

# Analysis of the Gated IPACT Scheme for EPONs

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**Abstract**—Interleaved Polling with Adaptive Cycle Time (IPACT) is one of the earliest proposed polling schemes for dynamic bandwidth allocation in Ethernet Passive Optical Networks (EPONs) and has been extensively used as a benchmark by many subsequent allocation schemes. In this paper, we attempt to construct a mathematical model of the IPACT scheme under the gated service discipline. For  $N = 1$  ONU, we derive closed-form expression for the steady state grant size. For  $N > 1$  ONUs, we need to consider separately a small and a large load-distance ratio. For the former case, the  $N = 1$  ONU model holds even for  $N > 1$ . For the latter case, we find a closed form expression for the grant size. Our model shows a reasonable match with the values obtained from simulation for the steady state queue size and hence the throughput and delay.

## I. INTRODUCTION

An *Ethernet Passive Optical Network (EPON)* is a point-to-multipoint, bidirectional, high rate optical network for data communication. The EPON link is shared by multiple users. Each user connects to the EPON link through a device known as an *Optical Network Unit (ONU)*. Since the link is shared, link use must be centrally arbitrated. This function is performed by a single special device called the *Optical Line Terminator (OLT)*. The direction of communication from the ONUs to the OLT is known as *upstream* direction whereas the direction from the OLT to the ONUs is known as the *downstream* direction. The data rate in each direction is set to 1 Gbps by the IEEE EPON standard [1]. Overall, the link exhibits a tree topology with the OLT at the root of the tree and the ONUs at the leaves. The EPON link is shared by all users in the upstream direction. The OLT decides which ONU is allowed to transmit data and for how many bytes. The OLT uses a special control message called a *Gate* to grant transmission opportunities to ONUs. Appended to the data traffic, the ONU also transmits a control message containing a *Report* of the number of bytes buffered in its queue, waiting for a subsequent transmission opportunity. An algorithm implemented in the OLT, which uses these reports and gate messages to construct a transmission schedule is known as a dynamic bandwidth allocation (DBA) algorithm.

## II. THE IPACT PROTOCOL

Interleaved Polling Scheme with Adaptive Cycle Time (IPACT) is a DBA scheme for EPON proposed by Kramer et al. [2][3]. IPACT is one of the earliest dynamic bandwidth allocation schemes for EPONs and has been extensively used as a benchmark by many subsequent allocation schemes [4][5][6][7][8][9][10]. To our knowledge, this is the first

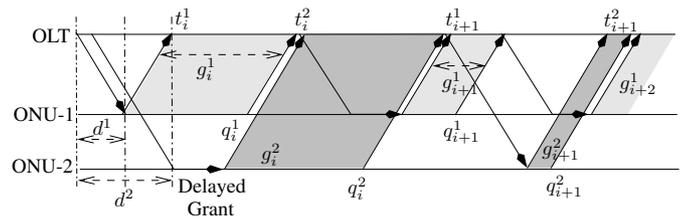


Fig. 1. An example with two ONUs illustrating notation used for the recursive model.

attempt to provide an analytical model for the IPACT scheme. The evaluation in [11] is based on simulations and focuses on service disciplines.

IPACT is an algorithm for interleaved polling of ONUs designed to minimize the walk times. For example, if the OLT sends a grant message to an ONU and then waits for the ONU to send data before sending a grant message to the next ONU, then, the waiting will result in wastage of a significant amount of data bandwidth thus decreasing link utilization. IPACT suggests interleaving the polling messages to consecutive ONUs in order of decreasing distance from the OLT such that transmissions arrive at the OLT as tightly packed as possible. As illustrated in Fig. 1 the OLT has sufficient information to schedule the transmissions for ONU-2 before the data from ONU-1 arrives at the OLT. This is because the OLT knows the distance to each ONU and also knows the size of the grant it allocated to each ONU. Hence the OLT can calculate the time of the arrival of the last bit from each ONU and can therefore schedule the transmission from the next ONU right after the one from the previous ONU has terminated. From Fig. 1 for example, at time  $t_i^1$ , the OLT knows the time at which the transmission from ONU-1 will end and can therefore send the Gate message to ONU-2 to schedule its transmission to arrive at the OLT at time  $t_i^2$ . In this way, the interleaving helps minimize link underutilization during walk times. However, this is not always possible depending on the distance of the ONU from the OLT. To address this issue, the original IPACT scheme also includes other optimizations as well as procedures for detecting arriving and departing ONUs. However, we do not include these portions in our analysis. We focus on the core IPACT bandwidth allocation algorithm which is a simple client-server protocol with quite general applicability.

Another parameter of the IPACT algorithm is the allocation policy of the “server”, i.e., the OLT. When the OLT receives a report of the queue length from an ONU, it need not

grant a single transmission slot in the current cycle for the buffered contents in their entirety. Instead, the OLT may fix a particular policy such as granting a time slot: a) of fixed length regardless of the reported queue length (static allocation), b) equal to the reported buffer length but bounded by a maximum (limited allocation), c) larger than the reported queue length in anticipation of future traffic (credit-based allocation) or d) equal to the reported queue length (gated allocation). Other variants are also possible. In this paper, for simplicity, we focus on the gated allocation discipline. In the following sections, we analyze this scheme using a recursive model.

### III. BACKGROUND AND RELATED WORK

Polling systems have been researched widely for a number of years [12][13][14]. A general analysis of a polling system seems to be a challenging problem and the results available are a product of complex analyses and therefore involve many simplifying assumptions and approximations. The analysis is further complicated if one wishes to include a realistic traffic model. In this paper, we attempt to derive a model of the IPACT scheme from its basic definition. Such a clean-slate analysis can provide clearer insight into the dynamics of the scheme with a minimal set of assumptions and simplifications. Our main goal is to provide a clear and simple yet detailed model of the IPACT scheme derived from its definition. In the end, we hope to obtain simple closed-form expressions relating the grant size (and hence the delay and utilization) to other parameters such as the load and round-trip time. We believe that our work can provide useful guidelines in the design of new bandwidth allocation schemes—currently an area of intense research [4][5][6][7][8][9][10]. Recently, Park et al. [15] obtained new results about the performance of the IPACT scheme under the gated allocation policy. Their approach is novel and comprehensive and provides strong results. However, their assumptions and results are different and their method, more sophisticated and complex.

### IV. A RECURSIVE MODEL FOR IPACT

An accurate model for the IPACT scheme must account for the recursive relationship between the transmission times, queue lengths and the grant sizes in successive scheduling cycles. In this section, we develop such a model. We will use the notation as defined in Table I and illustrated in Fig. 1.

Our goal is to express queue lengths, grant sizes and transmission times in terms of parameters such as the input traffic rate ( $\lambda$ ), link rate ( $\delta^{-1}$ ), number of ONUs ( $N$ ), and the distances of individual ONUs ( $d^j$ ) from the OLT.

#### A. Calculation of grant size $g_i^j$

The size of the grant issued by the OLT to an ONU is based on the queue length of the ONU. The ONU informs the OLT about the length of its queue in a Report message. However, the ONU must be granted a transmission slot to transmit the Report message as well. As per the IPACT protocol, the ONU always appends a Report message to each transmission. Let  $r$  denote the length of a Report message specified by the IEEE

TABLE I  
DEFINITION OF NOTATION USED

Notation	Definition	Units (value)
$t_i^j$	The time of arrival of the $i^{\text{th}}$ transmission by ONU- $j$ at the OLT	seconds
$q_i^j$	The queue length at ONU- $j$ after completion of the $i^{\text{th}}$ data transmission	bits
$g_i^j$	The size of the grant allocated by OLT to ONU- $j$ for its $i^{\text{th}}$ transmission	bits
$d^j$	$\frac{1}{2}$ RTT to ONU- $j$	seconds
$\lambda$	The average arrival bit-rate	bits/second
$\delta$	The time to transmit one bit over the EPON	seconds/bit ( $10^{-9}$ )
$r$	The size of the Report message	bits (512)
$b$	The size of the guard band	seconds ( $2 \times 10^{-6}$ )
$m$	The size of the Gate message	bits (512)
$N$	Total number of ONUs in the EPON	—

EPON standard [1]. The OLT receives information about the current size of the queue length of an ONU only at the end of a current transmission by that ONU. The OLT can use this information only to decide the size of the next transmission slot to be granted to that ONU. Under the gated service discipline, the OLT always allocates exhaustive grants, i.e., the transmission slots are always large enough to transmit all the data in the ONU queue reported to the OLT. Then, for gated service, the size of a grant is equal to the queue size reported in the previous grant, with an additional  $r$  bits allocated for the next Report message. Thus,

$$g_i^j = q_{i-1}^j + r, \quad i > 1. \quad (1)$$

What happens when an ONU is granted its first transmission opportunity? The OLT has no information about the queue size of the ONU, since the ONU has not transmitted any Report messages to the OLT at all. In this case, we assume that the OLT grants the ONU a transmission slot of a minimum size large enough to transmit a Report message. Thus, for any ONU, the size of its first transmission opportunity will be

$$g_1^j = r. \quad (2)$$

#### B. Calculation of initial queue length $q_1^j$

Next, we calculate the initial queue length at any ONU, as required by (1). Recall that for any ONU- $j$ , the first transmission comprises only a Report message and none of the buffered data as per (2). Traffic arrives at the rate of  $\lambda$  bits/second. Suppose an ONU- $j$  is  $d^j$  seconds away from the OLT. Suppose that the first bit of the ONU's first transmission reaches the OLT at time  $t_1^j$ . Since ONU- $j$  is  $d^j$  seconds away from the OLT, it must have transmitted the first bit of its first transmission at time  $t_1^j - d^j$ . The amount of traffic accumulated at ONU- $j$  during this interval will therefore be  $\lambda(t_1^j - d^j)$  which will be the queue length reported in the first Report message by ONU- $j$ . Thus,

$$q_1^j = \lambda \cdot (t_1^j - d^j). \quad (3)$$

The first Gate message from the OLT to poll the first ONU will reach ONU-1 at time  $d^1$ . Therefore, the first Report message from ONU-1 will reach the OLT at time  $t_1^1 = 2d^1 + \delta m$ . Thus,

$$q_1^1 = \lambda(d^1 + \delta m). \quad (4)$$

For the first cycle, i.e., all transmissions  $t_1^j$ , the length of each transmission will be equal to the length of the Report message. Thus, an ONU- $j$ ,  $j > 1$ , will have to wait at least  $\delta r$  after  $t_1^{j-1}$  before transmitting its own Report message. On the other hand, an ONU- $j$  cannot transmit its Report message until it has been polled by the OLT. This takes time at least  $d^j$ . Hence, for any ONU  $1 < j \leq N$ , the time at which its Report message arrives at the OLT will be

$$t_1^j = \max(t_1^{j-1} + \delta r + b, j\delta m + 2d^j). \quad (5)$$

### C. Calculation of queue length $q_i^j$ after $i > 1^{th}$ transmission

The queue length  $q_i^j$  at the end of  $i^{th}$  transmission by device  $j$  is exactly equal to the new traffic arrived since the last queue length measurement. To find this queue length, we need to find out the times at which queue lengths are measured and then using the arrival rate  $\lambda$  obtain the queue length. From Fig. 1 we observe that  $q_{i-1}^j$  is the length of the queue at the time the Report message is transmitted at the ONU. Since any transmission by the ONU takes time  $d^j$  to reach the OLT and since the first bit of the  $i^{th}$  transmission reaches the OLT at time  $t_i^j$ , the first bit must be transmitted at the ONU at time  $t_i^j - d^j$ . Further, since the grant size is  $g_i^j$ , it must take  $\delta g_i^j$  time for the transmission to complete. Thus, the time at which the ONU sends its last bit must be  $t_i^j - d^j + \delta g_i^j$ . The queue length is measured  $\delta r$  time before this time at the ONU. Thus, the time at which the queue length is measured after the  $i^{th}$  transmission by device  $j$  is

$$t_i^j - d^j + \delta g_i^j - \delta r.$$

Thus, the queue length after the previous i.e.,  $i - 1^{th}$  transmission would be measured at time

$$t_{i-1}^j - d^j + \delta g_{i-1}^j - \delta r.$$

Therefore, the amount of new traffic accumulated since the last queue measurement upto the current queue measurement will be

$$\begin{aligned} q_i^j &= \lambda \cdot [(t_i^j - d^j + \delta g_i^j - \delta r) - (t_{i-1}^j - d^j + \delta g_{i-1}^j - \delta r)] \\ &= \lambda \cdot [(t_i^j - t_{i-1}^j) + \delta \cdot (g_i^j - g_{i-1}^j)], \quad i > 1 \end{aligned} \quad (6)$$

### D. Calculation of transmission times $t_i^j$

Next, we calculate the transmission times for any ONU. If the EPON consists of  $N = 1$  ONU, then the transmission time  $t_i^j$  depends only on the transmissions in the previous grants. A new transmission can only begin after the previous one has finished and the new Gate message sent out by the OLT reaches the ONU. A message from the OLT takes  $d^j$  time to reach an ONU- $j$ . The new transmission from ONU- $j$  will in turn take time  $d^j$  to reach the OLT. Thus, consecutive transmissions from the ONU are separated by  $2d^j$  seconds.

An ONU's previous transmission beginning at time  $t_{i-1}^j$  will end at time  $t_{i-1}^j + \delta g_{i-1}^j$ . Thus, a new transmission, in the single ONU case will begin at

$$t_i^j = t_{i-1}^j + \delta g_{i-1}^j + 2d^j + \delta m + b, \quad N = 1 \quad (7)$$

If  $N > 1$ , then considering indices modulo  $N$ , the transmission from ONU- $j$  cannot begin unless the one from the previous ONU- $(j-1)$  has ended, unless, ONU- $j$  is sufficiently far away from the OLT. In this latter case, the time  $d^j$  taken by the Gate message to reach ONU- $j$  sufficiently delays the transmission from ONU- $j$  as illustrated by transmission  $t_2^2$  of ONU-2 in Fig. 1. Suppose ONU- $(j-1)$  transmits at time  $t_i^{j-1}$ . Its transmission will finish at  $t_i^{j-1} + \delta g_i^{j-1}$ . No other transmission can begin before this time. Now suppose that ONU- $j$  last transmitted in the previous cycle at time  $t_{i-1}^j$ . Then,  $d^j$  must be large enough so that ONU- $j$ 's next transmission beginning at  $t_i^j$  is sufficiently delayed past the time of completion  $t_i^{j-1} + \delta g_i^{j-1}$  of the previous ONU. Thus, we arrive at the following condition for the transmission time:

$$t_i^j = \max\left(t_{i-1}^j + \delta(g_{i-1}^j + m) + 2d^j, t_i^{j-1} + \delta g_i^{j-1}\right) + b \quad (8)$$

where device index  $j$  is counted modulo  $N > 1$ . This completes the specification of the recursive model.

## V. CLOSED-FORM SOLUTION OF THE RECURSIVE MODEL

In the previous section, we expressed the queue length reported by the ONU in its  $i^{th}$  transmission in terms of other parameters and variables of our model. In this section, we attempt to derive closed form expressions for the throughput and response time for an ONU. We divide our derivation into separate cases for  $N = 1$  (Sec.V-A) and  $N > 1$  (Sec.V-B) ONU(s).

### A. Single ONU

Consider an EPON with a single ONU. Thus,  $N = 1$  in the recursive model developed in Sec. IV. From, (7) we have

$$t_i^j - t_{i-1}^j = \delta g_{i-1}^j + 2d^j + \delta m + b \quad (9)$$

Substituting in (6) and simplifying gives,

$$q_i^j = \lambda \cdot (2d^j + \delta g_i^j + \delta m + b), \quad i > 1, N = 1.$$

Substituting (1) and simplifying gives,

$$g_{i+1}^j = \lambda \cdot (\delta g_i^j + 2d^j + \delta m + b) + r. \quad (10)$$

Solving this recurrence gives the steady state grant size,

$$g_s = \frac{\lambda \cdot (2d^j + \delta m + b) + r}{1 - \lambda \delta}. \quad (11)$$

The response time  $R_{1-ONU}$  for a single ONU when  $\lambda \delta < 1$  will be

$$R_{1-ONU} = \delta m + 2d^j + \delta g_s + b = \frac{2d^j + \delta(m+r) + b}{1 - \lambda \delta} \quad (12)$$

Fig. 5 shows the match between the steady state grant size predicted by (11) and that obtained by simulation. We have also verified that the response time predicted by (12) matches simulations but omit the graph due to space limitations.

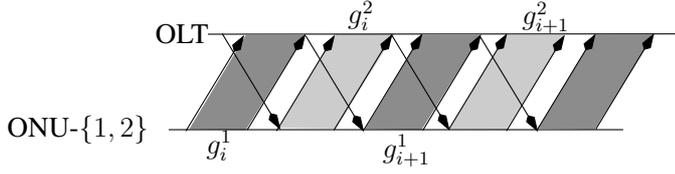


Fig. 2. The low load-distance ratio

### B. Multiple ONUs at an identical distance

For  $N > 1$ , we also assume that all  $N$  ONUs are located at an identical distance

$$d = d^j. \quad (13)$$

We consider two cases based on the ratio of the load to the RTT to any ONU.

1) *Low Load-Distance ratio*: First, consider the case where for  $N > 1$  ONUs, the l.h.s. term of the two expressions compared in (8) is the maximum, i.e.,

$$t_i^{j-1} + \delta g_i^{j-1} < t_{i-1}^j + \delta(g_{i-1}^j + m) + 2d. \quad (14)$$

Then by (8) and (14):

$$t_i^j - t_{i-1}^j = \delta(g_{i-1}^j + m) + 2d + b, \quad (15)$$

and,

$$\begin{aligned} t_i^j &> t_i^{j-1} + \delta g_i^{j-1} + b, \\ &> t_i^{j-2} + \delta g_i^{j-2} + \delta g_i^{j-1} + 2b, \end{aligned}$$

and so on. Continuing, we can write:

$$t_i^j > t_{i-1}^j + \sum_{k=j}^N \delta g_{i-1}^k + \sum_{k=1}^{j-1} \delta g_i^k + Nb. \quad (16)$$

In other words, using (14), (15) and (16), we can write:

$$\sum_{k=j}^N \delta g_{i-1}^k + \sum_{k=1}^{j-1} \delta g_i^k + Nb \leq \Delta t = \delta(g_{i-1}^j + m) + 2d + b. \quad (17)$$

where  $\Delta t = t_i^j - t_{i-1}^j$  is the cycle time. This corresponds to the example shown in Fig. 2 with two ONUs. Clearly, if the grant size to each ONU is smaller than the RTT to the ONU, then neither ONU will ever have to wait for the other ONU. The derivation would proceed after (15) exactly as for the single ONU case in the previous section, resulting in the same steady state grant size as arrived at in (11), i.e.,

$$g^j = g_s = \frac{\lambda \cdot (2d + \delta m + b) + r}{1 - \lambda \delta}. \quad (18)$$

Since the steady state grant size  $g^j$  is the same for all  $N$  ONUs, using (17) and (18), we can write:

$$N \cdot (\delta g^j + b) \leq \delta(g^j + m) + 2d + b, \text{ or } \quad (19)$$

$$(N - 1) \cdot (\delta g^j + b) \leq 2d + \delta m. \quad (20)$$

Substituting Eqn. (18) shows that the steady state grant size  $g^j$  from (18) holds when:

$$\lambda \leq \frac{1}{N\delta} \left( 1 - \frac{(N-1)(\delta r + b)}{2d + \delta m} \right), \quad (21)$$

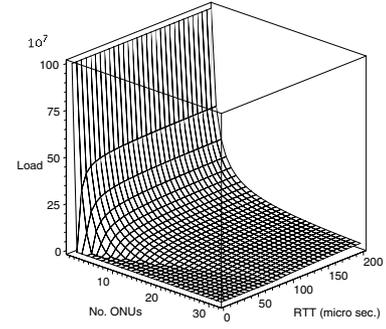


Fig. 3. Steady state grant size for non-negative load points beneath the surface is given by (18) and for those above is given by (26).

i.e., our assumption (14) translates into the above condition. In other words, the single-ONU model applies to the multi-ONU case for the load-distance relationship described by (21). The steady state grant size of any non-negative load point beneath the surface shown in Fig. 3 will be given by (21).

2) *High Load-Distance ratio*: Alternatively, consider the r.h.s. term of the two expressions compared in (8) to be larger, i.e.,

$$t_{i-1}^j + \delta(g_{i-1}^j + m) + 2d < t_i^{j-1} + \delta g_i^{j-1}. \quad (22)$$

Then by (8) and (22):

$$t_i^j - t_{i-1}^j = \sum_{k=j}^N \delta g_{i-1}^k + \sum_{k=1}^{j-1} \delta g_i^k + Nb, \quad (23)$$

and, by similar reasoning as before,

$$\delta(g_{i-1}^j + m) + 2d + b \leq \Delta t = \sum_{k=j}^N \delta g_{i-1}^k + \sum_{k=1}^{j-1} \delta g_i^k + Nb. \quad (24)$$

If all ONUs are located at the same distance, the cycle time  $\Delta t$  is solely comprised of transmission grants of other ONUs, i.e., those that transmitted after ONU- $j$  in the previous cycle ( $i-1$ ) and those that will transmit before ONU- $j$  in the current cycle ( $i$ ). (See the example with two ONUs in Fig. (4).) Substituting (23) into (6) and simplifying gives

$$q_i^j = \lambda \delta \left( \sum_{k=j+1}^N g_{i-1}^k + \sum_{k=1}^j g_i^k \right) + \lambda Nb. \quad (25)$$

Assume that a steady state exists (see appendix for details) and that the steady state grant size for each ONU is identical. Then, in steady state,

$$\begin{aligned} g^j - r &= \lambda \delta N g^j + \lambda Nb \text{ or,} \\ g^j &= \frac{\lambda Nb + r}{1 - N\lambda \delta}. \end{aligned} \quad (26)$$

Again, from (24) we know that (26) holds when

$$\delta(g^j + m) + 2d + b < N(\delta g^j + b). \quad (27)$$

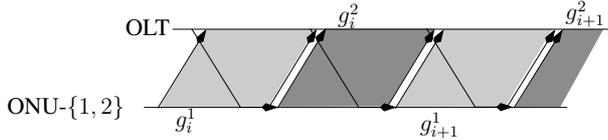


Fig. 4. High load-distance ratio.

Substituting (26) and simplifying shows that (26) holds for the remaining loads

$$\lambda > \frac{1}{\delta N} \left( 1 - \frac{(N-1)(\delta r + b)}{2d + \delta m} \right). \quad (28)$$

This completes the solution of our recursive model for IPACT under the gated service scheme.

## VI. SIMULATIONS AND A COMPARISON

Simulations were conducted in order to validate the above model of the IPACT scheme under a gated service policy. The simulations were conducted for  $N \in \{1, 2, 4, 20\}$  ONUs  $d \in \{25, 50, 100\}$   $\mu\text{s}$  away from the OLT for loads  $0 \leq \lambda \leq N^{-1}$ . Each run simulated 10 seconds of real time. For simulations with self-similar traffic, the duration of the simulation must be very large (of the order of hundreds to a few thousands of seconds). Depending on the load to be generated and the number of ONUs, such simulations can be computationally intensive in both time and space. More rigorous simulations of much longer duration are currently in progress. Fig. 5 shows the results of the simulation. The self-similar traffic curve consists of 3000 points, one obtained from each run for a specific load. Fifteen load values were used. Traffic injected into the ONUs was generated by an aggregated source with a measured Hurst parameter of 0.8 [16]. The source generated bursts whose “ON” and “OFF” period lengths followed the Pareto distribution. The integer lengths of the packets in each burst were drawn uniformly at random from the interval [64, 1518] bytes.

Fig. 5 shows the match between the steady state queue size predicted by our model and that measured from simulation with both Poisson as well as self-similar/LRD [16] traffic models. The match against the Poisson model is better than that against the self-similar model since our model is based simply on the average load and hence cannot account for the infinite variance exhibited by the bursty self-similar traffic. Nonetheless, the model shows a reasonably and sufficiently good match with both models as far as verifying the correctness of the model is concerned.

## VII. CONCLUSIONS AND FUTURE WORK

In this paper, we attempted to analyze the IPACT protocol for allocation of EPON bandwidth using a recursive formulation. IPACT is one of the earliest and most widely used benchmarks for bandwidth allocation in EPONs. However, there was no analytical model describing the IPACT scheme. In this paper, we developed such a simple model. We derived closed-form expressions for the grant size allocated by IPACT

under the gated service scheme as a function of the input load, ONU RTT and other protocol parameters.

The main assumption of our analysis is that the fluid traffic arriving in an interval  $t$  is  $\lambda t$  where  $\lambda$  is the average traffic arrival rate, is known and fixed. However, just as with other approaches, this is a drawback of our analysis as well. For a well-behaved traffic source such as Poisson traffic, a small sample from a small interval  $t$  will converge to the actual mean  $\lambda$  with high probability. However, for self-similar, heavy tailed traffic, this does not hold. Since the tail decays only polynomially, a small sample can contain a large deviation from the mean. In such cases, the predictions of the average value from our analysis will exhibit error. Thus, the assumption that the queue reported at the end of a transmission of size  $t$  will be  $\lambda t$  will introduce considerable error for a heavy tailed traffic source.

## ACKNOWLEDGMENTS

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## APPENDIX

Consider (25). We can write a similar equation for ONU- $j-1$  (using modulo  $N$  indexing):

$$q_i^{j-1} = \lambda \delta \left( \sum_{k=j}^N g_{i-1}^k + \sum_{k=1}^{j-1} g_i^k \right) + \lambda N b. \quad (29)$$

Subtracting, (29) from (25) gives:

$$q_i^j = q_i^{j-1} + \lambda \delta (q_{i-1}^j - q_{i-2}^j), \quad (30)$$

which can be rewritten as:

$$a_n = a_{n-1} + \lambda \delta (a_{n-N} - a_{n-2N}). \quad (31)$$

Equation (31) can be interpreted as the recurrence describing the IPACT scheme under gated service and high load-distance ratio (as defined by (28)) since it captures the essential behavior of the bandwidth allocation process of the scheme. The characteristic equation of (31) is

$$p(x) = x^{2N} - x^{2N-1} - c x^N + c = 0, \quad (32)$$

where  $c = \lambda \delta$ . By showing that all the roots of (32) will be no greater than unity for  $c < N^{-1}$ , we can argue that a steady state exists in the high load-distance ratio regime as well.

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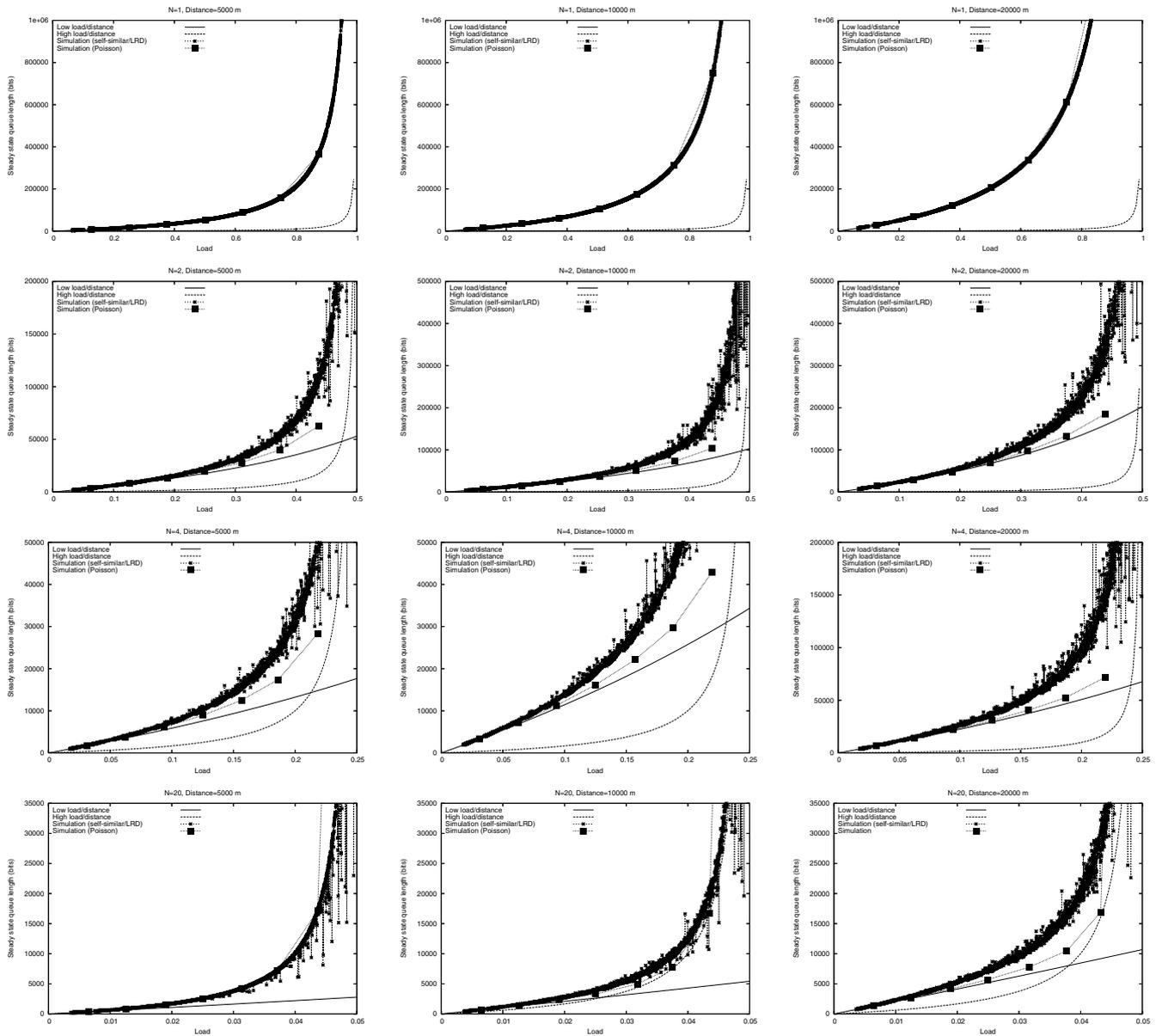


Fig. 5. Simulation results with Poisson and self-similar traffic for a few selected points from Fig. 3.

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