# First-order Logic

# Logic



A logic is a formal system:

- syntax: defines sentences
- semantics: relation to world
- inference rules: reaching new conclusions

three layers: proof, models, reality

flexible, general, and principled form of KR

# First-Order Logic



1. Things:
   - constants: *John*, *Chair23*
   - functions (thing $\rightarrow$ thing): *MotherOf(John)*, *SumOf(1,2)*
2. Relations:
   - predicates (objects $\rightarrow$ T/F): *IsWet(John)*, *IsSittingOn(MotherOf(John),Chair23)*
3. Complex sentences:
   - connectives: *IsWet(John)* $\vee$ *IsSittingOn(MotherOf(John),Chair23)*
   - quantifiers and variables: $\forall person \ldots$, $\exists person \ldots$

# More First-Order Logic

$$\forall person\ \forall time\quad (ItIsRaining(time) \wedge \neg\exists umbrella\ Holding(person, umbrella, time)) \rightarrow IsWet(person, time)$$

John loves Mary.

All crows are black.

Dolphin are mammals that live in the water.

Everyone loves someone.

Mary likes the color of one of John's ties.

I can't hold more than one thing at a time.

# The Joy of Power



1. Indirect knowledge: *Tall(MotherOf(John))*
2. Counterfactuals: $\neg Tall(John)$
3. Partial knowledge (disjunction):
   $IsSisterOf(b, a) \vee IsSisterOf(c, a)$
4. Partial knowledge (indefiniteness): $\exists x IsSisterOf(x, a)$

# Reasoning in First-order Logic

# Example



1. Cats like fish.
2. Cats eat everything they like.
3. Joe is a cat.

Prove: Joe eats fish.

# Clausal Form



1. Eliminate $\rightarrow$ using $\neg$ and $\vee$
2. Push $\neg$ inward using de Morgan's laws
3. Standardize variables apart
4. Eliminate $\exists$ using Skolem functions
5. Move $\forall$ to front
6. Move all $\wedge$ outside any $\vee$ (CNF)
7. Can finally remove $\forall$ and $\wedge$

# Break



- asst 5
- asst 6, 7
- project idea sharing next Thursday
- preliminary project proposals due Tue Mar 11 in class
- final project proposals due Tue Apr 1 in class

# Unifying Two Terms



1. if one is a constant and the other is
2.     a constant: if the same, done; else, fail
3.     a function: fail
4.     a variable: **substitute** *constant* for *var*
5. if one is a function and the other is
6.     a different function: fail
7.     the same function: unify the two arguments lists
8.     a variable: if *var* occurs in *function*, fail
9.         otherwise, **substitute** *function* for *var*
10. otherwise, **substitute** one variable for the other

Carry out substitutions on all expressions you are unifying!
Build up substitutions as you go, carrying them out before checking expressions?
See handout on website.

# Example



1. Anyone who can read is literate.
2. Dolphins are not literate.
3. Some dolphins are intelligent.
4. Prove: someone intelligent cannot read.

Skolem, standardizing apart

# Tricky Cases



don't unify $x$ and $f(x)$!
as in $P(x, x)$ meets $\neg P(z, f(z))$

note resolvent of $P(f(x))$ and $\neg P(z) \vee P(f(z))$

Semi-decidable: if yes, will terminate

# The Basis for Refutation



Recall KB $\models \alpha$ iff $\alpha$ true in every model of KB.

1. Assume KB $\models \alpha$.
2. So if a model $i$ satisfies KB, then $i$ satisfies $\alpha$.
3. If $i$ satisfies $\alpha$, then doesn't satisfy $\neg\alpha$.
4. So no model satisfies KB and $\neg\alpha$.
5. So KB $\wedge\neg\alpha$ is unsatisfiable.

The other way:

1. Suppose no model that satisfies KB also satisfies $\neg\alpha$. In other words, KB $\wedge\neg\alpha$ is unsatisfiable ($=$ inconsistent $=$ contradictory).
2. In every model of KB, $\alpha$ must be true or false.
3. Since in any model of KB, $\neg\alpha$ is false, $\alpha$ must be true in all models of KB.

# Completeness



Gödel's Completeness Theorem (1930) says a complete set of inference rules exists for FOL.

Herbrand base: substitute all constants and combinations of constants and functions in place of variables. Potentially infinite!

Herbrand's Theorem (1930): If a set of clauses $S$ is unsatisfiable, then there exists a finite subset of its Herbrand base that is also unsatisfiable.

Ground Resolution Thm: If a set of ground clauses is unsatisfiable, then the resolution closure of those clauses contains $\perp$.

Robinson (1965): If there is a proof on ground clauses, there is a corresponding proof in the original clauses.

FOL is semi-decidable: if entailed, will eventually know

# EOLQs



Please write down the most pressing question you have about the course material covered so far and put it in the box on your way out.

*Thanks!*