CS 758/858: Algorithms

Applications

LPs

http://www.cs.unh.edu/~ruml/cs758

Applications

- Max-Flow Thm
- **■** Segmentation
- Matching
- Scheduling
- Break

LPs

Applications of Cuts and Flows

Max-Flow Min-Cut Theorem

Applications

■ Max-Flow Thm

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LPs

value of a flow = flow across any cut

any flow value \leq capacity of cut

Theorem: these are the same:

- 1. f is a maximum flow
- 2. the residual network G_f contains no augmenting paths
- 3. there exists a cut whose capacity is the value of f

1=2: FF is correct; 1=3: FF also finds minimum cuts

Image Segmentation

Applications

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LPs

an image as a graph!

maximize

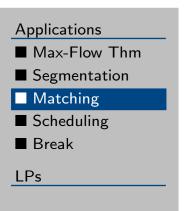
$$\sum_{i \in A} a_i + \sum_{i \in B} b_i - \sum_{e \ cutby \ A} p_{i,j}$$

minimize

$$\sum_{i \in A} b_i + \sum_{i \in B} a_i + \sum_{e \ cutby \ A} p_{i,j}$$

cut crosses three types of edges: s_i , t_i , and $p_{i,j}$

Maximum Matching



bipartite graphs: jobs/machines, classes/instructors, . . .

Maximum Matching

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bipartite graphs: jobs/machines, classes/instructors, . . .

unit capacities

flow = matching

FF guarantees integer flow

running time? (hint: bound $|f^*|$)

Scheduling

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LPs

does a feasible schedule exist using only 3 machines (allowing preemption)?

job	1	2	3	4
processing time	1.5	1.25	2.1	3.6
release date	3	1	3	5
due date	5	4	7	9

Scheduling

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LPs

does a feasible schedule exist using only 3 machines (allowing preemption)?

job	1	2	3	4
processing time	1.5	1.25	2.1	3.6
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due date	5	4	7	9

arcs from s to jobs labeled with job size

intervals: 1-3-4-5-7-9

arcs from job to feasible intervals labeled with length of interval

arcs from interval to t labeled with total achievable work (nummachines times length of interval)

Break

Applications

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- asst 11
- permissible research

Applications LPs LPs Examples Example Example Example Example Descriptions

Linear Programming

Linear Programming

Applications

LPs

■ LPs

- **■** Examples
- **■** Example
- **■** Example
- **■** Example
- **■** Example
- Beyond LPs
- **■** Others
- EOLQs

real variables, linear constraints, linear objective

eg, cheapest diet that meets nutrition guidelines

- \blacksquare decision variables: $x_{broccoli}$, $x_{carrots}$, ...
- \blacksquare minimize: $cost = 4.99x_{broccoli} + 2.67x_{carrots} \dots$
- subject to: $500 < y_{vitaminA} = 2.3x_{broccoli} + 1.7x_{carrots} \dots$
- $300 < y_{vitaminC} = 1.7x_{broccoli} + 5.2x_{carrots} \dots$

polynomial time (ellipsoid, Karmarkar's), but simplex method is popular

CPLEX, Gurobi, Ipsolve

Example LPs

Applications

LPs

LPs

Examples

Example

Example

Example

Example

Example

Others

Others

EOLQs

- shortest paths
- max flow obeying capacities
- min-cost flow meeting demand
- multicommodity flow meeting demands
- earliest finish time subject to job durations

Example: Max Flow

Applications

LPs

■ LPs

■ Examples

Example

■ Example

■ Example

■ Example

■ Beyond LPs

■ Others

■ EOLQs

maximize $\sum_{v} f(s, v) - \sum_{v} f(v, s)$ subject to

$$f(u,v) \geq 0 \text{ for every edge } (u,v)$$

$$\sum_{v} f(u,v) = \sum_{v} f(v,u) \text{ for every vertex } u$$

$$f(u,v) \leq c(u,v) \text{ for every edge } (u,v)$$

Example: Shortest Path

Applications

LPs

- LPs
- **■** Examples
- **■** Example
- Example
- **■** Example
- **■** Example
- Beyond LPs
- Others
- EOLQs

maximize d_t subject to

$$d_v \leq d_u + w(u, v)$$
 for every edge (u, v)
 $d_s = 0$

Min-cost Flow Meeting Demand

Applications

LPs

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- **■** Example
- **■** Example
- Example
- **■** Example
- Beyond LPs
- **■** Others
- **■** EOLQs

minimize $\sum_{(u,v)} a(u,v) f(u,v)$ subject to

$$f(u,v) \geq 0 \text{ for every edge } (u,v)$$

$$\sum_v f(u,v) = \sum_v f(v,u) \text{ for every vertex } u$$

$$f(u,v) \leq c(u,v) \text{ for every edge } (u,v)$$

$$\sum_v f(s,v) - \sum_v f(v,s) = d$$

Multicommodity Flow

Applications

LPs

- LPs
- **■** Examples
- **■** Example
- **■** Example
- **■** Example
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minimize 0 subject to

$$\sum_{v} f_i(u, v) - \sum_{v} f_i(v, u) = 0$$

$$f_i(u, v) \geq 0$$

$$\sum_{i=1}^k f_i(u, v) \leq c(u, v)$$

$$\sum_{v} f_i(s_i, v) - \sum_{v} f_i(v, s_i) = d_i$$

Beyond Linear Programming

Applications

LPs

LPs

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Example

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Example

Example

Example

Beyond LPs

■ Others

■ EOLQs

convex programming: constraints and objective are convex polynomial time

quadratic programming: constraints and objective are quadratic

some forms are polynomial time

integer linear programming: integer variables, linear constraints and objective

NP-complete

0-1 ILP: 0-1 variables, linear constraints, linear objective (BIP) NP-complete

combinatorial optimization: variables are discrete

Others

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Others
EOLQs

selection

multicommodity flow is NP-hard for integer flows. Use LP for fractional flows.

EOLQs

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Example

Example

Example

Example

Example

Others

EOLQs

For example:

- What's still confusing?
- What question didn't you get to ask today?
- What would you like to hear more about?

Please write down your most pressing question about algorithms and put it in the box on your way out.

Thanks!