When does Weighted A* Fail?

Christopher Wilt

University of New Hampshire
Weighting the heuristic is one of the most important techniques in heuristic search.
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When A* fails to find a solution in the required time, next we typically try Weighted A*
Weighted A* expands nodes in $f'(n) = w \cdot h(n) + g(n)$ order.
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A weight of 1 is equivalent to A*.
Weighted A* expands nodes in $f'(n) = w \cdot h(n) + g(n)$ order.

A weight of 1 is equivalent to A*.

A weight of $\infty$ is equivalent to greedy search.
What happens when $w$ increases?

14 disk Hanoi

Total Nodes Expanded

A* 1.1 1.2 2.5 5 10 20 G

TopSpin(3)

Grid Navigation

Total Nodes Expanded

A* 1.1 1.2 2.5 5 10 20 G

Unit Tile

Total Nodes Expanded

A* 1.1 1.2 2.5 5 10 20 G

Dynamic Robot

Total Nodes Expanded

A* 1.1 1.2 2.5 5 10 20 Gdy

40 Pancake Problem

Total Nodes Expanded

A* 1.1 1.2 2.5 5 10 20 G
City Navigation

Weighted Search
- Overview
- Terminology
- What happens when \( w \) increases?
- City Navigation
- Not Always
- Conclusion

The Problem

Hypotheses

Conclusion
Not Always

Weighted Search
- Overview
- Terminology
- What happens when w increases?
- City Navigation
- Not Always
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The Problem

Hypotheses

Conclusion

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We sometimes weight the heuristic in to try and make best-first searches go faster.
We sometimes weight the heuristic in to try and make best-first searches go faster.

This often results in fewer node expansions, but not always.
The Problem
Weighting may or may not help.
How do we differentiate between these?

TopSpin(3)

- Total Nodes Expanded
- A*, 1.1, 1.2, 2.5, 5, 10, 20, G

TopSpin(4)

- Total Nodes Expanded
- A*, 1.1, 1.2, 2.5, 5, 10, 20, G
Hypotheses

- Heuristic % Error
- Local Minimum Size
- $h^*(n) - h(n)$
- Correlation
- $d^*(n) - h(n)$
- Correlation

Conclusion
Heuristic % Error

Heuristic % Error is \( \frac{h^*(n) - h(n)}{h^*(n)} \)
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<table>
<thead>
<tr>
<th>Domain</th>
<th>Error</th>
</tr>
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<tbody>
<tr>
<td>Citynav 4 4</td>
<td>37.41</td>
</tr>
<tr>
<td>Unit Tiles</td>
<td>33.37</td>
</tr>
<tr>
<td>Inverse Tiles</td>
<td>29.49</td>
</tr>
<tr>
<td>Hanoi</td>
<td>29.47</td>
</tr>
<tr>
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Heuristic % Error

- Heuristic % Error is \( \frac{h^*(n) - h(n)}{h^*(n)} \)

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- Knowing % Heuristic Error does not let us predict whether or not greedy search will fail.
Greedy search might do well in domains where local minima are small.
Local Minimum Size

Weighted Search

The Problem

Hypotheses
- Heuristic % Error
- Local Minimum Size
  - \( h^*(n) - h(n) \)
  - Correlation
- \( d^*(n) - h(n) \)
  - Correlation

Conclusion

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Local Minimum Size

Weighted Search

The Problem

Hypotheses
- Heuristic % Error
- Local Minimum Size
  - $h^*(n) - h(n)$
  - Correlation
  - $d^*(n) - h(n)$
  - Correlation

Conclusion
Greedy search can do well when local minima are large, and poorly when local minima are small.
Correlation

- $h^*(n) - h(n)$ Correlation

Weighted Search

The Problem

Hypotheses
- Heuristic % Error
- Local Minimum Size
- $h^*(n) - h(n)$ Correlation
- $d^*(n) - h(n)$ Correlation

Conclusion
$h^*(n) - h(n)$ Correlation

- $h^*(n) - h(n)$ Correlation

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</tr>
<tr>
<td>Citynav 4 4</td>
<td>0.7077</td>
</tr>
<tr>
<td>Unit Tiles</td>
<td>0.7064</td>
</tr>
<tr>
<td>Inverse Tiles</td>
<td>0.6722</td>
</tr>
<tr>
<td>TopSpin(3)</td>
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$h^*(n) - h(n)$ \textbf{Correlation}

- $h^*(n) - h(n)$ Correlation

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- $h^*(n) - h(n)$ Correlation cannot be used to identify domains where greedy search works poorly.
\[ d^*(n) - h(n) \] Correlation

- Heuristic % Error
- Local Minimum Size
- Correlation
- Correlation

Conclusion
### Weighted Search

**The Problem**

**Hypotheses**
- Heuristic % Error
- Local Minimum Size
- $h^*(n) - h(n)$ Correlation
- $d^*(n) - h(n)$ Correlation

**Conclusion**

#### $d^*(n) - h(n)$ Correlation

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<tr>
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<tr>
<td>Citynav 3 3</td>
<td>0.3670</td>
</tr>
<tr>
<td>Citynav 4 4</td>
<td>0.2827</td>
</tr>
<tr>
<td>TopSpin(4)</td>
<td>0.0246</td>
</tr>
<tr>
<td>Inverse Tiles</td>
<td>0.0853</td>
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- $d^*(n) - h(n)$ Correlation can be used to identify domains where greedy search works poorly.
Conclusion
Greedy search fails when the correlation between $d^*(n) - h(n)$ is weak.