Assignment 14: Algorithm Design
CS 758/858, Fall 2024
Due at **11:30pm** on **Wed Dec 4**

Written Problems

Solve the following problems. When you are asked to 'give' an algorithm, please be sure to prove that it is correct and exhibits the desired running time!

1. You are an usher in a theater with n balconies for a children's matinee performance with n children in attendance. You are given a list of m statements of the form "i hates j." If i hates j, then you do not want to seat i above or in the same balcony as j, otherwise i will throw popcorn at j instead of watching the play. Give an algorithm that assigns balconies to children (or determines that no feasible assignment exists) in time $O(m + n)$. We can assume that balconies are numbered ascending with height.

2. Let's assume that UNH IT has a pretty good idea of where people like to use their laptops' WiFi. Given a set of n points on campus representing demand for WiFi bandwidth and a set of b points representing some proposed wireless access point locations, each with a fixed total bandwidth T (in number of users) and radio range R, give an algorithm (polynomial in n and b) for determining whether this proposed access point layout will satisfy all the users.

3. You are consulting for a cell phone company in Arizona. Their territory includes a long straight road that has some houses at known locations along it. The company wants to build the minimum number of cell towers such that every house is within four miles of a tower. Give an efficient algorithm to place the towers and prove that it finds optimal solutions.

4. Give an $O(n)$ algorithm to determine the laziest way to dial an n-digit number on a telephone keypad using two fingers. The two fingers start on the * and # keys, respectively. You can assume that the effort required to move a finger from one button to another is proportional to the Euclidean distance between them.

5. It is midnight and pitch black, but you are holding a dim lighted candle. You are facing a high wall that stretches infinitely in both directions. There is a door in the wall, but you don't know where. The candlelight is only sufficient to enable you to see the door when you are right in front of it. Give an algorithm that will enable you to find the door in $O(n)$ steps, where n is the number of steps that you would take if you knew the location of the door and walked directly to it.

6. (Those in 858 only) The target value search problem is: given two vertices s and t in a graph G and a number k, find a path from s to t in G with cost k. Either give an efficient algorithm for this problem or prove that it is NP-complete.

7. What suggestions do you have for improving this assignment in the future?

Submission

Submit your work electronically as usual.