CS 758/858: Algorithms

Searching

Hash Tables

Hash Functions

Related

http://www.cs.unh.edu/~ruml/cs758

Searching

■ Dictionaries

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Searching

Dictionaries



'associative array', 'map', 'look-up table', 'set'

Dictionaries

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'associative array', 'map', 'look-up table', 'set' n items, key length k

Structure	Find	Insert	Delete
List (unsorted)			
List (sorted)			
Array (unsorted)			
Array (sorted)			
Heap			
Hash table			
Binary tree (unbalanced)			
Binary tree (balanced)			

Searching

Hash Tables

- Hash Tables
- Time Complexity
- More Collisions
- Open Addressing
- Break

Hash Functions

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applications:

- dictionaries
- object method tables
- string matching
- set operations: \cup , \cap , -

first methods:

- direct-address tables: small key range. eg, bit vectors.
- chaining: deletion?

Time Complexity

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n items in m buckets time complexity of search =

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n items in m buckets time complexity of search = number of items per bucket assume nice hash: P(h(i)=x)=1/m

Time Complexity

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n items in m buckets time complexity of search = number of items per bucket assume nice hash: P(h(i)=x)=1/m let X_i be 1 iff h(i)=x, 0 otherwise

$$E\left[\sum_{i=1}^{n} X_{i}\right] = \sum_{i=1}^{n} E\left[X_{i}\right]$$

$$= \sum_{i=1}^{n} 1/m$$

$$= n/m$$

let $\alpha=\frac{n}{m}$ 'load factor' expected number of items per bucket is α expected time is $\Theta(1+\alpha)$

More Collisions

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probability that k of n elements land in same of m bins: let $\alpha = \frac{n}{m}$ 'load factor'

$$\binom{n}{k} \left(\frac{1}{m}\right)^k \left(1 - \frac{1}{m}\right)^{n-k} \approx \frac{\alpha^k}{e^{\alpha}k!}$$

probability

Open Addressing

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- 1. linear probing: $h(k,i) = (h_1(k) + i) \mod m$ for increasing i
 - the runs
- 2. double hashing: $h(k,i) = (h_1(k) + ih_2(k)) \mod m$ for increasing i
 - requires: $h_2 \neq 0$, $h_2(k)$ and m relatively prime
 - \blacksquare eg, m prime and $h_2(k) < m$
 - \blacksquare or, $m=2^x$ and $h_2(k)$ odd
- 3. cuckoo hashing: lookups ${\cal O}(1)$, insertions amortized expected ${\cal O}(1)$

moral: low load factor

deletion?

Break

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- asst 2
- asst 3

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- $h: key \rightarrow 0..m 1$
- 1. mediocre is easy, good takes effort
- 2. want time (at most) linear in key size
- 3. perfect hashing is possible (and efficient) if keys known
 - linear time to construct, linear space to store
- 4. minimal perfect hashing is possible!

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- $h: key \rightarrow 0..m-1$
- 1. mediocre is easy, good takes effort
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 - linear time to construct, linear space to store
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bad news:

- \blacksquare if $|keys| \ge m$, there must be collisions
- \blacksquare if $|keys| \ge n \cdot m$, then \exists set of n that map to same bin

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Desiderata:

- make collisions unlikely
 - spread keys across all hashes
 - for each key, each hash equally likely
- similar keys get different hashes
 - all bits of key affect the hash
 - every bit of key affects every bit of hash
- no input always gives worst-case behavior
- fast to compute
- low memory requirement
- easy to implement

Basic Multiplicative Hashing

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- 1. $hash \leftarrow 0$
- 2. for each byte of key
- 3. $hash \leftarrow (hash \times multiplier) + byte$
- 5. return $hash \mod m$

want *multiplier* to smear bits, not shift them (to avoid interaction with table size)

multiplier = 31 or 127

Tabulation Hashing

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assume we have an array of 256 random integers

- 1. $hash \leftarrow 0$
- 2. for each byte of key
- 3. rotate the bits in *hash* by 1
- 4. $hash \leftarrow hash \times array[byte]$
- 5. return hash mod m

each byte affects all bits rotate makes order matter

universal class of hash functions : for randomly chosen keys, randomly chosen function from class has P(collision) = 1/m

good on average case (over inputs) \neq good average case on any input

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- Bloom Filters
- Merkle Trees
- Blockchain
- EOLQs

Related Data Structures

Bloom Filters

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set of n items using m bits with false positives set addition: hash element to bit index, set to true do this with k hash functions

no removal

for false positive rate ϵ :

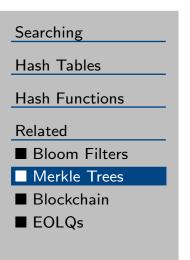
$$m/n = -\frac{\lg \epsilon}{\ln 2} \approx -1.44 \lg \epsilon$$
 and $k = -\lg \epsilon$

independent of n or m!

for 1% false positive rate:

- = 9.6 bits per elt
- = 7 hash functions

Merkle Trees



a leaf node represents a data block, contains its hash internal nodes contain hash of hashes of children to verify that node's data is part of n block tree, $\lg n$ hashes are required

Blockchain

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block contains hash of previous block, timestamp, transaction data

linked list version of Merkle tree

EOLQs

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- What's still confusing?
- What question didn't you get to ask today?
- What would you like to hear more about?

Please write down your most pressing question about algorithms and put it in the box on your way out.

Thanks!