1 handout: slides
730W journal entries were due

First-order Logic

Logic

A logic is a formal system:

- syntax: defines sentences
- semantics: relation to world
- inference rules: reaching new conclusions

three layers: proof, models, reality

flexible, general, and principled form of KR

1. Things:
   - constants: *John*, *Chair23*
   - functions (thing $\rightarrow$ thing): *MotherOf(John)*, *SumOf(1,2)*
2. Relations:
   - predicates (objects $\rightarrow$ T/F): *IsWet(John)*, *IsSittingOn(MotherOf(John),chair23)*
3. Complex sentences:
   - connectives: *IsWet(John)* $\vee$ *IsSittingOn(MotherOf(John),Chair23)*
   - quantifiers and variables: $\forall person \ldots$, $\exists person \ldots$

More First-Order Logic

$$\begin{aligned}\forall person\ \forall time \quad & (ItIsRaining(time) \wedge \\ & \neg\exists umbrella\ \ Holding(person, umbrella, time)) \rightarrow \\ & IsWet(person, time)\end{aligned}$$

John loves Mary.

All crows are black.

Dolphin are mammals that live in the water.

Everyone loves someone.

Mary likes the color of one of John's ties.

I can't hold more than one thing at a time.

The Joy of Power

1. Indirect knowledge: *Tall(MotherOf(John))*
2. Counterfactuals: $\neg Tall(John)$
3. Partial knowledge (disjunction):
   $IsSisterOf(b, a) \vee IsSisterOf(c, a)$
4. Partial knowledge (indefiniteness): $\exists x IsSisterOf(x, a)$

Reasoning in First-order Logic

Clausal Form

1. Eliminate $\rightarrow$ using $\neg$ and $\vee$
2. Push $\neg$ inward using de Morgan's laws
3. Standardize variables apart
4. Eliminate $\exists$ using Skolem functions
5. Move $\forall$ to front
6. Move all $\wedge$ outside any $\vee$ (CNF)
7. Can finally remove $\forall$ and $\wedge$

Example

1. Cats like fish.
2. Cats eat everything they like.
3. Joe is a cat.

Prove: Joe eats fish.

Break

- asst 1
- asst 2
- office hours

Unifying Two Terms

1. if one is a constant and the other is
2. a constant: if the same, done; else, fail
3. a function: fail
4. a variable: **substitute** *constant* for *var*
5. if one is a function and the other is
6. a different function: fail
7. the same function: unify the two arguments lists
8. a variable: if *var* occurs in *function*, fail
9. otherwise, **substitute** *function* for *var*
10. otherwise, **substitute** one variable for the other

Carry out substitutions on all expressions you are unifying!
Build up substitutions as you go, carrying them out before checking expressions?
See handout on website.

Example

1. Anyone who can read is literate.
2. Dolphins are not literate.
3. Some dolphins are intelligent.
4. Prove: someone intelligent cannot read.

Skolem, standardizing apart

Models

A *model* is:

Propositional: a truth assignment for symbols. Exponential number of models.

First-order: a set of objects and an interpretation for constants, functions, and predicates (fixing referent of every term). Unbounded number of models.

No unique names assumption: constants not distinct.
No closed world assumption: unknown facts not false.

α valid iff true in every model
$\alpha \models \beta$ iff β true in every model of α

FOL is semi-decidable: if entailed, will eventually know

The Basis for Refutation

Recall $\alpha \models \beta$ iff β true in every model of α.

1. Assume KB $\models \alpha$.
2. So if a model i satisfies KB, then i satisfies α.
3. If i satisfies α, then doesn't satisfy $\neg\alpha$.
4. So no model satisfies KB and $\neg\alpha$.
5. So KB $\wedge\neg\alpha$ is unsatisfiable.

The other way:

1. Suppose no model that satisfies KB also satisfies $\neg\alpha$. In other words, KB $\wedge\neg\alpha$ is unsatisfiable ($=$ inconsistent $=$ contradictory).
2. In every model of KB, α must be true or false.
3. Since in any model of KB, $\neg\alpha$ is false, α must be true in all models of KB.

Completeness

Gödel's Completeness Theorem (1930) says a complete set of inference rules exists for FOL.

Herbrand base: substitute all constants and combinations of constants and functions in place of variables. Potentially infinite!

Herbrand's Theorem (1930): If a set of clauses S is unsatisfiable, then there exists a finite subset of its Herbrand base that is also unsatisfiable.

Ground Resolution Thm: If a set of ground clauses is unsatisfiable, then the resolution closure of those clauses contains $\perp$.

Robinson (1965): If there is a proof on ground clauses, there is a corresponding proof in the original clauses.

EOLQs

Please write down the most pressing question you have about the course material covered so far and put it in the box on your way out.

Thanks!