CS 730/830: Intro AI

Propositional Logic

First-Order Logic

1 handout: slides

Propositional Logic

Logic

A logic is a formal system:

■ syntax: defines sentences

■ semantics: relation to world

■ inference rules: reaching new conclusions

three layers: proof, models, reality

soundness, completeness

flexible, general, principled (Advice Taker, 1958)

Propositional Reasoning

computing entailment

soundness, completeness

$\alpha \models \beta$ iff $\alpha \wedge \neg\beta$ is unsatisfiable

determining satisfiability is NP-complete

[NP-hard = polytime to verify certificate of 'yes']

therefore, verification that β is not entailed is polytime

said another way:

$\alpha \models \beta$ iff $\alpha \rightarrow \beta$ is valid

determining validity/tautology is co-NP-complete

[co-NP-hard = polytime to verify certificate of 'no']

therefore, verification that β is not entailed is polytime

Reasoning Methods

- variable elimination: Davis-Logemann-Loveland

    exhaustively branch on variable assignments

- model finding: WalkSAT

    tweak assignment until satisfying

- modus ponens, resolution: resolution refutation theorem proving

    derive new clauses until query is proved

An Example of Propositional Reasoning

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Prove: the unicorn is magical.

Resolution Refutation Proofs

Given KB, is α entailed?

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(Is it true in all models of the KB?)

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Is KB $\wedge \neg \alpha$ unsatisfiable?

Resolution Refutation Proofs

Given KB, is α entailed?
(Is it true in all models of the KB?)
Is KB $\wedge \neg\alpha$ unsatisfiable?

Resolution is refutation complete.

Conversion to Conjunctive Normal Form

Syntax: $\wedge, \vee, \neg, \rightarrow (\supset, \Rightarrow), \leftrightarrow$

1. eliminate $\leftrightarrow$
2. eliminate $\rightarrow$
3. move $\neg$ inward: $\neg\neg x$, $\neg(x \wedge y)$, , $\neg(x \vee y)$
4. distribute $\vee$: $x \vee (y \wedge z)$

Break

- asst 5

- asst 6

- projects: share thoughts next week

First-Order Logic

First-Order Logic

Gottlob Frege (1848-1925)

PhD at 25

Begriffsschrift, 1879 (concept script)
"a formula language, modelled on that of arithmetic, of pure thought."

First-Order Logic

$$\forall person\ ItIsRaining() \rightarrow IsWet(person)$$

1. Things:

   - constants: *John*, *Chair23*
   - functions (thing → thing): *MotherOf(John)*, *SumOf(1,2)*

2. Relations:

   - predicates (objects → T/F): *IsWet(John)*, *IsSittingOn(MotherOf(John),Chair23)*

3. Complex sentences:

   - connectives: *IsWet(John)* ∨ *IsSittingOn(MotherOf(John),Chair23)*
   - quantifiers and variables: $\forall person IsWet(person)...$, $\exists person...$

First-Order Logic

1. constants: objects
2. predicates: relations between objects
3. variables
4. quantifiers
5. functions
6. connectives

$$\forall person\ \forall time\quad (ItIsRaining(time) \wedge$$
$$\neg \exists umbrella\ Holding(person, umbrella, time)) \rightarrow$$
$$IsWet(person, time)$$

John loves Mary.

All crows are black.

Dolphin are mammals that live in the water.

Everyone loves someone.

Mary likes the color of one of John's ties.

I can't hold more than one thing at a time.

EOLQs

Please write down the most pressing question you have about the course material covered so far and put it in the box on your way out.

Thanks!