Bayesian Networks
Approx. Inference
Exact Inference

Bayesian Networks
- Models
- Example
- The Joint
- Independence
- Example
- Break

Approx. Inference

Exact Inference

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Approx. Inference

Exact Inference

MDPs:
Naive Bayes:
k-**Means:**

Representation: variables, connectives
Inference: approximate, exact

The Alarm Domain

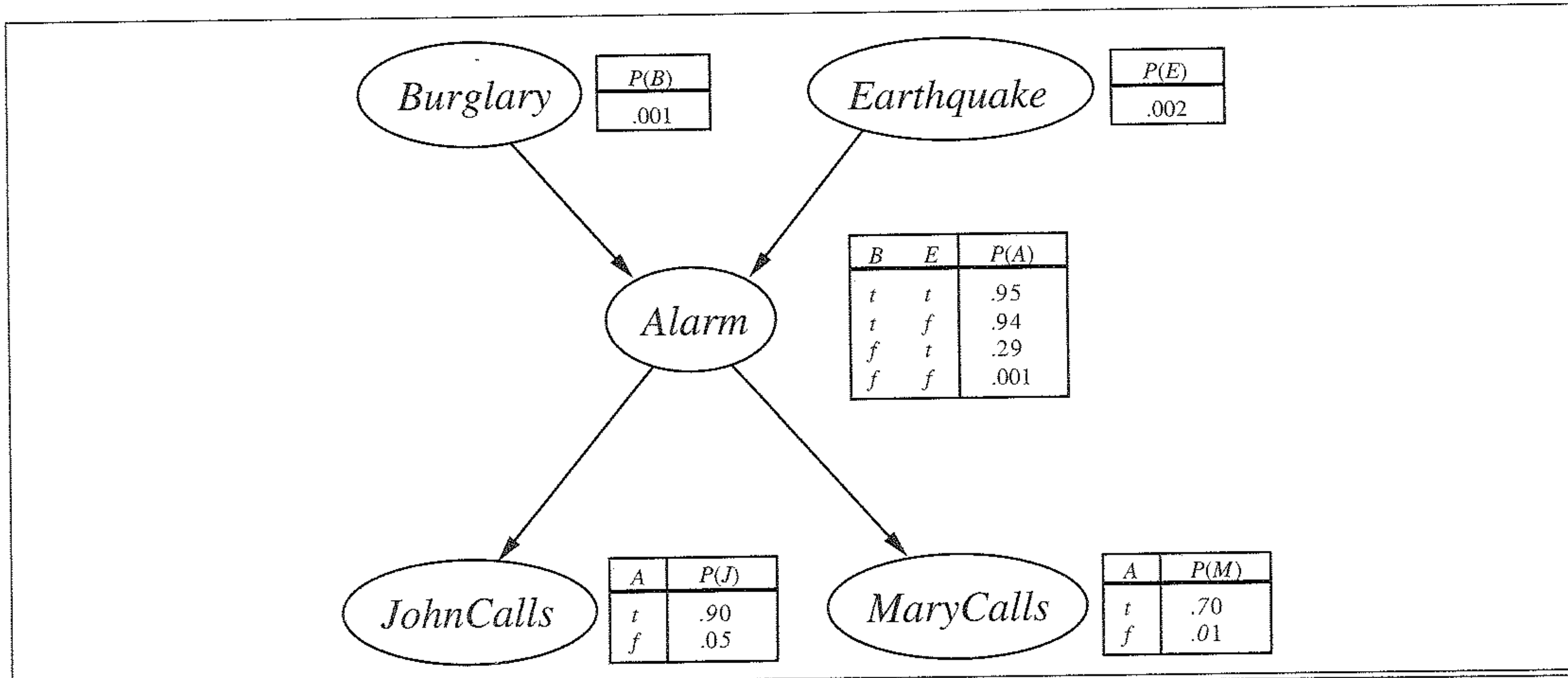

Figure 14.2 A typical Bayesian network, showing both the topology and the conditional probability tables (CPTs). In the CPTs, the letters B, E, A, J, and M stand for *Burglary*, *Earthquake*, *Alarm*, *JohnCalls*, and *MaryCalls*, respectively.

The Full Joint Distribution

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Approx. Inference

Exact Inference

ultimate power: knowing the probability of every possible atomic event (combination of values)

The Full Joint Distribution

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Approx. Inference

Exact Inference

ultimate power: knowing the probability of every possible atomic event (combination of values)

simple inference via enumeration over the joint:

what is distribution of X given evidence e and unobserved Y

$$P(X|e) = \frac{P(e|X)P(X)}{P(e)} = \alpha P(X,e) = \alpha \sum_y P(X,e,y)$$

Bayes Net $=$ joint probability distribution

The Magic of Independence

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Approx. Inference

Exact Inference

In general:

$$P(x_1, \ldots, x_n) = P(x_n | x_{n-1}, \ldots, x_1) P(x_{n-1}, \ldots, x_1)$$

The Magic of Independence

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Approx. Inference

Exact Inference

In general:

$$
\begin{aligned}
P(x_1, \ldots, x_n) &= P(x_n | x_{n-1}, \ldots, x_1) P(x_{n-1}, \ldots, x_1) \\
&= \prod_{i=1}^{n} P(x_i | x_{i-1}, \ldots, x_1)
\end{aligned}
$$

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$$\begin{aligned} P(x_1, \ldots, x_n) &= P(x_n | x_{n-1}, \ldots, x_1) P(x_{n-1}, \ldots, x_1) \\ &= \prod_{i=1}^{n} P(x_i | x_{i-1}, \ldots, x_1) \end{aligned}$$

A Bayesian net specifies independence:

$$P(X_i | X_{i-1}, \ldots, X_1) = P(X_i | parents(X_i))$$

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In general:

$$\begin{aligned} P(x_1, \ldots, x_n) &= P(x_n | x_{n-1}, \ldots, x_1) P(x_{n-1}, \ldots, x_1) \\ &= \prod_{i=1}^{n} P(x_i | x_{i-1}, \ldots, x_1) \end{aligned}$$

A Bayesian net specifies independence:

$$P(X_i | X_{i-1}, \ldots, X_1) = P(X_i | parents(X_i))$$

So joint distribution can be computed as

$$P(x_1, \ldots, x_n) = \prod_{i=1}^{n} P(x_i | parents(X_i))$$

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Approx. Inference

Exact Inference

In general:

$$\begin{aligned} P(x_1,\ldots,x_n) &= P(x_n|x_{n-1},\ldots,x_1)P(x_{n-1},\ldots,x_1) \\ &= \prod_{i=1}^{n} P(x_i|x_{i-1},\ldots,x_1) \end{aligned}$$

A Bayesian net specifies independence:

$$P(X_i|X_{i-1},\ldots,X_1) = P(X_i|parents(X_i))$$

So joint distribution can be computed as

$$P(x_1,\ldots,x_n) = \prod_{i=1}^{n} P(x_i|parents(X_i))$$

For n b-ary variables with p parents, that's nb^p instead of b^n!

Example

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Break

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Approx. Inference

Exact Inference

- asst 12
- project

Approximate Inference

Rejection Sampling

Bayesian Networks
Approx. Inference
Sampling
Likelihood Wting
Exact Inference

What is distribution of X given evidence e and unobserved Y?

Draw worlds from the joint, rejecting those that do not match e.
Look at distribution of X.

sample values for variables, working top down
directly implements the semantics of the network
'generative model'
each sample is linear time, but overall slow if e is unlikely

Bayesian Networks
Approx. Inference
■ Sampling
■ Likelihood Wting
Exact Inference

Likelihood Weighting

What is distribution of X given evidence e and unobserved Y?

ChooseSample (e)
$w \leftarrow 1$
for each variable V_i in topological order:
  if $(V_i = v_i) \in e$ then
    $w \leftarrow w \cdot P(v_i|parents(v_i))$
  else
    $v_i \leftarrow$ sample from $P(V_i|parents(V_i))$

(afterwards, normalize samples so all w's sum to 1)

uses all samples, but needs lots of samples if e are late in ordering

Bayesian Networks
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- Enumeration
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- Var. Elim. 1
- Var. Elim. 2
- EOLQs

Exact Inference in Bayesian Networks

Enumeration Over the Joint Distribution

Bayesian Networks

Approx. Inference

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What is distribution of X given evidence e and unobserved Y?

$$
\begin{aligned}
P(X|e) &= \frac{P(e|X)P(X)}{P(e)} \\
&= \alpha P(X,e) \\
&= \alpha \sum_y P(X,e,y) \\
&= \alpha \sum_y \prod_{i=1}^{n} P(V_i|parents(V_i))
\end{aligned}
$$

Example

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$$\begin{aligned}
P(B|j,m) &= \frac{P(j,m|B)P(B)}{P(j,m)} \\
&= \alpha P(B,j,m) \\
&= \alpha \sum_e \sum_a P(B,e,a,j,m) \\
&= \alpha \sum_e \sum_a \prod_{i=1}^{n} P(V_i|parents(V_i)) \\
P(b|j,m) &= \alpha \sum_e \sum_a P(b)P(e)P(a|b,e)P(j|a)P(m|a) \\
&= \alpha P(b) \sum_e P(e) \sum_a P(a|b,e)P(j|a)P(m|a)
\end{aligned}$$

[draw tree]

Variable Elimination

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$$P(B|j,m) \quad = \quad \alpha P(B) \sum_e P(e) \sum_a P(a|B,e)P(j|a)P(m|a)$$

factors = tables = $f_{varsused}(dimensions)$.
eg: $f_A(A,B,E)$, $f_M(A)$
multiplying factors: table with union of variables
summing reduces table

Variable Elimination

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eliminating variables: eg $P(J|b)$

$$P(J|b) = \alpha P(b) \sum_e P(e) \sum_a P(a|b,e) P(J|a) \sum_m P(m|a)$$

all vars not ancestor of query or evidence are irrelevant!

EOLQs

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- What question didn't you get to ask today?
- What's still confusing?
- What would you like to hear more about?

Please write down your most pressing question about AI and put it in the box on your way out.
Thanks!