Probabilistic Machine Learning

Bayesian Nets, MCMC, and more

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4/18/2017

Based on: P. Murphy, K. (2012). Machine Learning: A Probabilistic Perspective. Chapter 10.

Conditional Independence

- Independent random variables

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- **Conditional** independence

$$X \perp Y | Z \Leftrightarrow \mathbb{P}[X, Y | Z] = \mathbb{P}[X | Z]\mathbb{P}[Y | Z]$$

- Use conditional independence in machine learning

Dependent but Conditionally Independent

Events with a possibly biased coin:

1. X: Your first coin flip is heads
2. Y: Your second flip is heads
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Conditional Independence in Machine Learning

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- Naive Bayes

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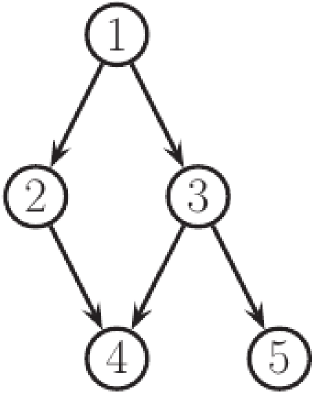
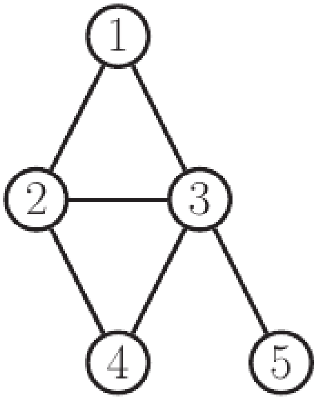
- Node is independent of all predecessors **conditional** on parent value

$$x_s \perp x_{pred(s)\setminus pa(s)} \mid x_{pa(s)}$$



Undirected Graphical Models

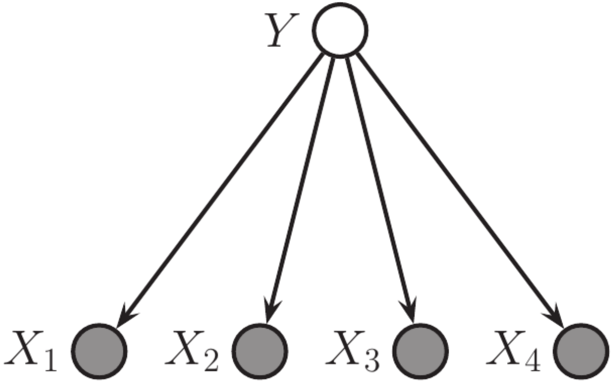
- Another (different) representation of conditional independence



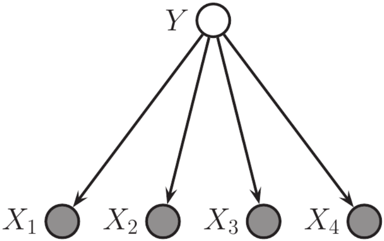
- Markov Random Fields

Naive Bayes Model

Closely related to QDA and LDA



Naive Bayes Model



- Chain rule

$$\mathbb{P}[x_1, x_2, x_3] = \mathbb{P}[x_1]\mathbb{P}[x_2|x_1]\mathbb{P}[x_3|x_1, x_2]$$

- Probability

$$\mathbb{P}[x, y] = \mathbb{P}[y] \prod_{j=1}^{D} \mathbb{P}[x_j|y]$$

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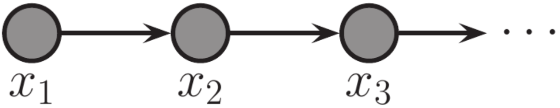
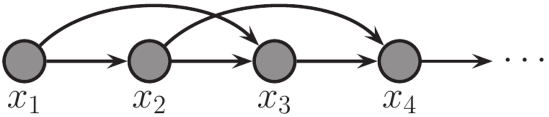
- Reduces bias or variance?

Markov Chain

- 1st order Markov chain:



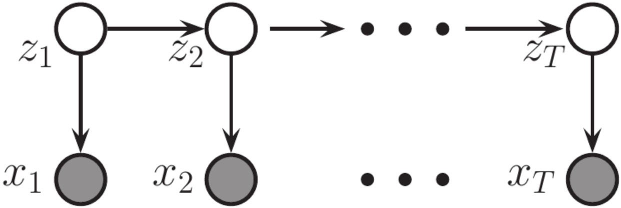
- 2nd order Markov chain:



Uses of Markov Chains

- Time series prediction
- Simulation of stochastic systems
- Inference in Bayesian nets and models
- Many others …

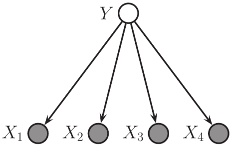
Hidden Markov Models



Used for:

- Speech and language recognition
- Time series prediction
- **Kalman filter**: version with normal distributions used in GPS's

Inference



- Inference of hidden variables (y)

$$\mathbb{P}[y|x_v, \theta] = \frac{\mathbb{P}[y, x_v|\theta]}{\mathbb{P}[x_v|\theta]}$$

- Eliminating <u>nuisance</u> variables (e.g. x_1 is not observed)

$$\mathbb{P}[y|x_2, \theta] = \sum_{x_1} \mathbb{P}[y, x_1|x_2, \theta]$$

- What is inference in linear regression?

Learning

- Computing conditional probabilities θ
- Approaches:
  1. Maximum A Posteriori (MAP)

$$\arg\max_{\theta} \log \mathbb{P}[\theta|x] = \arg\max_{\theta} \; (\log \mathbb{P}[x|\theta] + \log \mathbb{P}[\theta])$$

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- Fixed effects vs random effects (mixed effects models)

Inference in Practice

- Precise inference is often impossible
- Variational inference: approximate models
- Markov Chain Monte Carlo (MCMC):
  - Gibbs samples
  - Metropolis Hastings
  - Others

Probabilistic Modeling Languages

- Simple framework to describe a Bayesian model
- Inference with MCMC and parameter search
- Popular frameworks:
  - JAGS
  - BUGS, WinBUGS, OpenBUGS
  - **Stan**
- Examples:
  - Linear regression
  - Ridge regression
  - Lasso