NP-Completeness

SAT

`http://www.cs.unh.edu/~ruml/cs758`

NP-Completeness
- Terms
- Interchangability
- Reductions
- NPC Proofs
- C-SAT is in NP
- C-SAT is NP-Hard
- Break

SAT

NP-Completeness

Terms

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optimization vs decision: if opt were easy, decision would be too

P: solvable in polynomial time
NP: $\exists$ certificate verifiable in polynomial time
NP-Hard: as hard as any problem in NP (via polytime reduction)
NP-Complete: NP-Hard and in NP

reduce b to a: $b \rightarrow a$ in polytime $+$ decide a yields answer for b

a hard by reduction from b: if $b \rightarrow a$ in polytime and a were polytime, could solve b. so a must be hard!

The Power of Reduction

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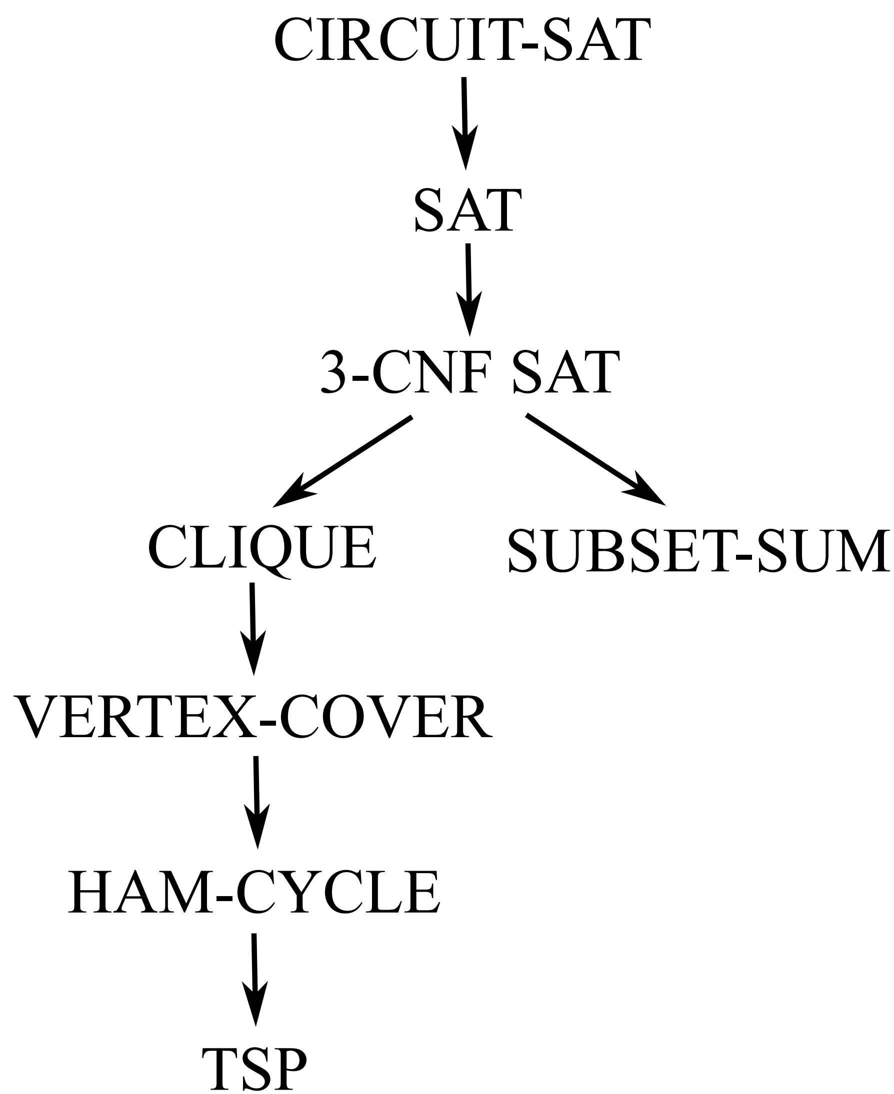
Theorem: If $B \leq_P A$ for some $B \in$ NPC, then A is NP-Hard.

Proof: Since B is NPC, we have $\forall C \in \text{NP}, C \leq_P B$. Since $B \leq_P A$, then $C \leq_P A$ which shows A is NP-Hard.

If also $A \in$ NP, then since $A \in$ NP, we have $A \in$ NPC.

Reductions


CIRCUIT-SAT
SAT
3-CNF SAT
CLIQUE
SUBSET-SUM
VERTEX-COVER
HAM-CYCLE
TSP

Framework for an NP-Completeness Proof

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To prove some problem A is NP-Complete:

1. Prove $A \in NP$
2. Pick a known NP-Complete problem B
3. Find a translation of instances of B into instances of A
4. Show that translated A version is accepted if and only if the original B version should be accepted.
5. Prove that the reduction runs in polynomial time.

Circuit-SAT is in NP

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Circuit-SAT: is circuit satisfiable? (otherwise, can be removed)

Certificate is value for every wire.
Verify that each gate is computed corrrectly and output is true.

Circuit-SAT is NP-Hard

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Need to construct reduction from any $L \in$ NP. Given input $x \in L$, resulting circuit $C \in$ Circuit-SAT iff $x \in L$. We'll make C so it's SAT iff $\exists y$ s.t. verification algorithm $A(x, y)$ for L gives true. Intuition: for input y, run $A(x, y)$.
Let $n = |x|$ and $T(n) = O(n^k)$ be bound on A's running time.
Let M be a circuit for a stored-program computer (including PC and storage). String $T(n)$ of them together to form C'.
C is C' with input hardwired to program for A and input x, and output hardwired to result of A. Input to C is y.

Iff y exists, C is satisfiable, so we have a reduction.
A is constant size and uses poly storage. M is poly size and needs poly steps to run A. y is poly sized. So C' and C have size polynomial in n and can be constructed in polynomial time.

Break

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SAT

- asst 12
- Wildcard Vote!

NP-Completeness

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- NPC Proofs
- Reduction
- 3-CNF SAT
- Reductions
- EOLQs

SAT

Framework for an NP-Completeness Proof

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Reduction from Circuit-SAT to SAT

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Consider formula with n variables and m connectives.

SAT $\in$ NP: given variables assignments, evaluate formula.

SAT is NP-Hard: Reduction from Circuit-SAT. Basic translation fails on shared subcircuits.
Instead, use one variable for each wire and one clause per gate.
Combine clauses with $\wedge$ and include $\wedge\ x_0$ (output).
SAT iff wires in circuit have legal values yielding true.

3-CNF SAT

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CNF where each clause has exactly 3 literals. Aka 3-SAT.

3-CNF SAT $\in$ NP: given variables assignments, evaluate formula.

3-CNF SAT is NP-Hard: Reduction from SAT. Construct expression tree and convert to binary branching.
Assign each node a variable.
Form clause for each internal node's variable, eg: $y_3 \leftrightarrow (y_1 \vee y_2)$
Clauses will have at most 3 literals.
Convert each clause to CNF: form complete truth table, form DNF for false rows, negate and push $\neg$ inward (using DeMorgan) to get CNF
For each binary clause $(l_1 \vee l_2)$, convert to $(l_1 \vee l_2 \vee p) \wedge (l_1 \vee l_2 \vee \neg p)$.
For each unit clause (l), convert to $(l \vee p \vee q) \wedge (l \vee p \vee \neg q) \wedge (l \vee \neg p \vee q) \wedge (l \vee \neg p \vee \neg q)$.
Each step preserves satisfiability and is polynomial time.

Reductions

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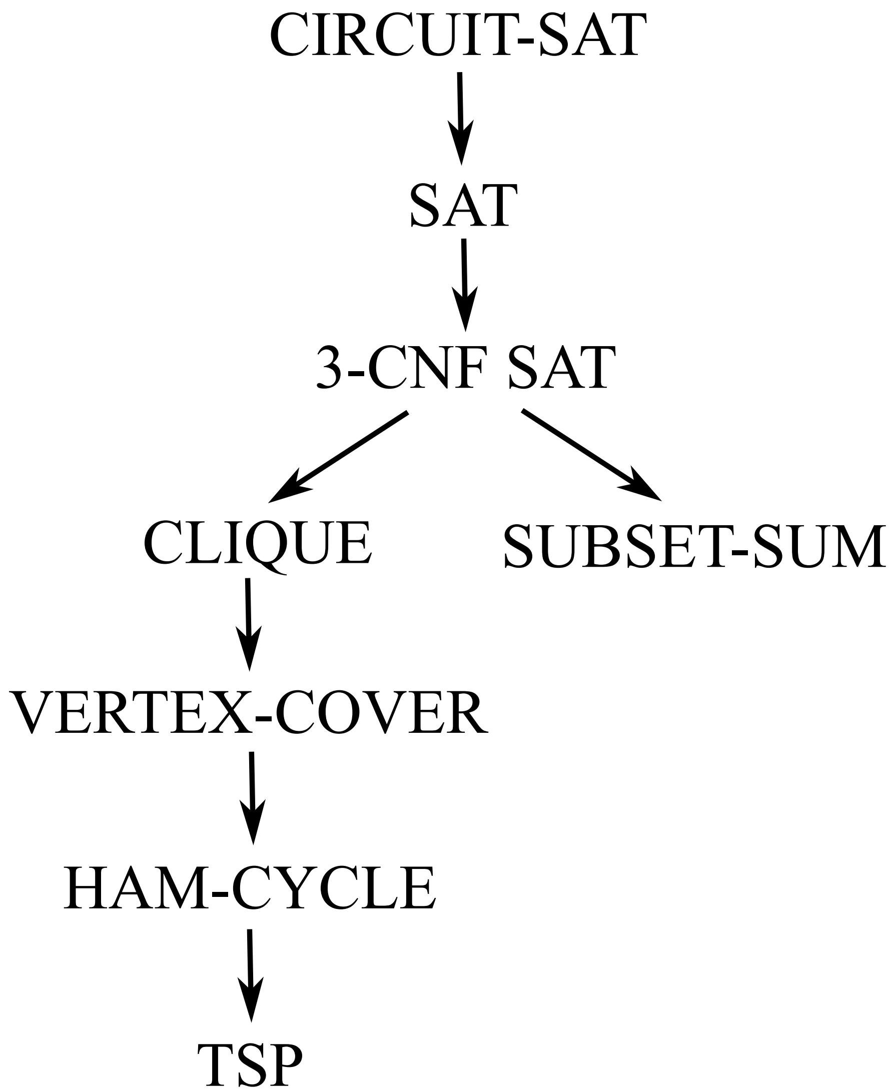

EOLQs

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For example:

- What's still confusing?
- What question didn't you get to ask today?
- What would you like to hear more about?

Please write down your most pressing question about algorithms and put it in the box on your way out.
Thanks!