http://www.cs.unh.edu/~ruml/cs758

1 handout: slides
- check your Wildcat Pass before coming to campus
- if you have concerns, let me know
Red-Black Trees
node: data, left, right, parent, color

1. every node is either red or black
2. the root is black
3. (consider nil to be black)
4. both children of a red node are black
5. from any node, all paths to leaves have the same 'black height'
Plain Binary Tree Deletion

4 cases of delete(z):

1. no left child, or no kids: substitute right subtree at parent.
2. no right child: substitute left subtree at parent.
3. successor y is z’s right child:
   (a) substitute y for z
   (b) attach z’s left subtree as y’s left subtree
4. successor y is deeper:
   (a) substitute y’s right subtree for y
   (b) attach z’s right subtree as y’s right subtree
   (c) as above, substitute y for z
   (d) as above, attach z’s left subtree as y’s left subtree

What if it’s a red-black tree?
Cases 1 and 2: Single Child

1. every node is either red or black
2. the root is black
3. (consider nil to be black)
4. both children of a red node are black
5. from any node, all paths to leaves have the same ‘black height’

Deleting \( z \) with single child \( x \)

1. \( x \) takes \( z \)’s place
Cases 1 and 2: Single Child

1. every node is either red or black
2. the root is black
3. (consider nil to be black)
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5. from any node, all paths to leaves have the same ‘black height’

Deleting $z$ with single child $x$

1. $x$ takes $z$’s place
2. book uses $y$ for $z$ for short code
3. if $y (= z)$ was black, we have ‘extra black’ at $x$, so call fixup routine at $x$
Case 3: Two Children, Successor is Child

1. every node is either red or black
2. the root is black
3. (consider nil to be black)
4. both children of a red node are black
5. from any node, all paths to leaves have the same ‘black height’

deleting $z$, successor $y$ is right child

1. $y$ takes $z$’s place and color
2. attach $z$’s left subtree as $y$’s left subtree
Case 3: Two Children, Successor is Child

1. every node is either red or black
2. the root is black
3. (consider nil to be black)
4. both children of a red node are black
5. from any node, all paths to leaves have the same ‘black height’

deleting $z$, successor $y$ is right child

1. $y$ takes $z$’s place and color
2. attach $z$’s left subtree as $y$’s left subtree
3. if $y$ was black, we need ‘extra black’ at $y$’s right child $x$, so call fixup routine at $x$
Case 4: Two Children, Successor is Deeper

Deleting $z$, successor $y$ is deep down

1. Substitute $y$’s right child $x$ for $y$
2. Attach $z$’s right subtree as $y$’s right subtree
   as in simpler case:
3. $y$ takes $z$’s place and color
4. Attach $z$’s left subtree as $y$’s left subtree
5. If $y$ was black, we need ‘extra black’ at $x$, so call fixup routine at $x$
asst 4: write verifier
Red-Black Tree Deletion Fixup
need to find a red node to make black

when \( x \) red or root, color black and terminate

\( x \) is non-root black node.

assume \( x \) is a left child (other cases symmetric).

Must have sibling \( w \), since \( x \) holds ‘extra blackness’.

4 cases:

1. \( w \) is red
2. \( w \) and both its children are black
3. \( w \) is black, its right child is black, its left child is red
4. \( w \) is black, its right child is red

fix-up loop invariant: all properties hold if ‘extra black’ at \( x \) is considered, heights of fringe (greek) nodes preserved
case 1: \( w \) is red. so parent and children must be black.

solution:
1. rotate and recolor to get black sibling (moves red horizontally)
2. fall through to case 2, 3, or 4
**Case 2**

**Red-Black Trees**

**Deletion Fixup**

- Fix-up Loop
- Case 1
- Case 2
- Case 3
- Case 4
- Complexity
- EOLQs

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**Case 2:** \(w\) and both its children are black

**Solution:**

1. Color \(w\) red. Subtree at parent now ‘black-balanced’.
2. Move \(x\)'s blackness (and \(w\)’s) up the tree
3. Recur at parent

If from case 1, \(x\) now red, so will terminate
Case 3

Case 3: \( w \) is black, its right is black, left is red

Solution:
1. rotate right and move red over to right child
2. fall through to case 4
case 4: $w$ is black, its right child is red

solution:

1. rotate and recolor to annihilate red with $x$’s black
2. set $x$ to root to force termination
Complexity

Red-Black Trees

Deletion Fixup

Find successor is $O(\lg n)$

One fixup iteration is constant time

Fixup loops only when moving up, so is $O(\lg n)$

How many rotations are performed?
For example:

- What’s still confusing?
- What question didn’t you get to ask today?
- What would you like to hear more about?

Please write down your most pressing question about algorithms and put it in the box on your way out.

Thanks!