“Spock had a big, big effect on me. I am so much more Spock-like today than when I first played the part in 1965 that you wouldn’t recognize me. I’m not talking about appearance, but thought processes. Doing that character, I learned so much about rational logical thought that it reshaped my life.”

– Leonard Nimoy

1 handout: slides asst 2 milestone was due
First-Order Inference

- Clausal Form
- Example
- Unification
- Tricky Cases
- Completeness
- Break
- Equality
- Specific Answers
- Res. Strategies
- Terminology

Logic in Practice
1. Eliminate $\rightarrow$ using $\neg$ and $\lor$
2. Push $\neg$ inward using de Morgan’s laws
3. Standardize variables apart
4. Eliminate $\exists$ using Skolem functions
5. Move $\forall$ to front
6. Move all $\land$ outside any $\lor$ (CNF)
7. Can finally remove $\forall$ and $\land$
Example

1. Anyone who can read is literate.
2. Dolphins are not literate.
3. Some dolphins are intelligent.

Skolem, standardizing apart
Unifying Two Terms

1. if one is a constant and the other is
2. a constant: if the same, done; else, fail
3. a function: fail
4. a variable: **substitute** constant for var
5. if one is a function and the other is
6. a different function: fail
7. the same function: unify the two arguments lists
8. a variable: if var occurs in function, fail
9. otherwise, **substitute** function for var
10. otherwise, **substitute** one variable for the other

Carry out substitutions on all expressions you are unifying!
Build up substitutions as you go, carrying them out before checking expressions?
See handout on website.
Tricky Cases

don’t unify $x$ and $f(x)$!
as in $P(x, x)$ meets $\neg P(z, f(z))$

note resolvent of $P(f(x))$ and $\neg P(z) \lor P(f(z))$

Semi-decidable: if yes, will terminate
Gödel’s Completeness Theorem (1930) says a complete set of inference rules exists for FOL.

Herbrand base: substitute all constants and combinations of constants and functions in place of variables. Potentially infinite!

Herbrand’s Theorem (1930): If a set of clauses is unsatisfiable, then there exists a finite subset of the Herbrand base that is also unsatisfiable.

Ground Resolution Theorem: If a set of ground clauses is unsatisfiable, then the resolution closure of those clauses contains \( \perp \).

Robinson’s Lifting Lemma (1965): If there is a proof on ground clauses, there is a corresponding proof in the original clauses.
Break

First-Order Inference
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Logic in Practice

- asst 2
- office hours, final projects
Equality: $\forall xy (\text{Holding}(x) \land \neg(x = y) \rightarrow \neg \text{Holding}(y))$

Unique: $\exists! x P(x) \equiv \exists x (P(x) \land \forall y (\neg(x = y) \rightarrow \neg P(y)))$
Use the “answer literal”:

1. FatherOf(Alice, Bob)
2. FatherOf(Caroline, Bob)
3. FatherOf(x, y) \rightarrow ParentOf(x, y)

Query: Who is Caroline’s parent?
Resolution Strategies

**Breadth-first:** all first-level resolvents, then second-level...
- Complete, slow

**Set of Support:** at least one parent comes from SoS
- Complete if non-SoS are satisfiable, nice

**Input Resolution:** at least one parent from the input set
- Complete for Horn KBs

Simplifications: remove tautologies, subsumed clauses, and pure literals.
**Terminology**

**Interpretation:** maps constant symbols to objects in the world, each function symbol to a particular function on objects, and each predicate symbol to a particular relation.

**Model of** $P$: an interpretation in which $P$ is true. Eg, $\text{Famous(BarbaraBush)}$ is true under the intended interpretation but not when the symbol $\text{BarbaraBush}$ maps to Joe Shmoe.

**Satisfiable:** $\exists$ a model for $P$. Eg, $P \land \neg P$ is not satisfiable.

**Entailment:** if $Q$ is true in every model of $P$, then $P \models Q$. Eg, $P \land Q \models P$.

**Valid:** true in any interpretation. Eg, $P \lor \neg P$. 
1. given $\exists$, can introduce new constant
2. given sentence with ground expression, can introduce $\exists$
3. given $\forall$, can introduce new constant
4. given sentence, can introduce $\forall$ over new free variable

$\land$ elimination/introduction:

$\lor$ introduction:

$\neg\neg$ elimination:
Modus Ponens:

Resolution:

Abduction:

Induction:

mathematical induction $\neq$ inductive reasoning
Please write down the most pressing question you have about the course material covered so far and put it in the box on your way out.

*Thanks!*