"Spock had a big, big effect on me. I am so much more Spock-like today than when I first played the part in 1965 that you wouldn't recognize me. I'm not talking about appearance, but thought processes. Doing that character, I learned so much about rational logical thought that it reshaped my life."

– Leonard Nimoy

1 handout: slides asst 2 milestone was due

First-Order Inference

Clausal Form

1. Eliminate $\rightarrow$ using $\neg$ and $\vee$
2. Push $\neg$ inward using de Morgan's laws
3. Standardize variables apart
4. Eliminate $\exists$ using Skolem functions
5. Move $\forall$ to front
6. Move all $\wedge$ outside any $\vee$ (CNF)
7. Can finally remove $\forall$ and $\wedge$

Example

1. Anyone who can read is literate.
2. Dolphins are not literate.
3. Some dolphins are intelligent.
4. Prove: someone intelligent cannot read.

Skolem, standardizing apart

Unifying Two Terms

1. if one is a constant and the other is
2.   a constant: if the same, done; else, fail
3.   a function: fail
4.   a variable: **substitute** *constant* for *var*
5. if one is a function and the other is
6.   a different function: fail
7.   the same function: unify the two arguments lists
8.   a variable: if *var* occurs in *function*, fail
9.   otherwise, **substitute** *function* for *var*
10. otherwise, **substitute** one variable for the other

Carry out substitutions on all expressions you are unifying!
Build up substitutions as you go, carrying them out before checking expressions?
See handout on website.

Tricky Cases

don't unify x and $f(x)$!
as in $P(x, x)$ meets $\neg P(z, f(z))$

note resolvent of $P(f(x))$ and $\neg P(z) \vee P(f(z))$

Semi-decidable: if yes, will terminate

Completeness

Gödel's Completeness Theorem (1930) says a complete set of inference rules exists for FOL.

Herbrand base: substitute all constants and combinations of constants and functions in place of variables. Potentially infinite!

Herbrand's Theorem (1930): If a set of clauses is unsatisfiable, then there exists a finite subset of the Herbrand base that is also unsatisfiable.

Ground Resolution Theorem: If a set of ground clauses is unsatisfiable, then the resolution closure of those clauses contains $\perp$.

Robinson's Lifting Lemma (1965): If there is a proof on ground clauses, there is a corresponding proof in the original clauses.

Break

- asst 2
- office hours, final projects

Equality

Equality: $\forall xy\,(Holding(x) \wedge \neg(x = y) \rightarrow \neg Holding(y))$

Unique: $\exists! x P(x) \equiv \exists x\,(P(x) \wedge \forall y(\neg(x = y) \rightarrow \neg P(y)))$

Specific Answers

Use the "answer literal":

1. FatherOf(Alice, Bob)
2. FatherOf(Caroline, Bob)
3. FatherOf(x, y) $\rightarrow$ ParentOf(x, y)

Query: Who is Caroline's parent?

Resolution Strategies

Breadth-first: all first-level resolvents, then second-level...

- Complete, slow

Set of Support: at least one parent comes from SoS

- Complete if non-SoS are satisfiable, nice

Input Resolution: at least one parent from the input set

- Complete for Horn KBs

Simplifications: remove tautologies, subsumbed clauses, and pure literals.

Terminology

Interpretation: maps constant symbols to objects in the world, each function symbol to a particular function on objects, and each predicate symbol to a particular relation.

Model of P: an interpretation in which P is true. Eg, *Famous(BarbaraBush)* is true under the intended interpretation but not when the symbol *BarbaraBush* maps to Joe Shmoe.

Satisfiable: $\exists$ a model for P. Eg, $P \wedge \neg P$ is not satisfiable.

Entailment: if Q is true in every model of P, then $P \models Q$. Eg, $P \wedge Q \models P$.

Valid: true in any interpretation. Eg, $P \vee \neg P$.

Logic in Practice

Natural Deduction

1. given $\exists$, can introduce new constant
2. given sentence with ground expresion, can introduce $\exists$
3. given $\forall$, can introduce new constant
4. given sentence, can introduce $\forall$ over new free variable

$\wedge$ **elimination/introduction:**
$\vee$ **introduction:**
$\neg\neg$ **elimination:**

Inference Rules

Modus Ponens:

Resolution:

Abduction:

Induction:

maathematical induction $\neq$ inductive reasoning

EOLQs

Please write down the most pressing question you have about the course material covered so far and put it in the box on your way out.

Thanks!