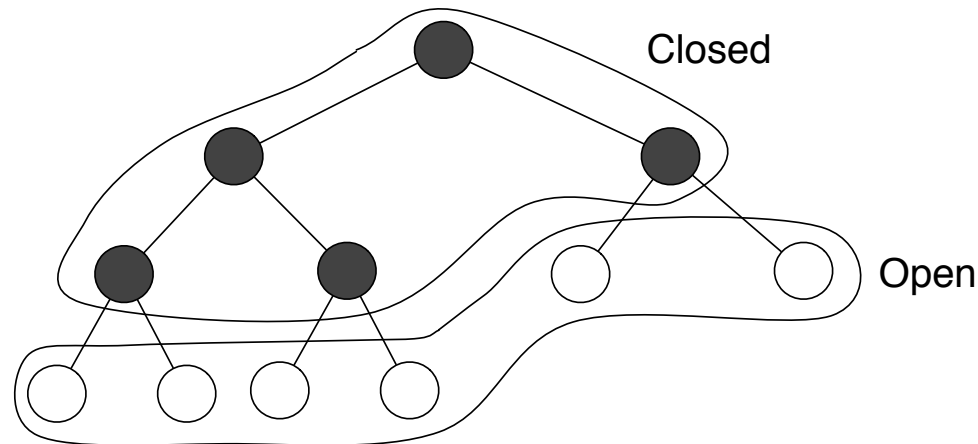
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- Closed List: random access!
- How to put this on disk?



- Two ideas: Buckets and Layers

Buckets

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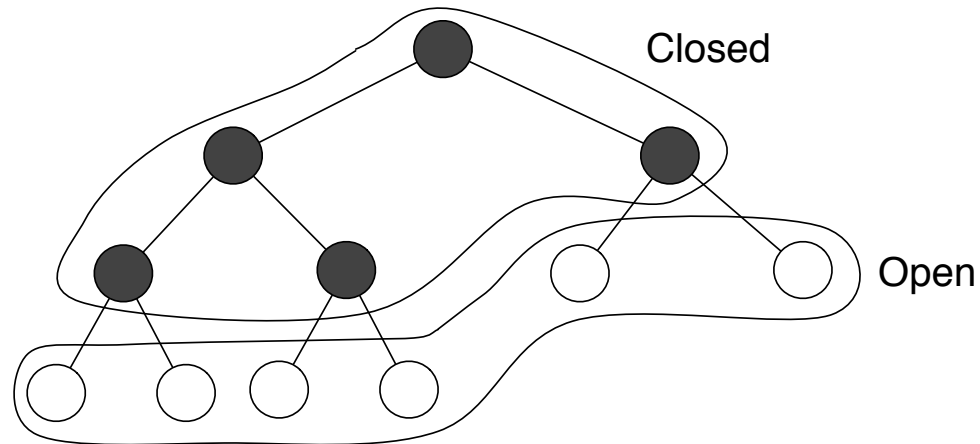
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- Use a hash function to partition the space
- Duplicate nodes will be in the same bucket



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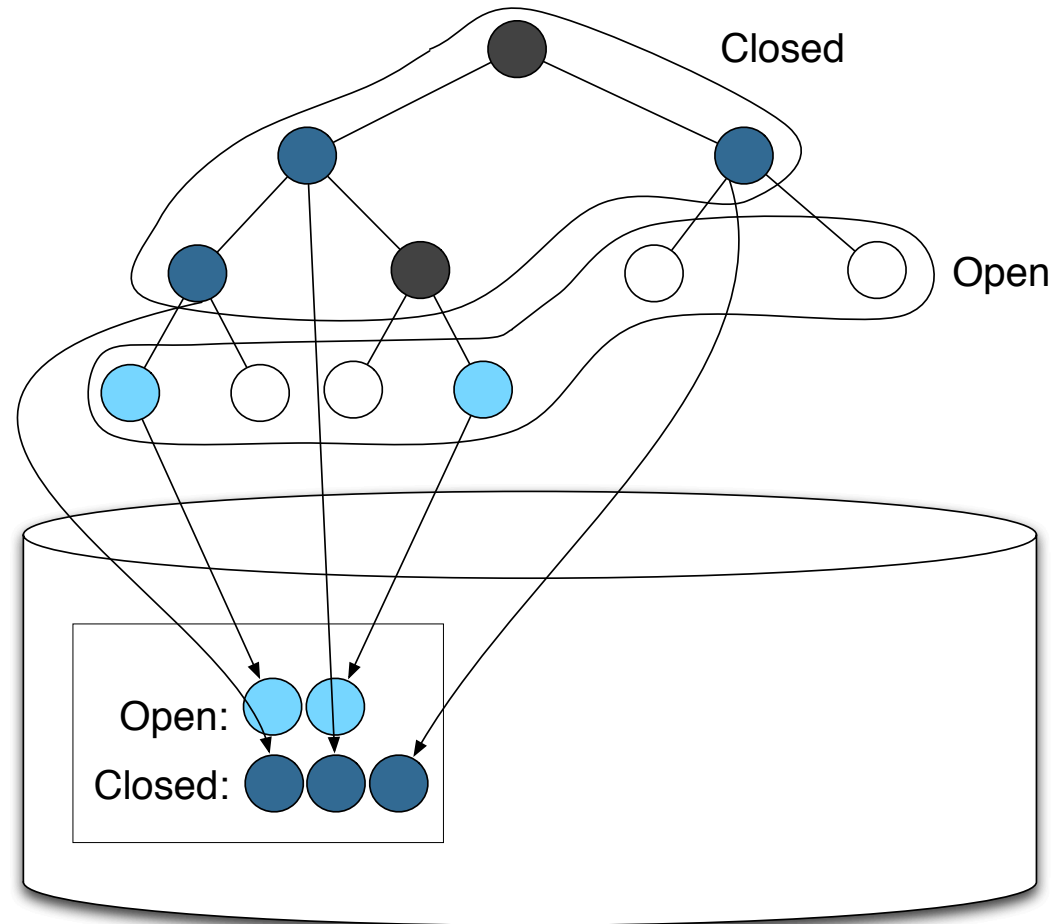
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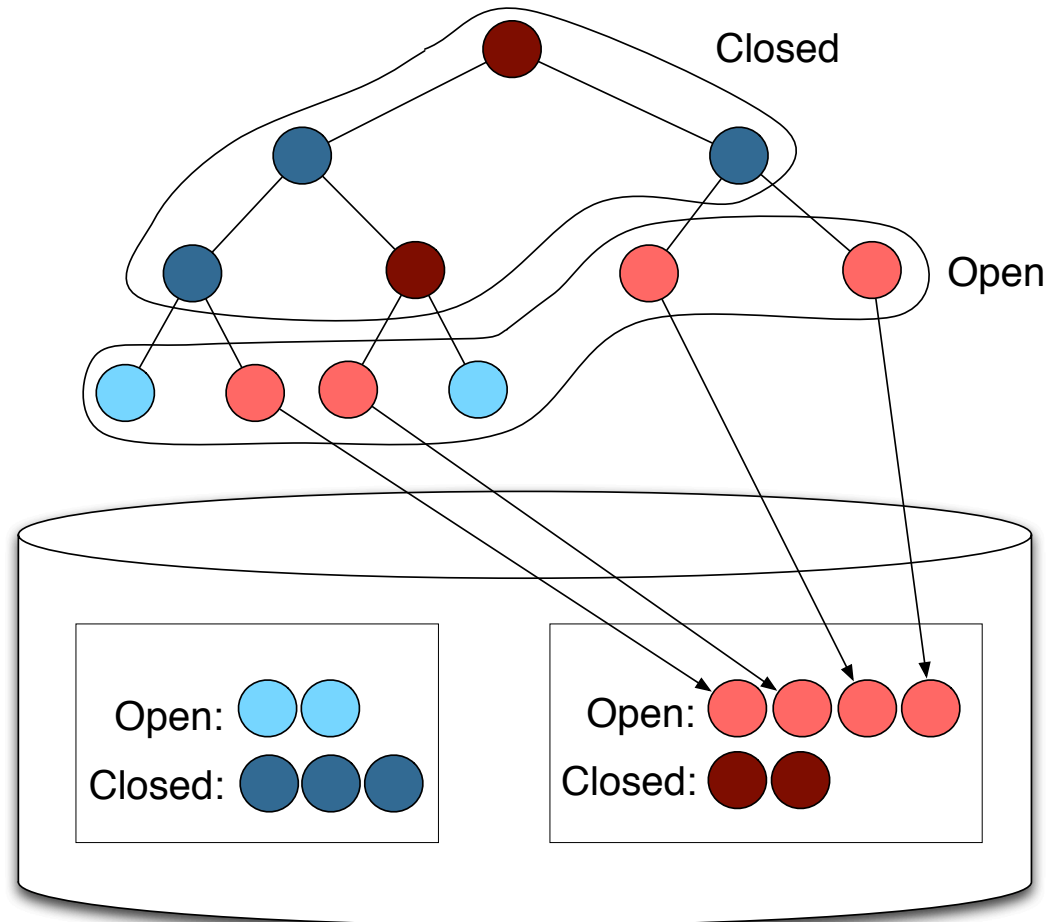
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- Duplicate nodes will be in the same bucket



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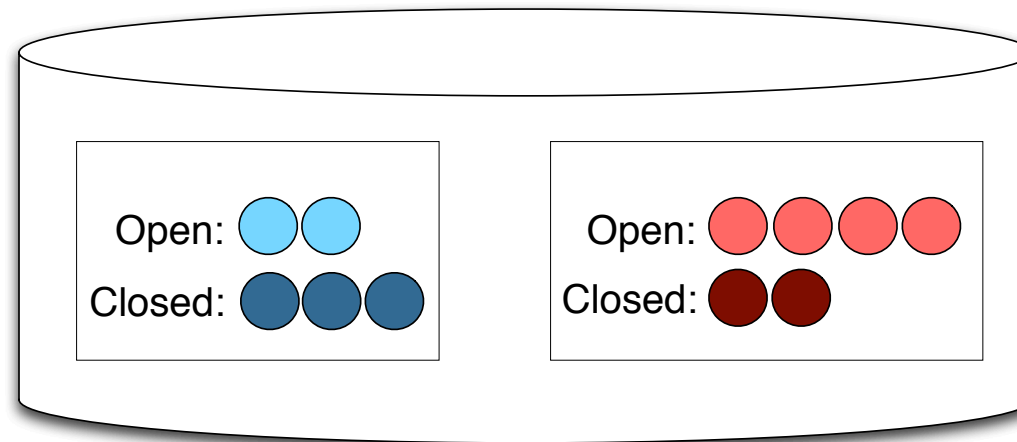
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Only one bucket need fit in RAM to expand

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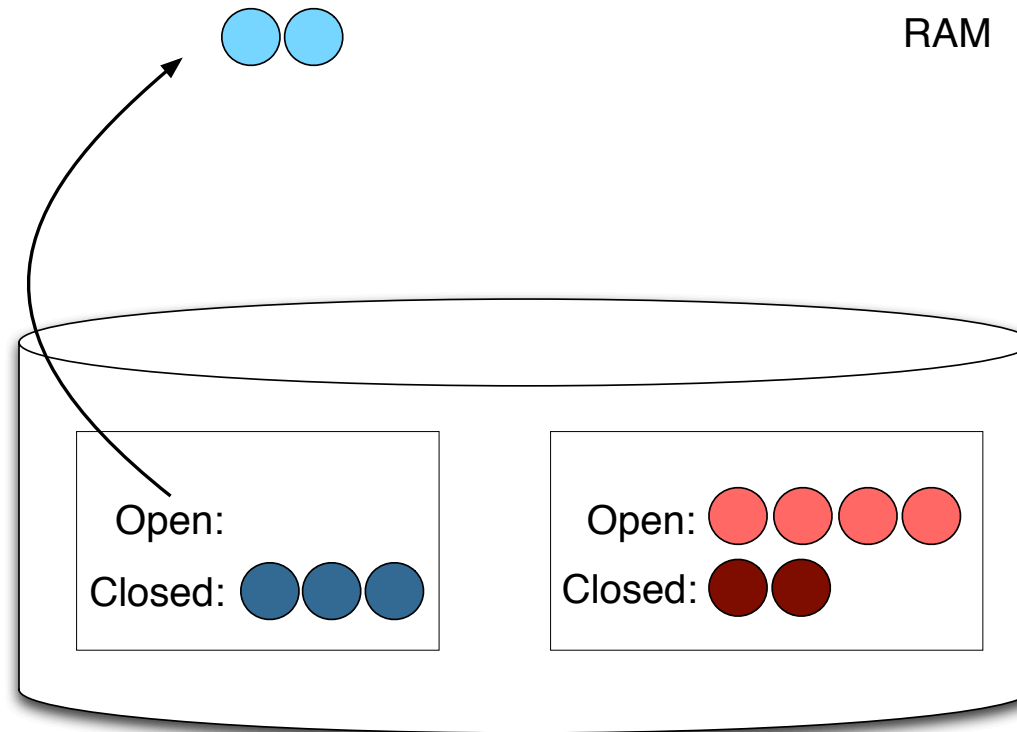
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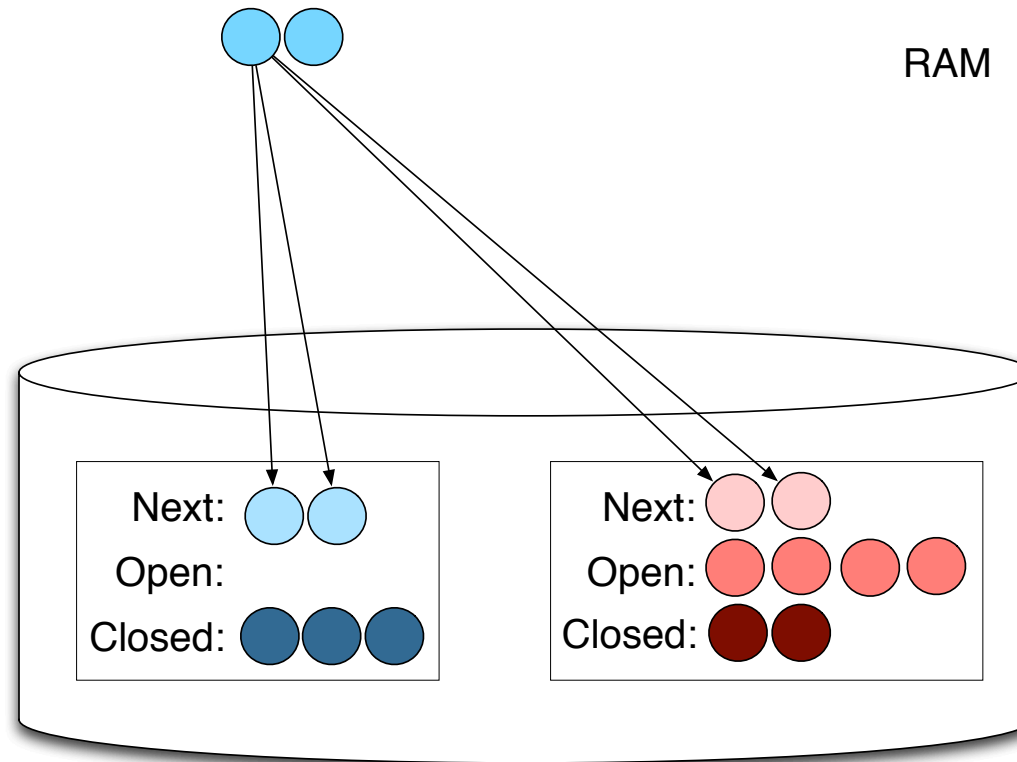
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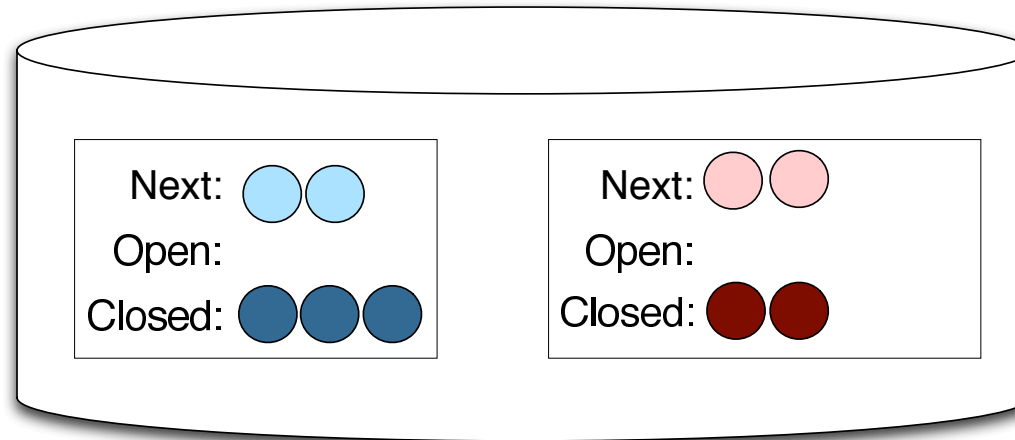
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Only one bucket need fit in RAM to merge duplicates

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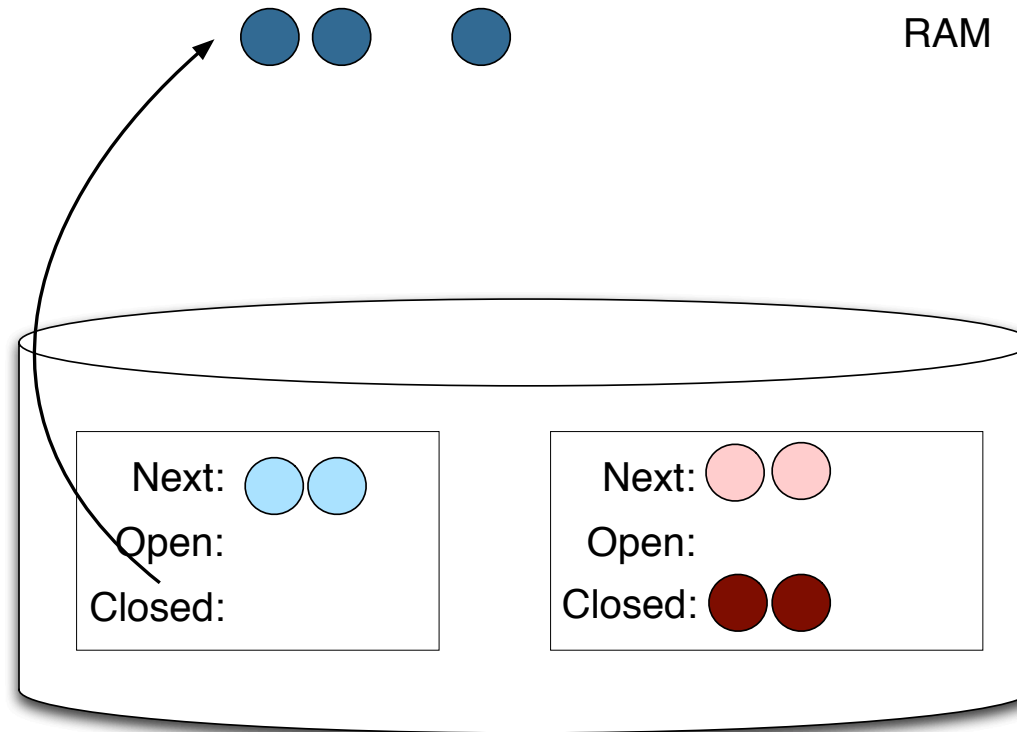
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Only one bucket need fit in RAM to merge duplicates

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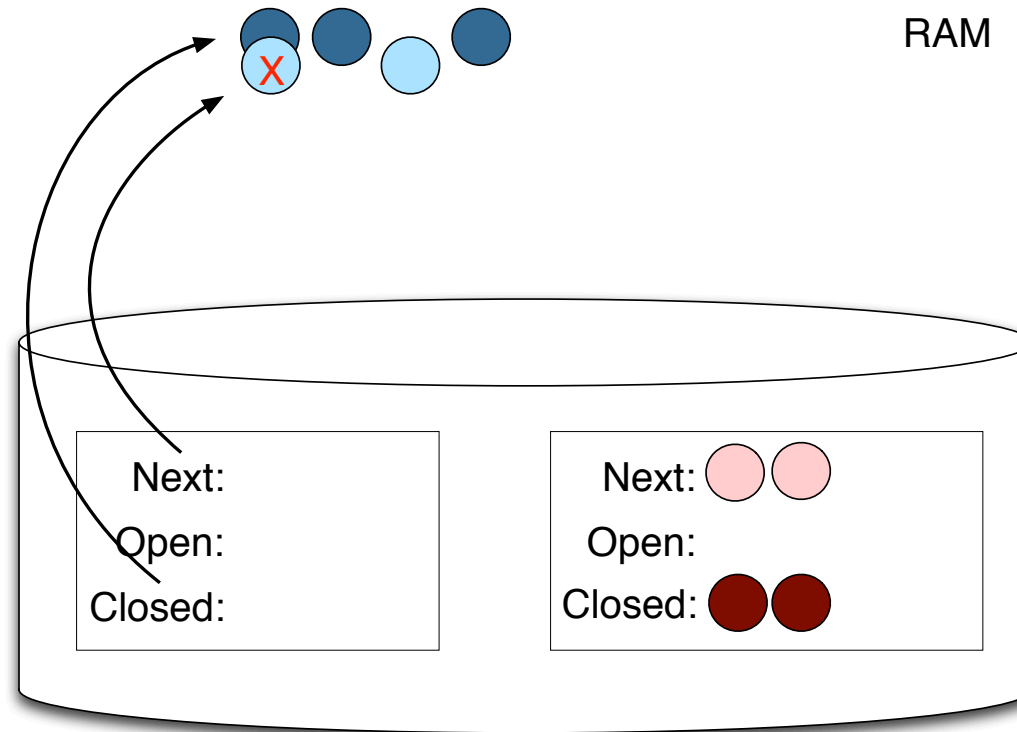
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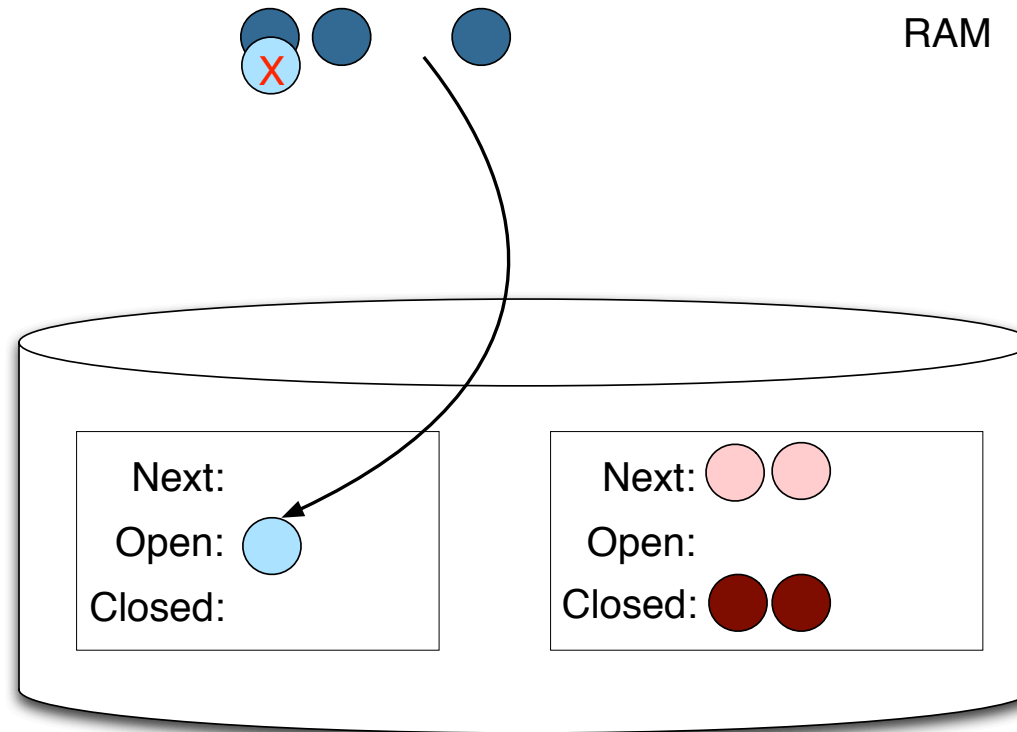
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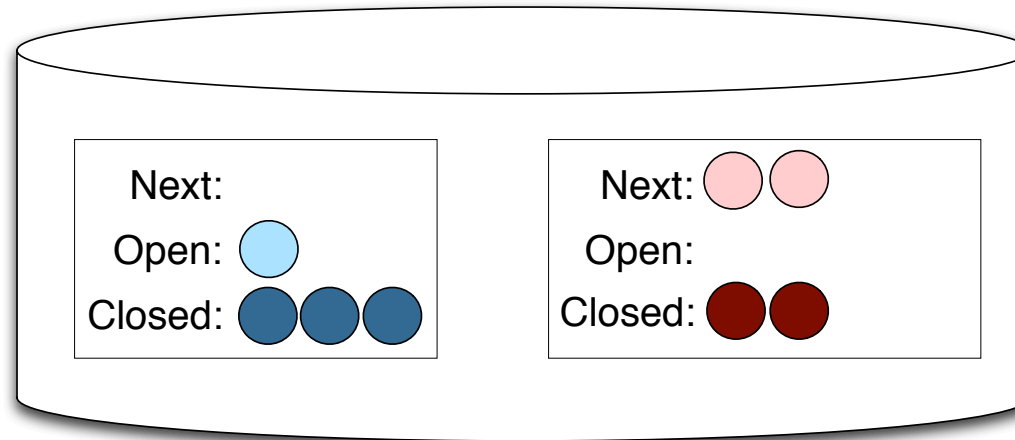
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- Depth based layers give breadth-first search
eg., Breadth-First Heuristic Search (Zhou, Hansen 2004)
- How do we perform best-first search?

Layers

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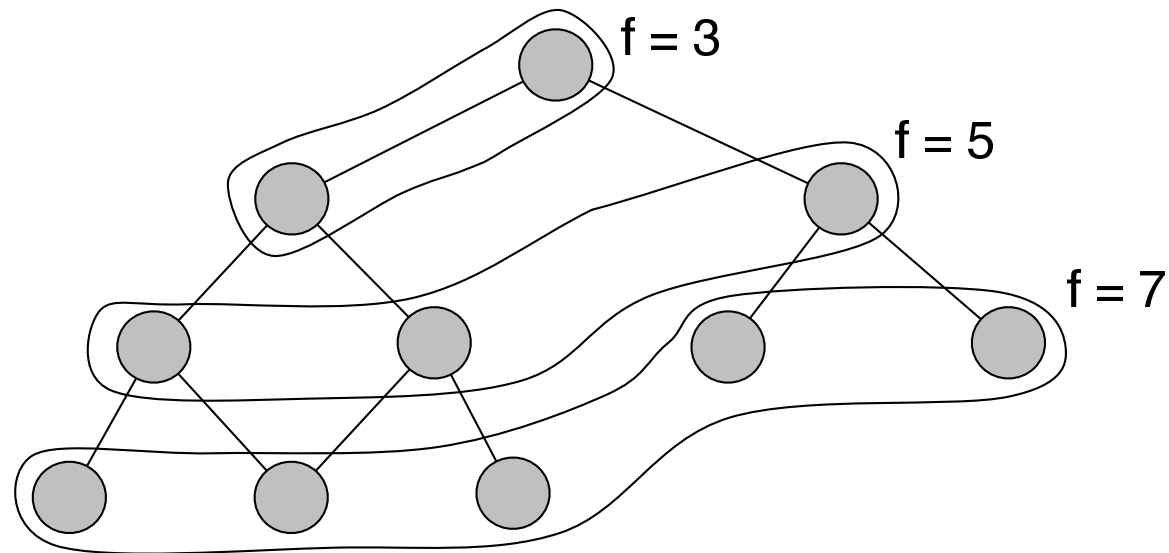
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- Depth based layers give breadth-first search
eg., Breadth-First Heuristic Search (Zhou, Hansen 2004)
- How do we perform best-first search?
 - ◆ Set an f limit and expand all nodes within the limit



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- Divide the search space into buckets
- Duplicate detection is localized per bucket
- Duplicate detection is done in a separate phase
- Layer the search by f

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- Real Costs Prob.
- Histogram Method
- PEDAL

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Real Costs Prob.

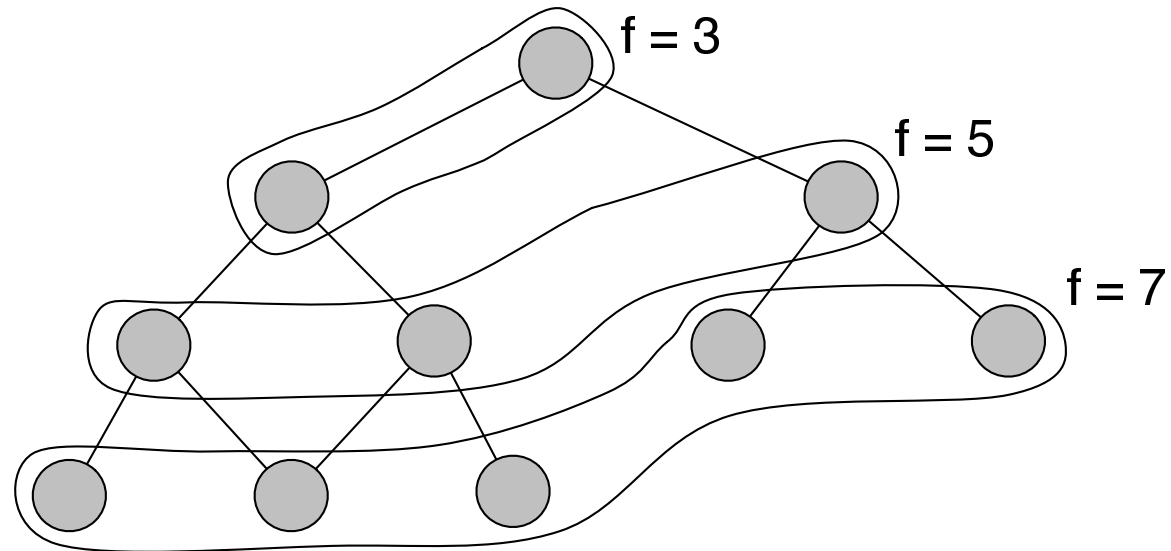
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Unit Costs



Layers grow, many nodes per layer

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Real Costs Prob.

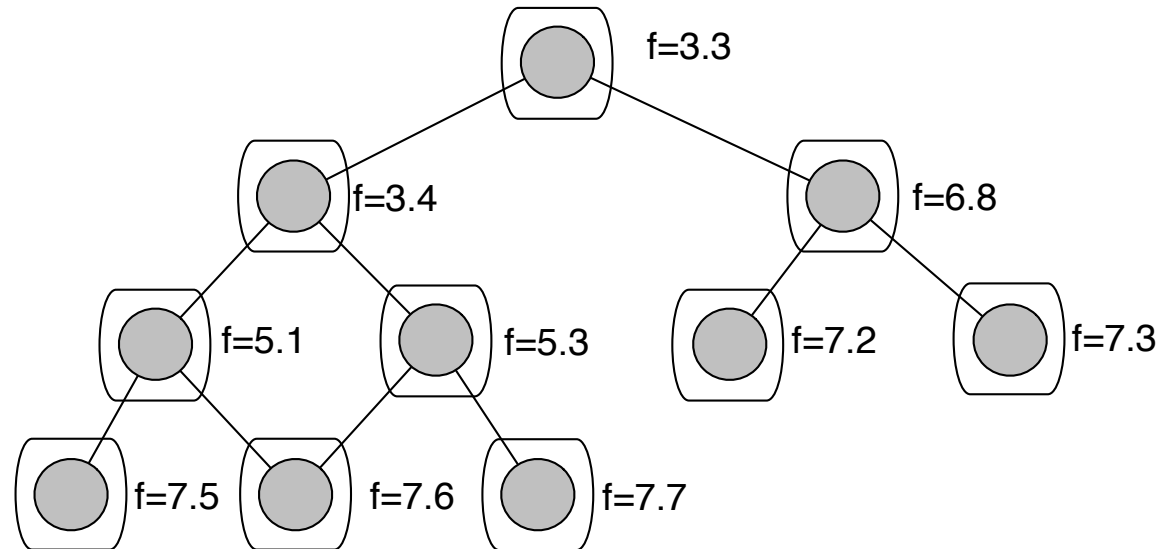
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Real Costs



Many layers, too few nodes per layer

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Real Costs Prob.

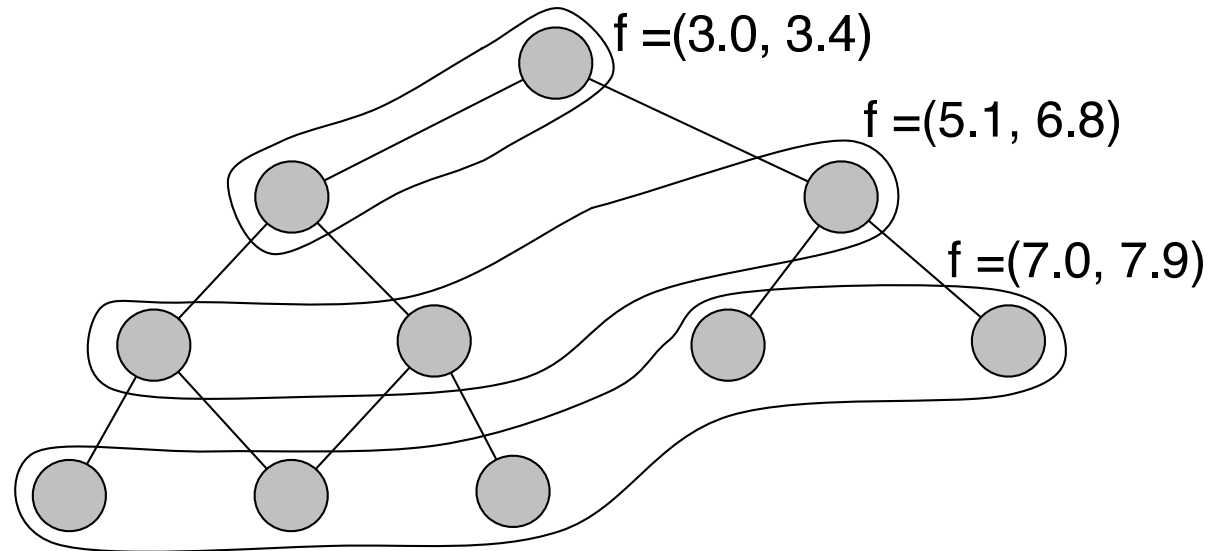
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Real Costs



Our solution is to inflate layers
But how should we inflate layers?

Using a Histogram to Inflate Layers

Bad

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Using a Histogram to Inflate Layers

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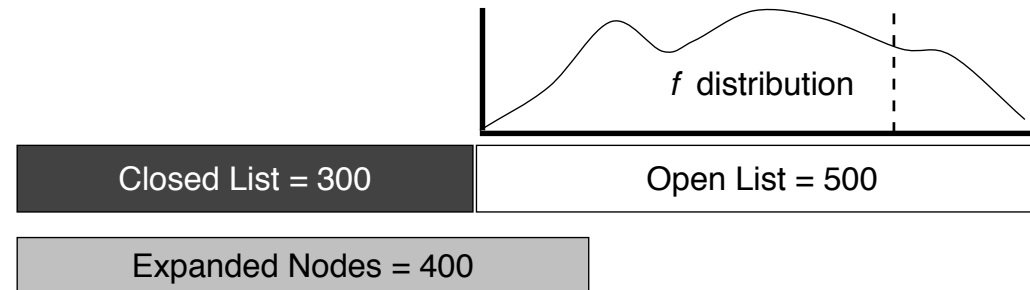
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Good



- Keep a distribution of all f -values on the frontier
- Choose a value in this distribution to guarantee I/O efficiency
- See paper for proof

Parallel External Dynamic A* Layering

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- Best-first:
 - ◆ layer search on f
- Real Costs:
 - ◆ dynamically inflate f layers for I/O efficiency
- Exploits Parallelism (see paper)
- Recursive expansions (see paper)

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- Three domains
- Dual quad-core
- 8GB of RAM
- 7 SATA disks

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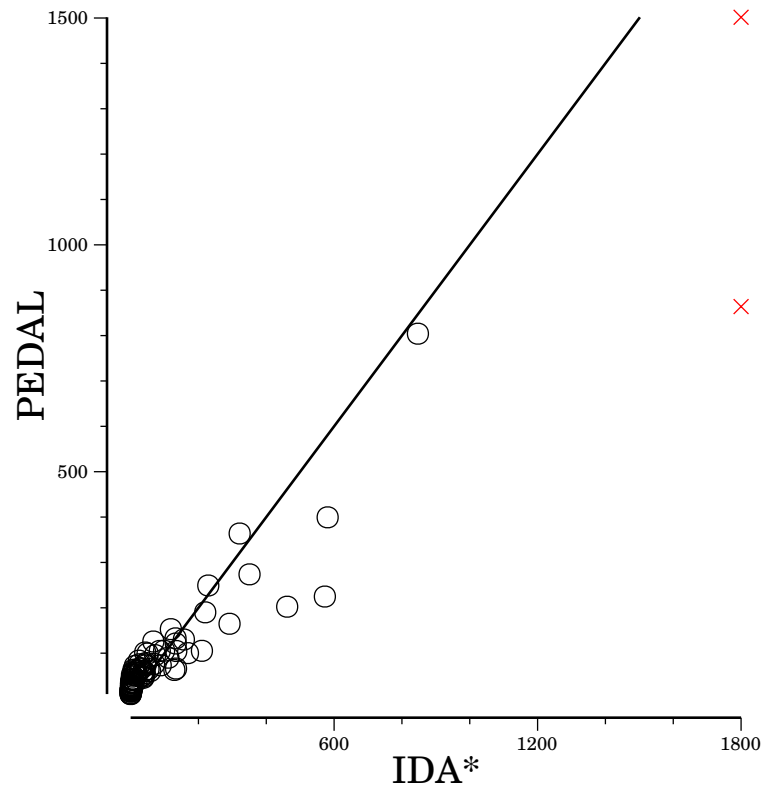
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- Unit cost sliding tile puzzle
- Classic benchmark
- Korf 100 (Korf 1985)
- A* is unable to solve all 100 with Manhattan Distance

6	2	5	14
3	15	4	
7	12	11	10
8	1	13	9

15-Puzzle: PEDAL vs. IDA* (Korf 1985)

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PEDAL uses disk but is faster than IDA*!

15-Puzzle: PEDAL vs. Breadth-First Heuristic Search

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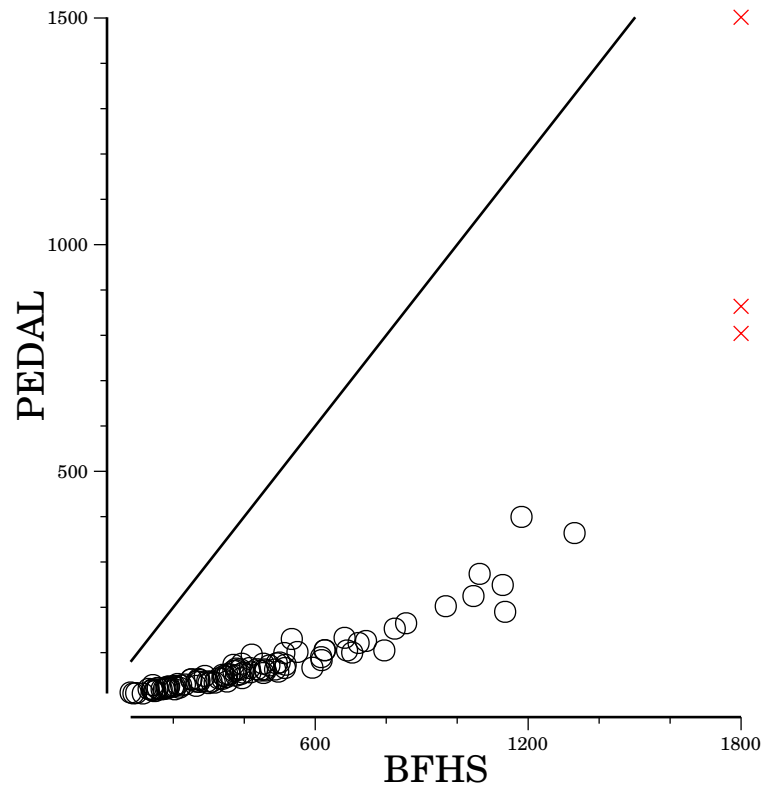
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Best-first is better than breadth-first

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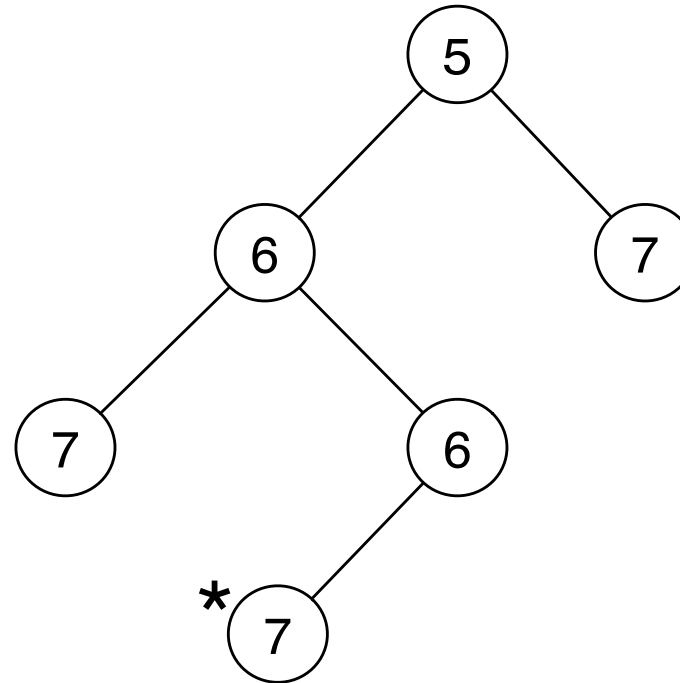
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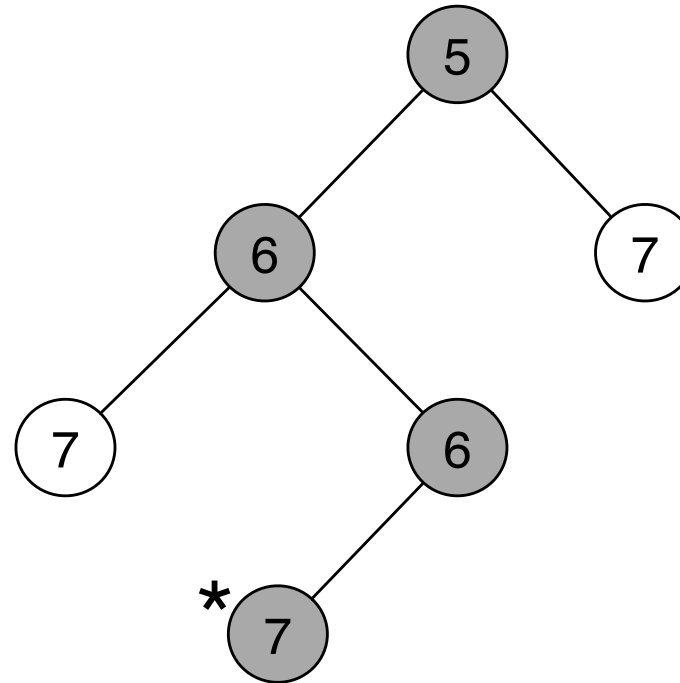
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A* with proper tie breaking

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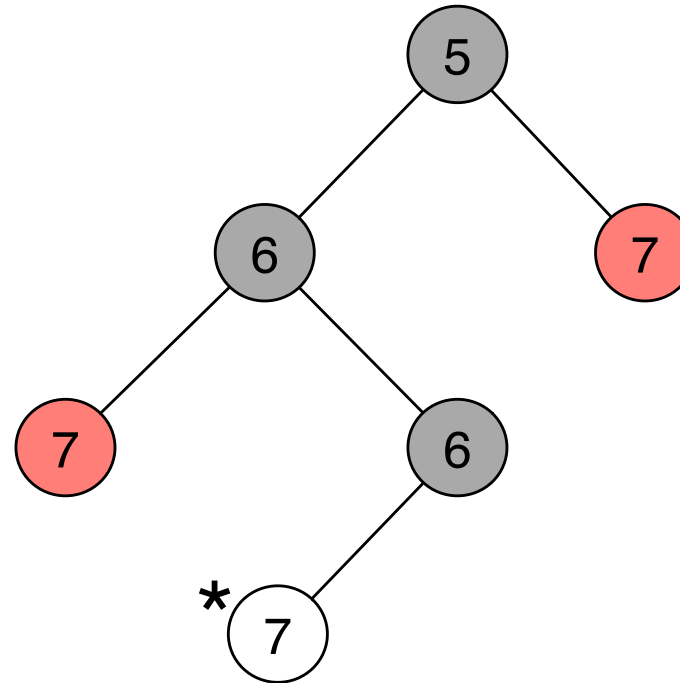
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BFHS must expand more nodes

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6	2	5	14
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$\sqrt{10}$

- Move costs square root of tile number
- Simple real-valued version of well understood benchmark
- Easy to reproduce

Sqrt Puzzle

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$\sqrt{10}$

- IDA* and BFHS did not solve any instances in time limit
- We compare against IDA*_{CR} and novel variant of BFHS

Sqrt. 15-Puzzle: PEDAL vs. IDA*_{CR}

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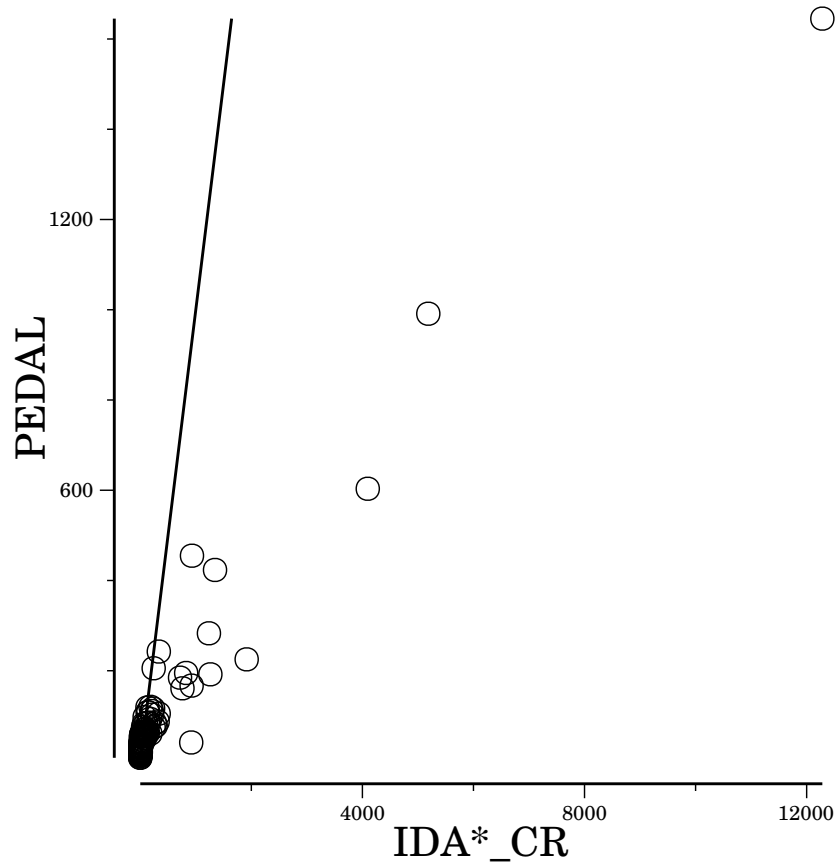
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Again duplicate checking and parallelism trump disk latency

Sqrt. 15-Puzzle: PEDAL vs. BFHS

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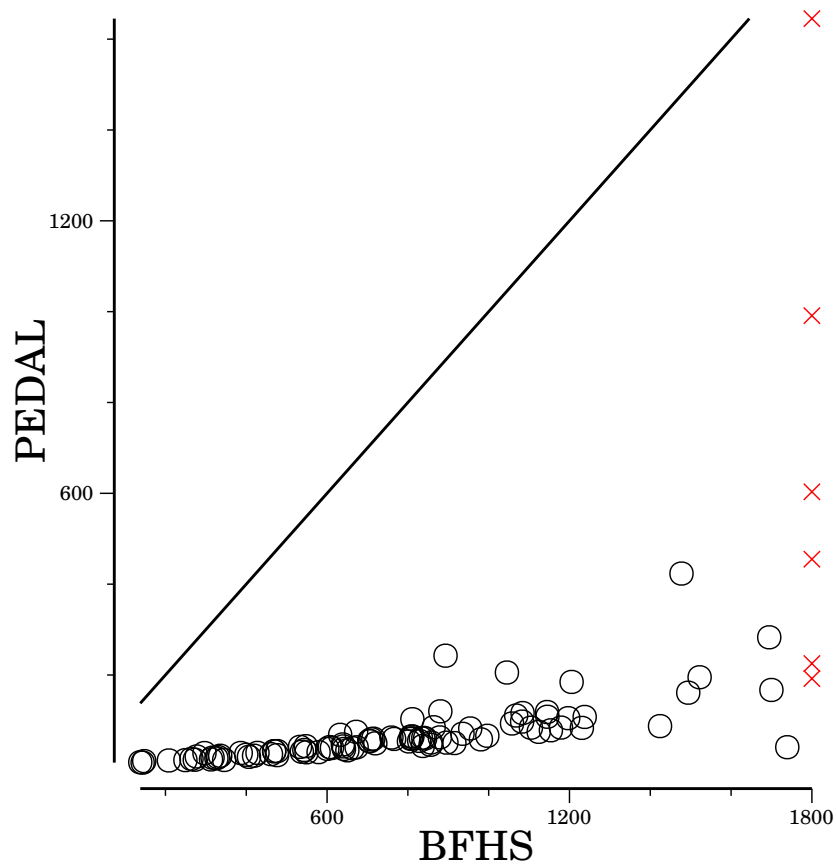
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Again best-first is better than breadth-first

Dockyard Robots

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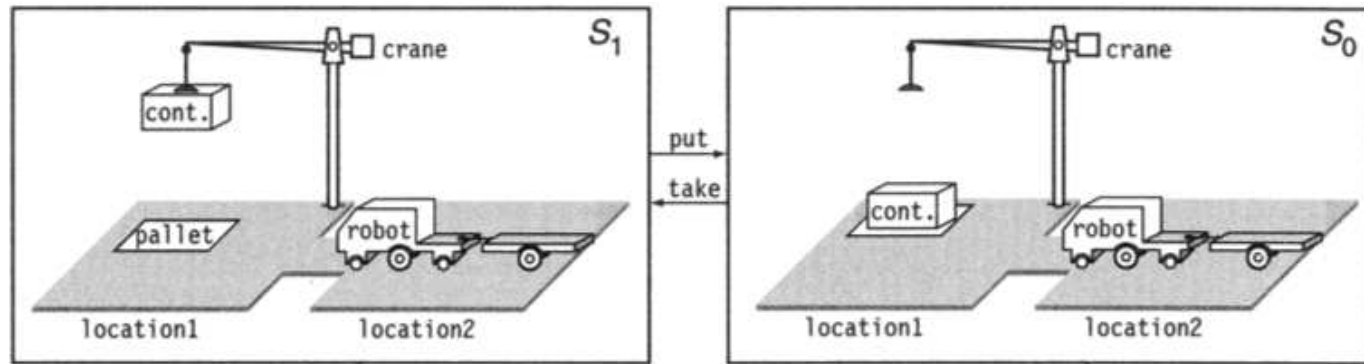
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- From Ghallab, Nau, Traverso (2004)
- All actions have real costs
- Many duplicate states
- IDA* and IDA*_{CR} fail on all instances

Dockyard Robots: PEDAL vs. BFHS

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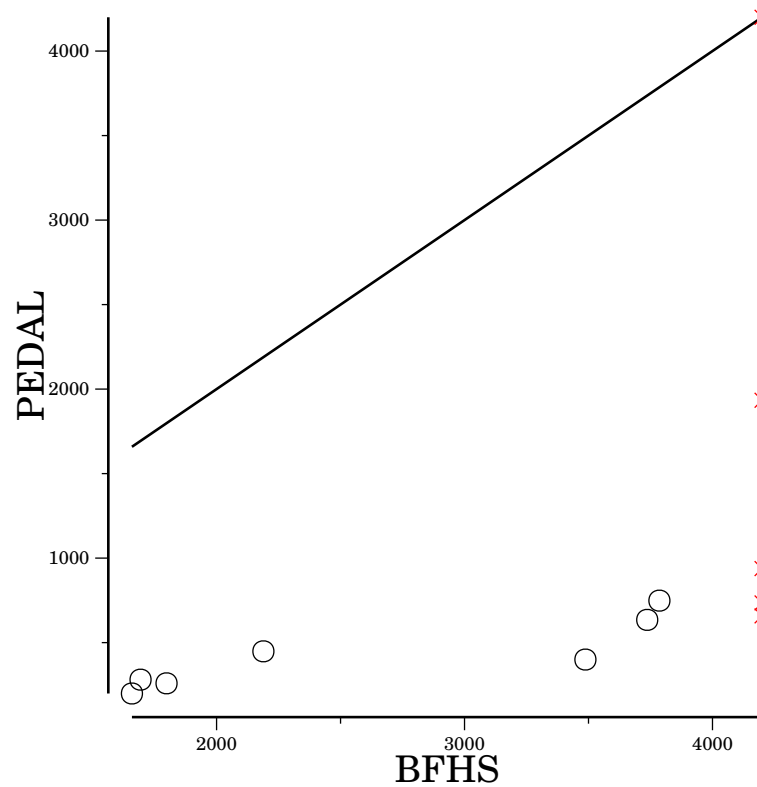
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PEDAL is the only viable alternative

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- Previous external-memory search:
 - ◆ fails on domains with real costs
 - ◆ suggests that breadth-first search is preferred

- PEDAL:
 - ◆ is best-first
 - ◆ I/O efficient for real costs

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External memory search doesn't have to be slow!

- ◆ faster than IDA* on standard 15-Puzzle!

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 - ◆ I/O efficient for real costs

External memory search doesn't have to be slow!

- ◆ faster than IDA* on standard 15-Puzzle!

General purpose best-first external memory algorithm

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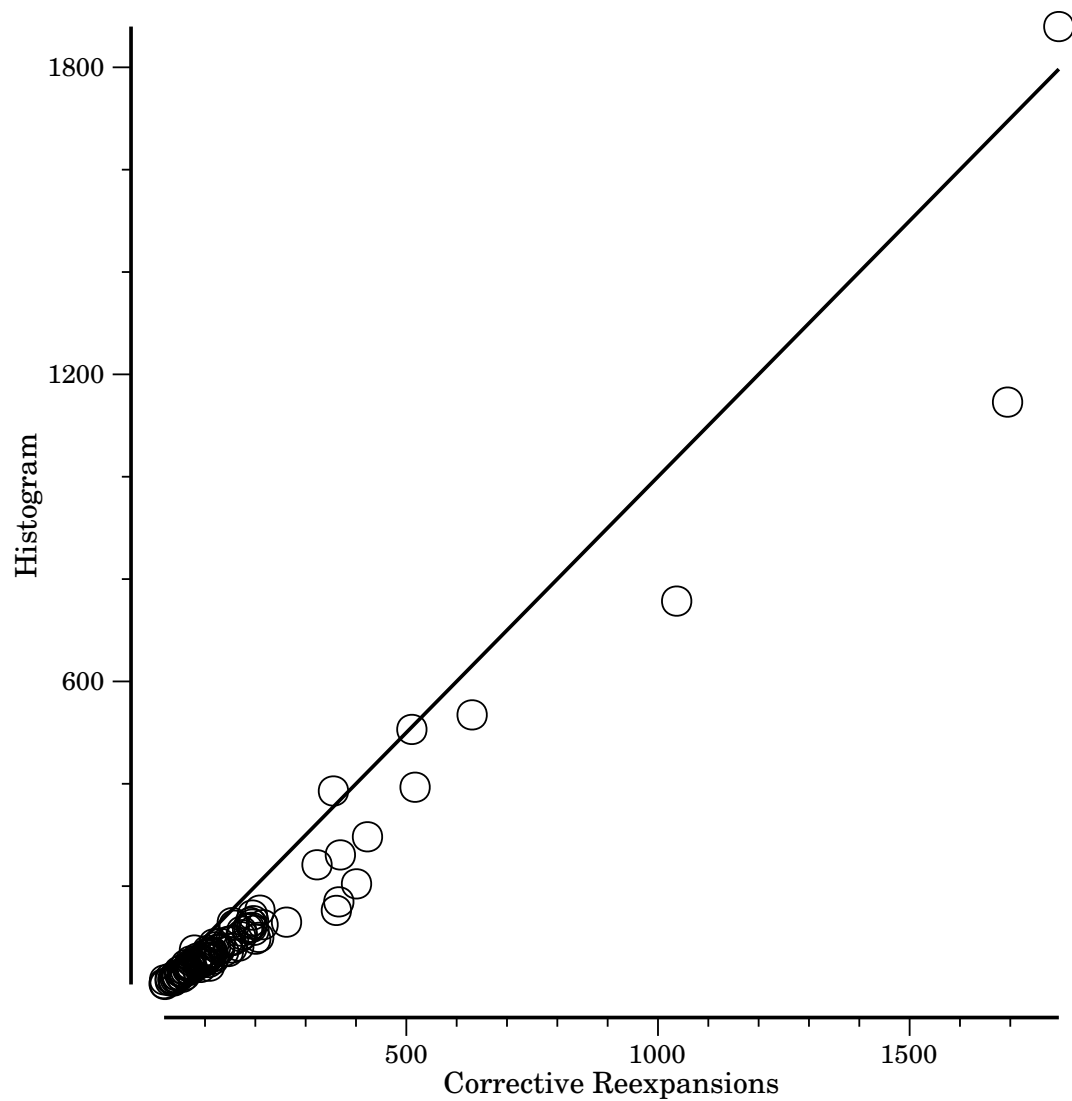
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Non-Recursive

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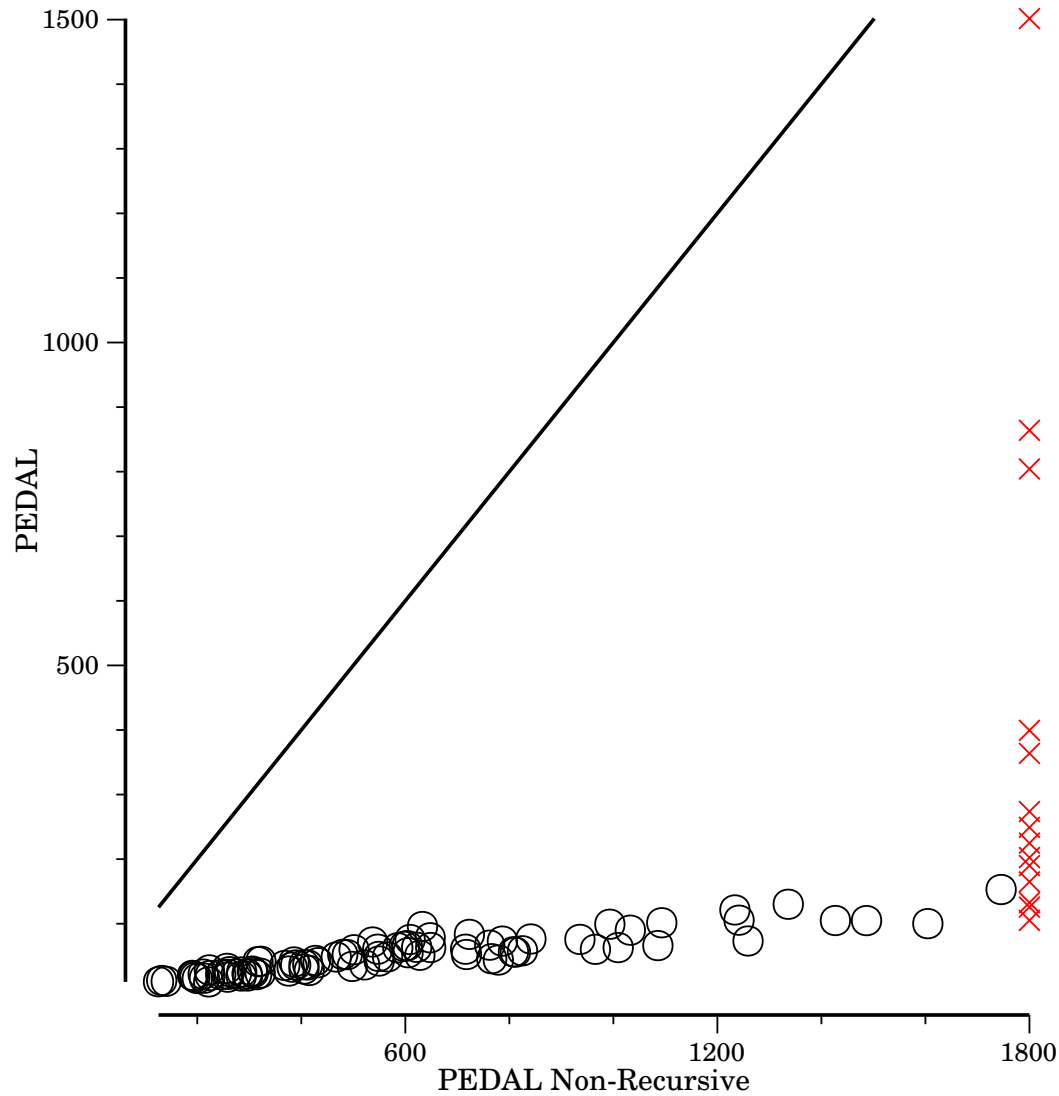
PEDAL CR vs. Histogram

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BFHS vs. PEDAL Non-Recursive

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