1 handout: slides
Propositional Logic
A logic is a formal system:

- syntax: defines sentences
- semantics: relation to world
- inference rules: reaching new conclusions

three layers: proof, models, reality

soundness, completeness

flexible, general, principled (Advice Taker, 1958)
Propositional Reasoning

computing entailment
soundness, completeness

\( \alpha \models \beta \) iff \( \alpha \land \neg \beta \) is unsatisfiable
determining satisfiability is NP-complete
[ NP-hard = polytime to verify certificate of ‘yes’ ]
therefore, verification that \( \beta \) is not entailed is polytime

said another way:
\( \alpha \models \beta \) iff \( \alpha \rightarrow \beta \) is valid
determining validity/tautology is co-NP-complete
[ co-NP-hard = polytime to verify certificate of ‘no’ ]
therefore, verification that \( \beta \) is not entailed is polytime
Reasoning Methods

- variable elimination: Davis-Logemann-Loveland
  exhaustively branch on variable assignments

- model finding: WalkSAT
  fix assignment until satisfying

- modus ponens, resolution: resolution refutation theorem proving
  derive new clauses until query is proved
An Example of Propositional Reasoning

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

Prove: the unicorn is magical.
Given KB, is $\alpha$ entailed?
Given KB, is $\alpha$ entailed? 
(Is it true in all models of the KB?)
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Is $KB \land \neg \alpha$ unsatisfiable?
Given KB, is $\alpha$ entailed?
(Is it true in all models of the KB?)
Is KB $\land \neg\alpha$ unsatisfiable?

Resolution is refutation complete.
Conversion to Conjunctive Normal Form

Syntax: $\land, \lor, \neg, \rightarrow$ ($\supset, \Rightarrow$), $\leftrightarrow$

1. eliminate $\leftrightarrow$
2. eliminate $\rightarrow$
3. move $\neg$ inward: $\neg \neg x, \neg(x \land y), \neg(x \lor y)$
4. distribute $\lor$: $x \lor (y \land z)$
■ Propositional Logic
  ■ Logic
  ■ Reasoning
  ■ Methods
  ■ Example
  ■ Refutation
  ■ CNF

■ Break

■ First-Order Logic

■ asst 5
■ projects: proposals due Oct 11
First-Order Logic
Gottlob Frege (1848-1925)
PhD at 25

Begriffsschrift, 1879 (concept script)
"a formula language, modelled on that of arithmetic, of pure thought."
∀person ItIsRaining() → IsWet(person)

1. Things:
   - constants: John, Chair23
   - functions (thing → thing): MotherOf(John), SumOf(1,2)

2. Relations:
   - predicates (objects → T/F): IsWet(John), IsSittingOn(MotherOf(John), Chair23)

3. Complex sentences:
   - connectives: IsWet(John) ∨ IsSittingOn(MotherOf(John), Chair23)
   - quantifiers and variables: ∀personIsWet(person)..., ∃person...
1. constants: objects
2. predicates: relations between objects
3. variables
4. quantifiers
5. functions
6. connectives
∀person ∀time (ItIsRaining(time) ∧ ¬∃umbrella Holding(person, umbrella, time)) → IsWet(person, time)

John loves Mary.

All crows are black.

Dolphin are mammals that live in the water.

Everyone loves someone.

Mary likes the color of one of John’s ties.

I can’t hold more than one thing at a time.
Please write down the most pressing question you have about the course material covered so far and put it in the box on your way out.

Thanks!