# Planning Algorithms: When Optimal Is Just Not Good Enough

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Joint work with the UNH AI Group. Support from NSF and DARPA.

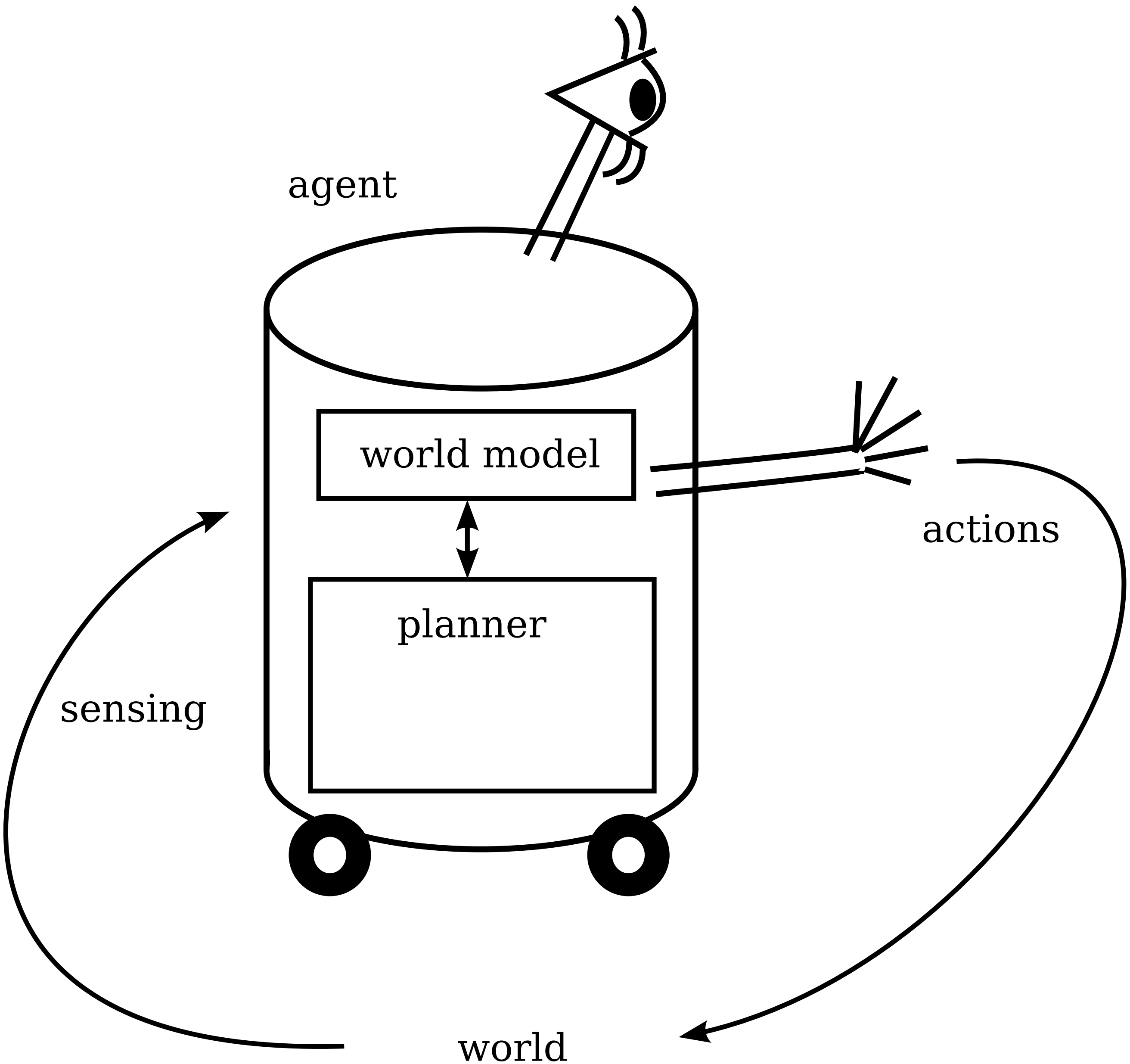
# Robot Time!





# The Role of Planning





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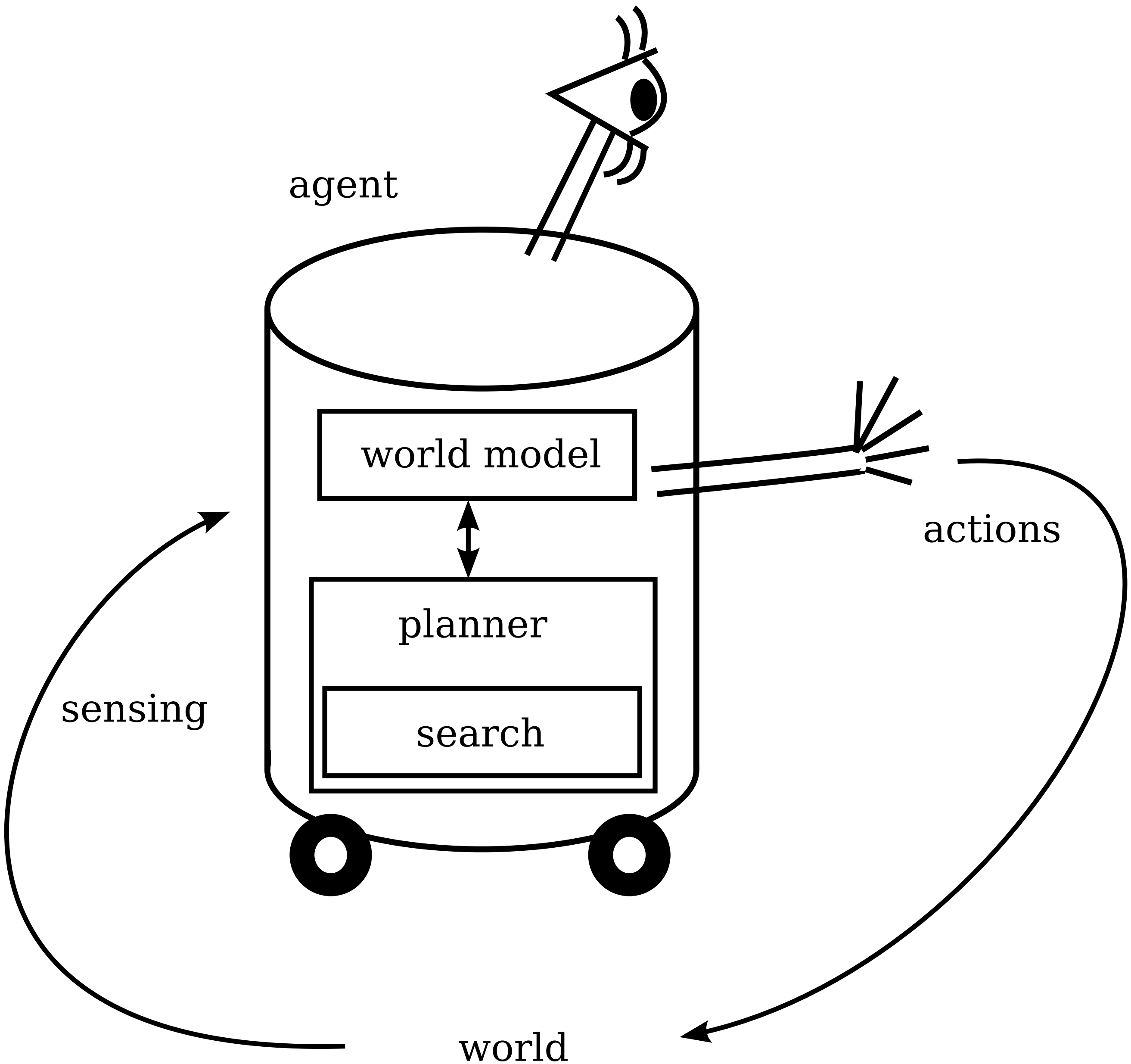


# The Roles of Planning



- autonomy
  - exploration
  - transportation
  - manufacturing
  - autonomic systems


- decision support
  - operations management
  - personal health
  - eldercare
  - education

# Planning as Heuristic Graph Search

# Planning



Given:

- current state of the world
- models of available actions

  *preconditions, effects, costs*
- desired state of the world (partially specified?)

Find:

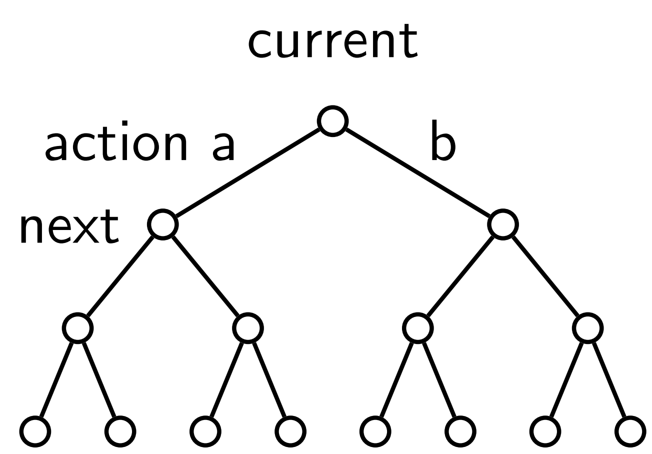
- cheapest plan

Given:

- start state: an explicit node
- expand function:
  lazily generate children and their costs
- goal test: predicate on nodes

Find:

- cheapest path to a goal node



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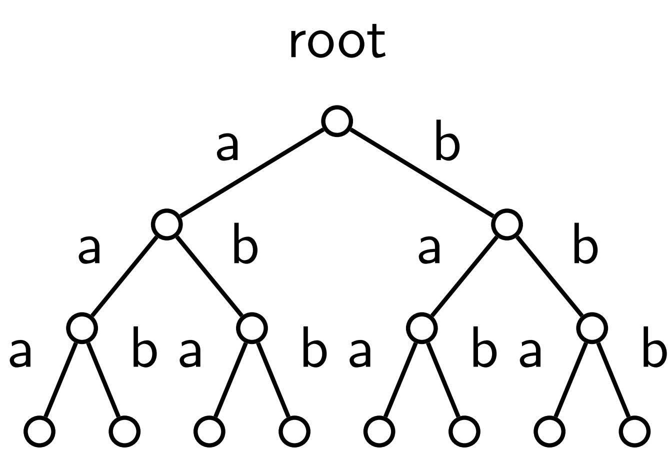

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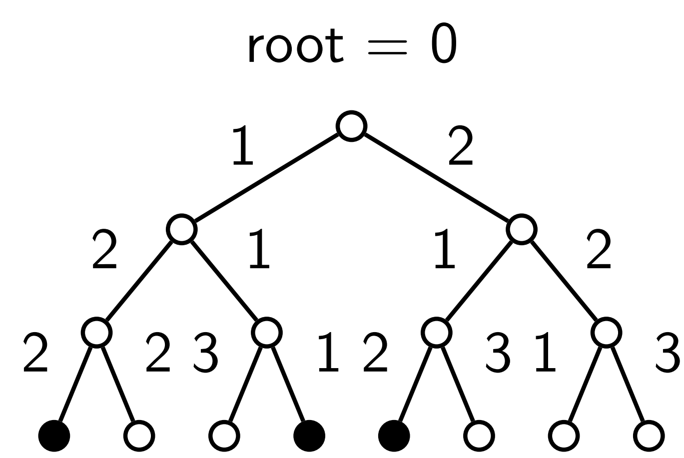
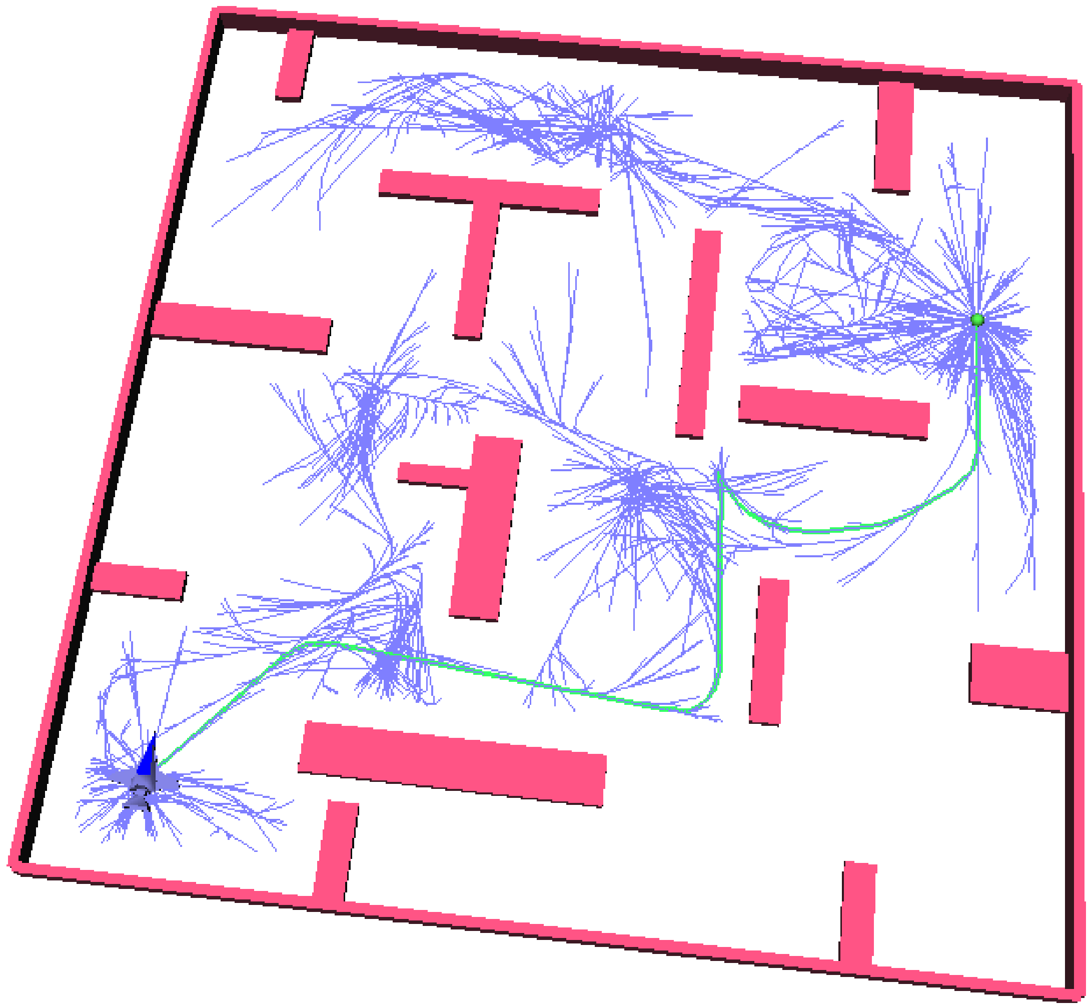


# Example: Motion Planning





(S. LaValle)

1. Uniform Cost Search (Dijkstra, 1959)

2. A* Search (Hart, Nilsson, and Raphael, 1968)

3. Explicit Estimation Search (Thayer and Ruml, 2011)

# Optimal Planning

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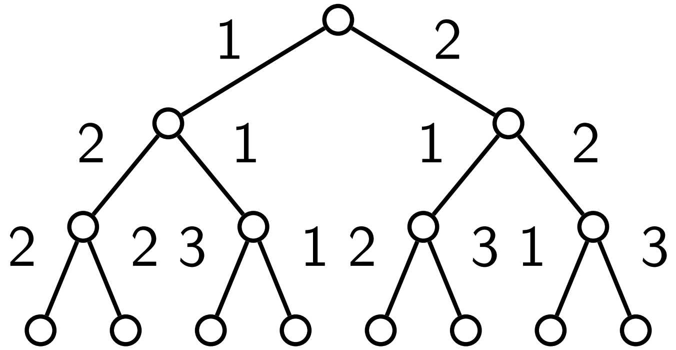
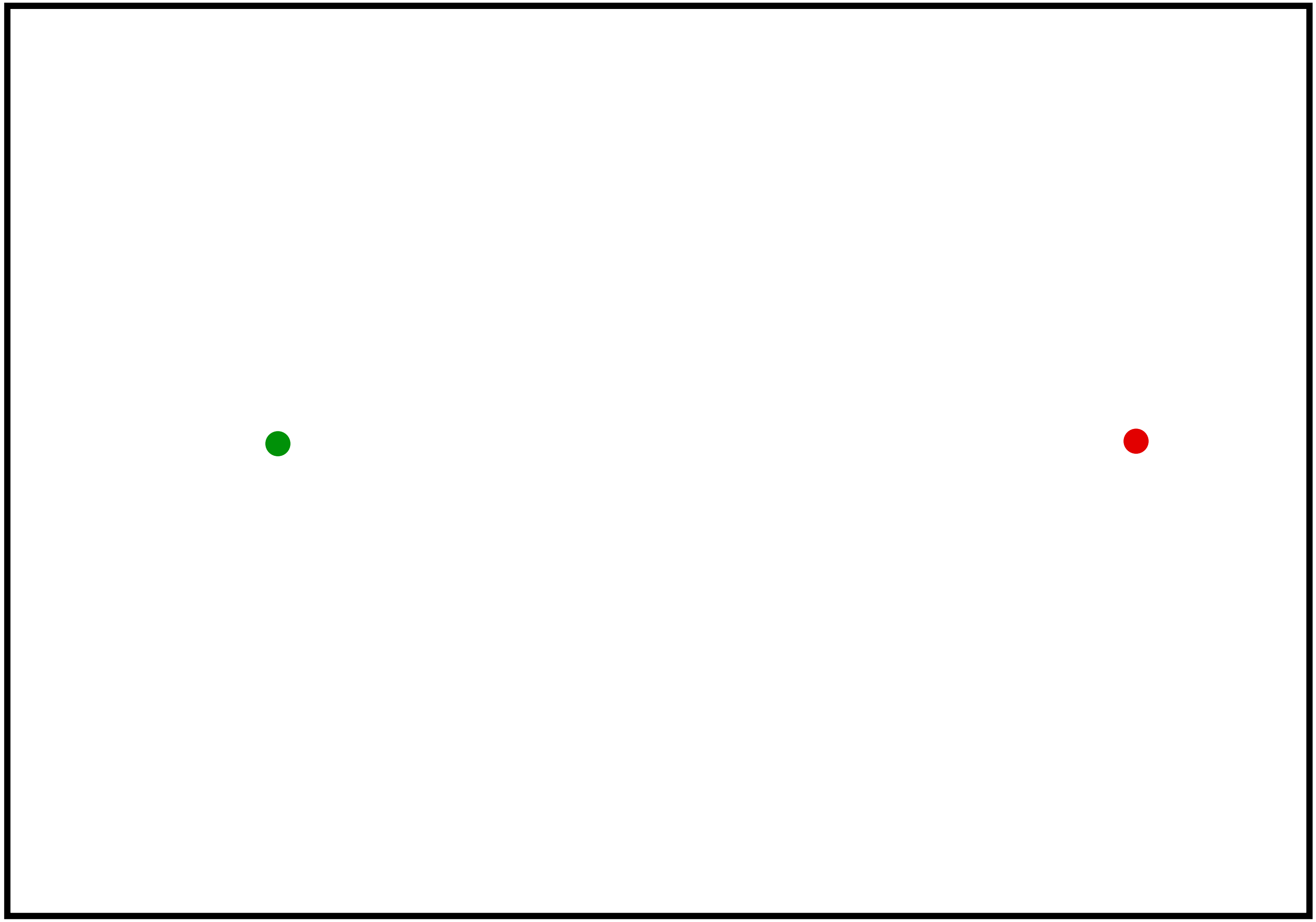


# Dijkstra Behavior





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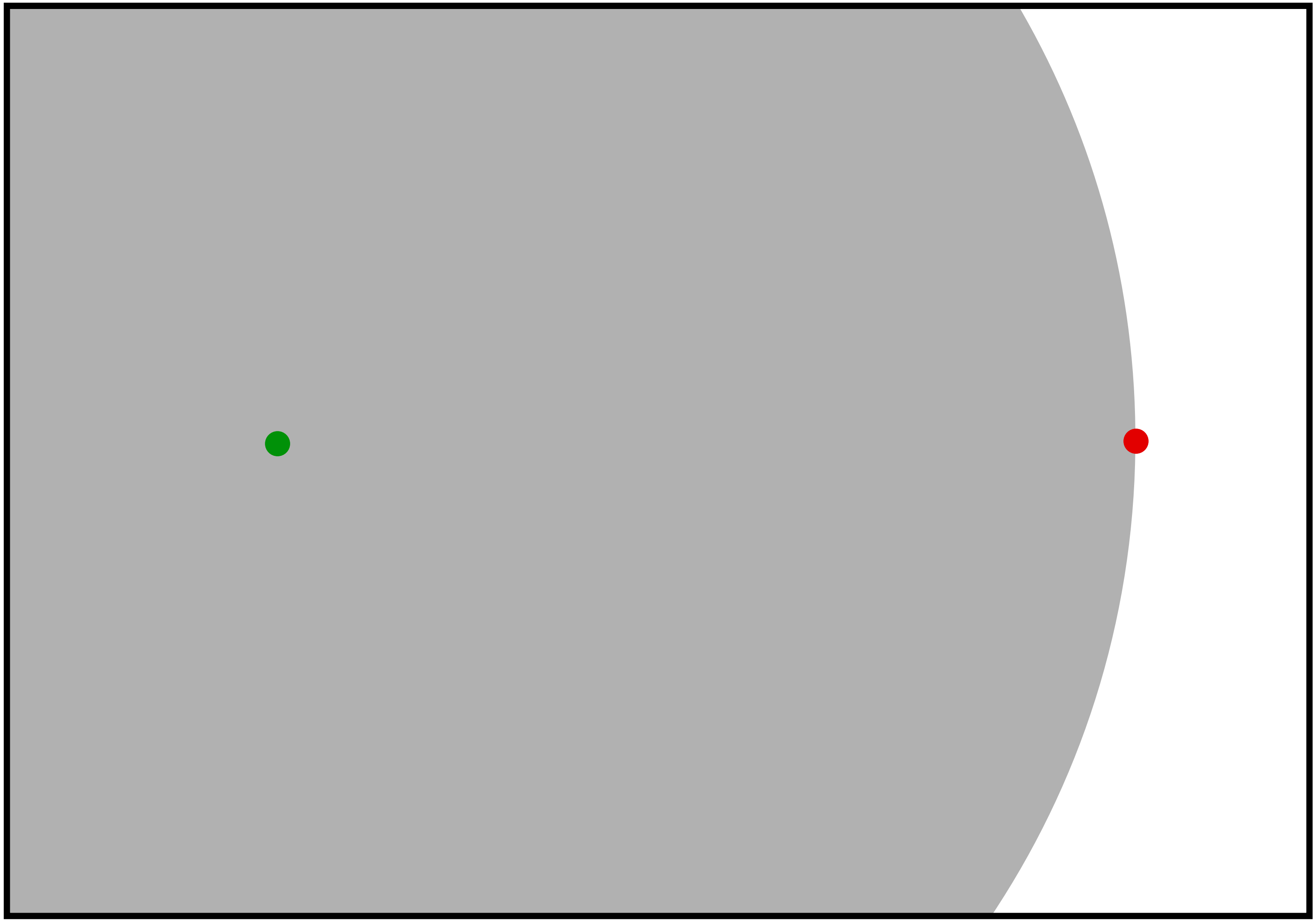

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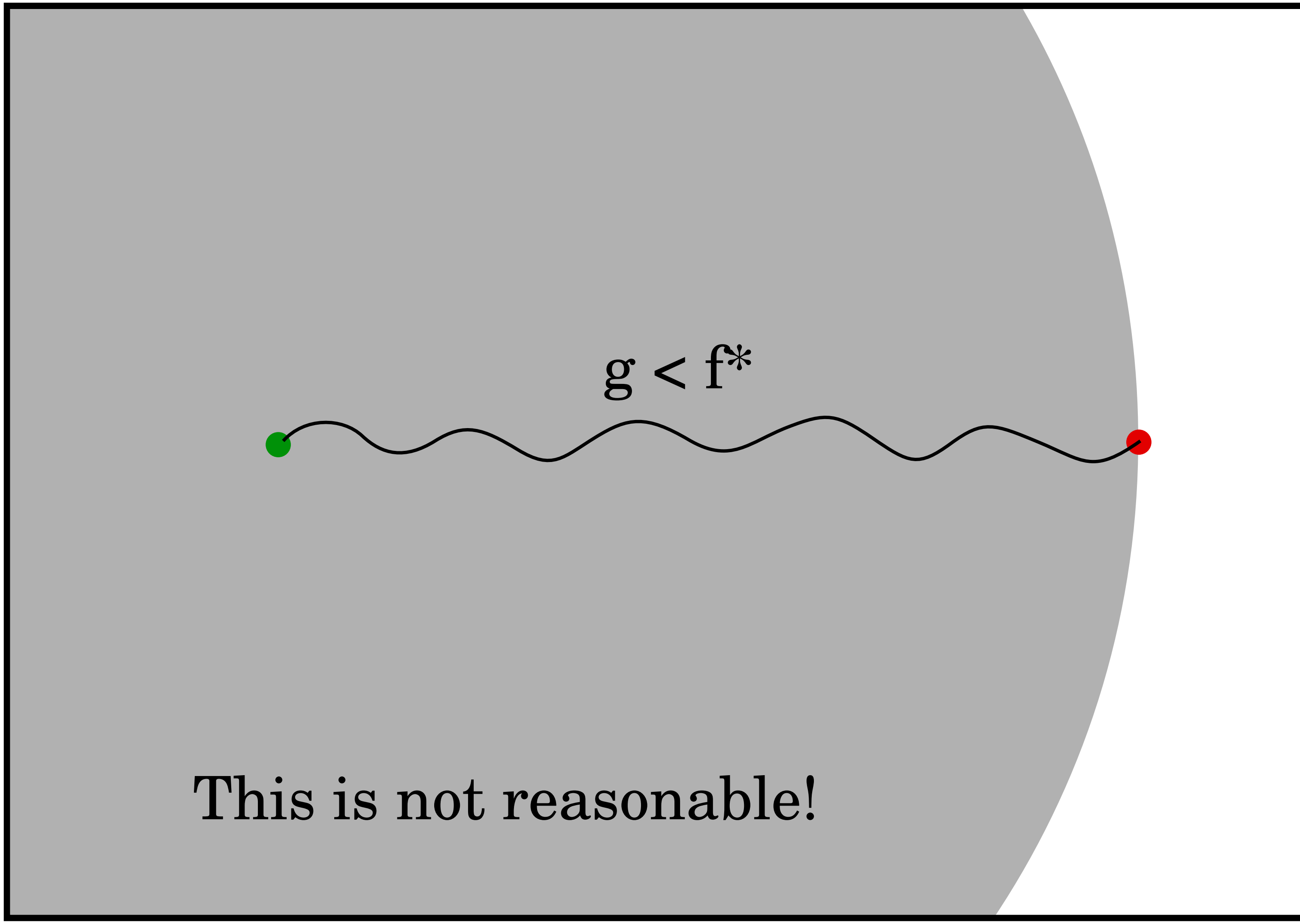
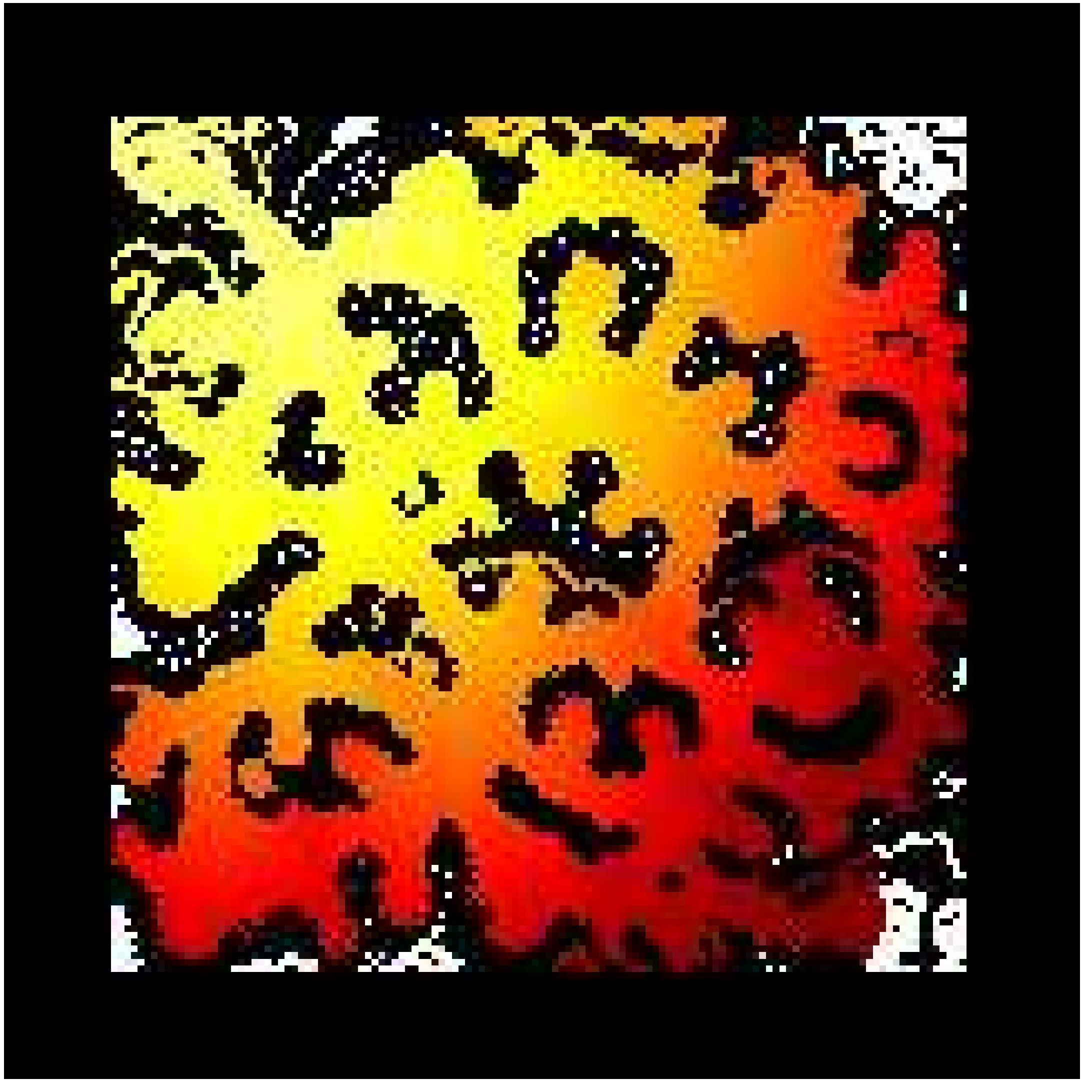


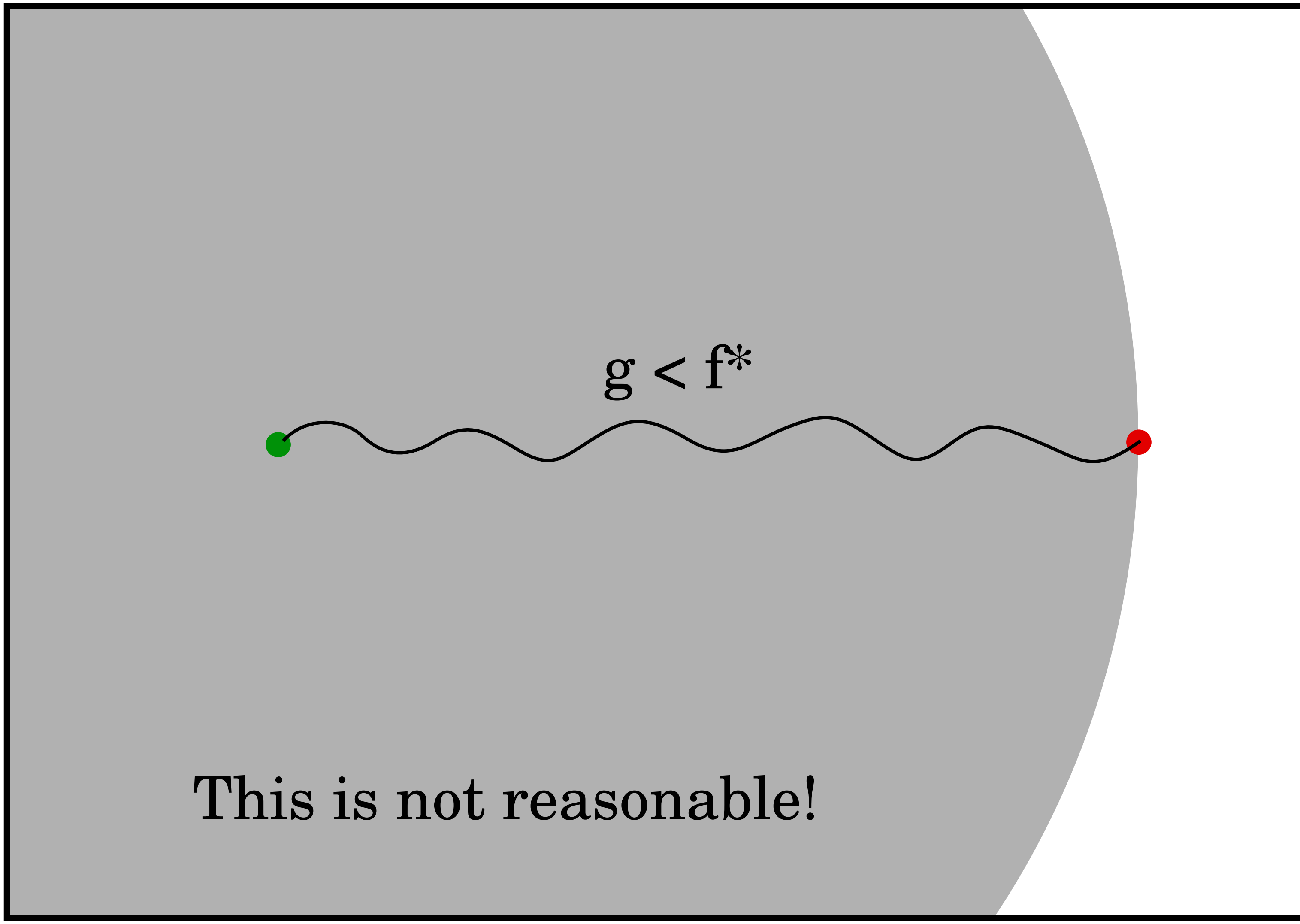
# Dijkstra: Pathfinding in Warcraft





# Problem-Specific Background Knowledge

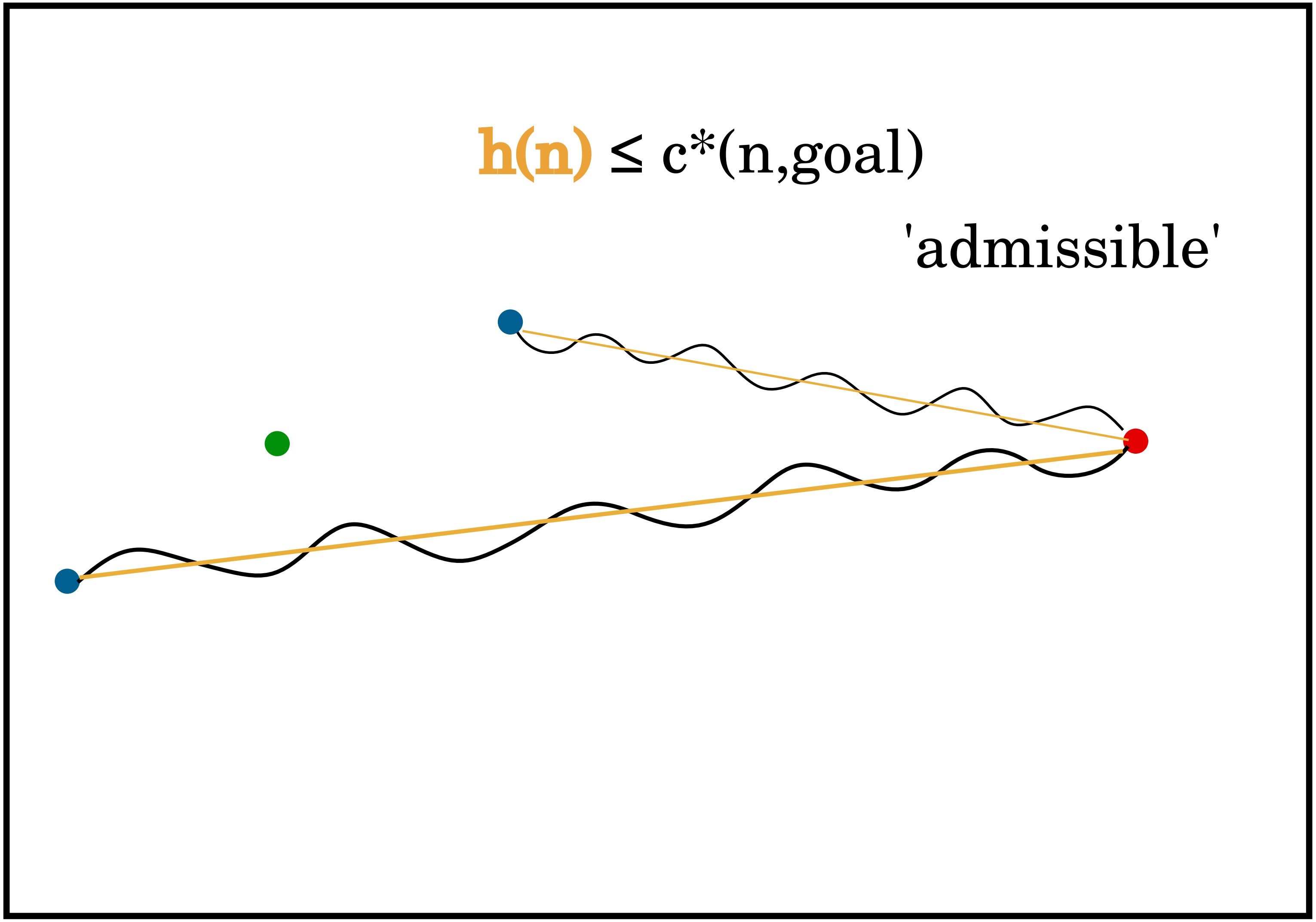





This is not reasonable!

# Heuristic Estimates of Cost-to-go





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Cost should include both cost-so-far and cost-to-go:

$g(n) =$ cost incurred so far
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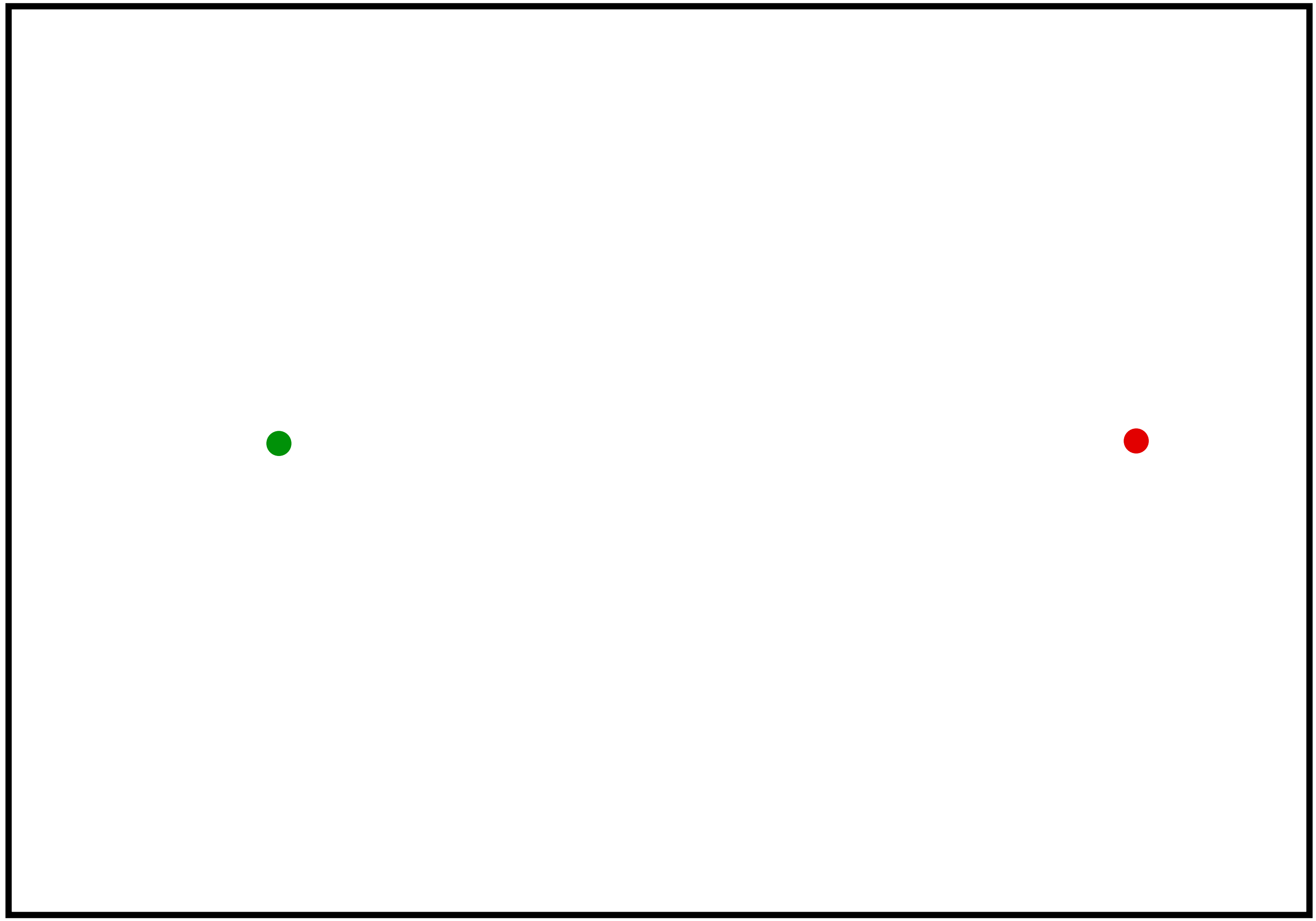
finds optimal solution if heuristic is admissible

# A* Behavior





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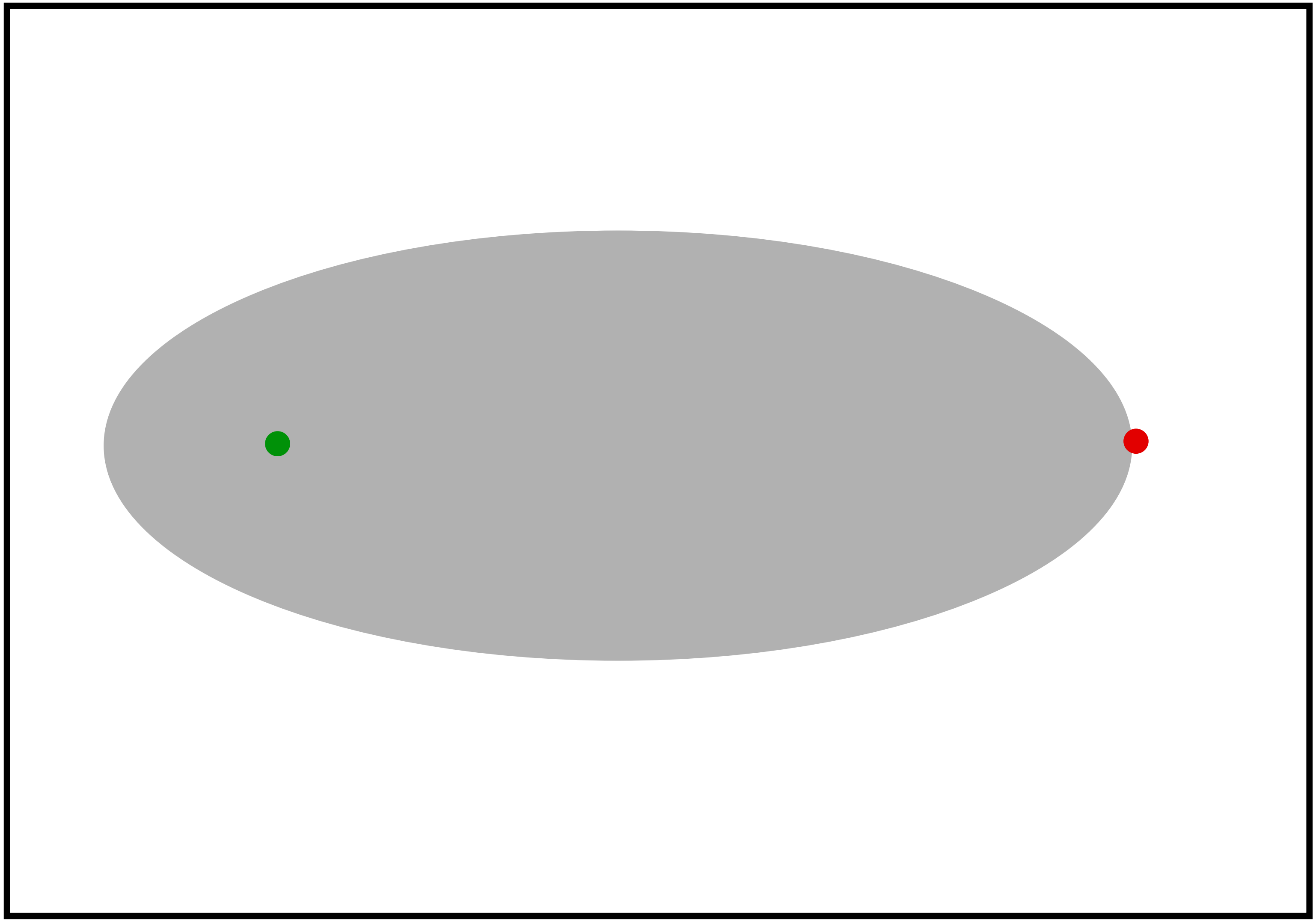
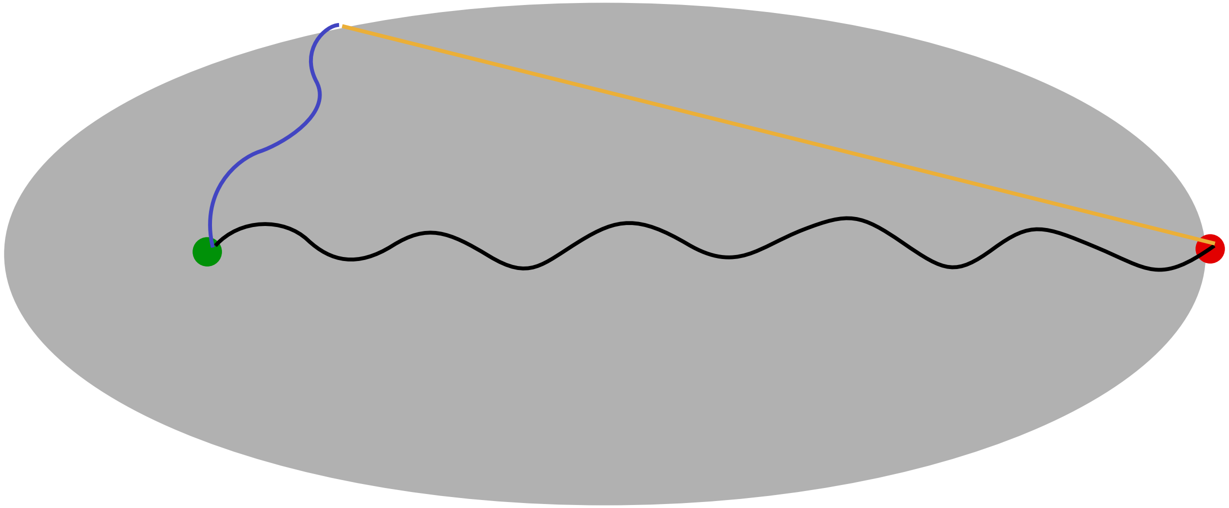


## A* Behavior

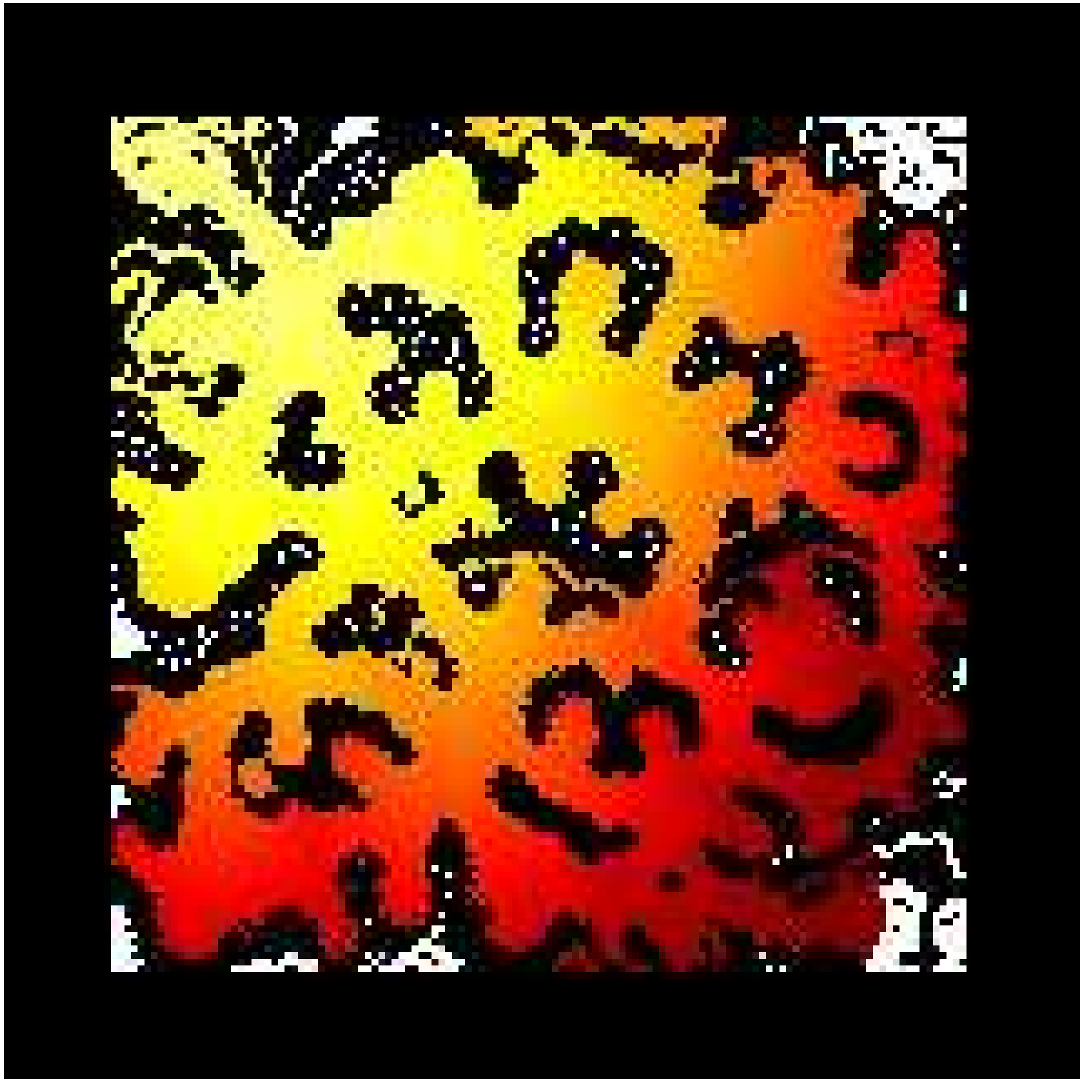


$$\mathbf{g} + \mathbf{h} < f^*$$



# Dijkstra vs A*: Pathfinding in Warcraft





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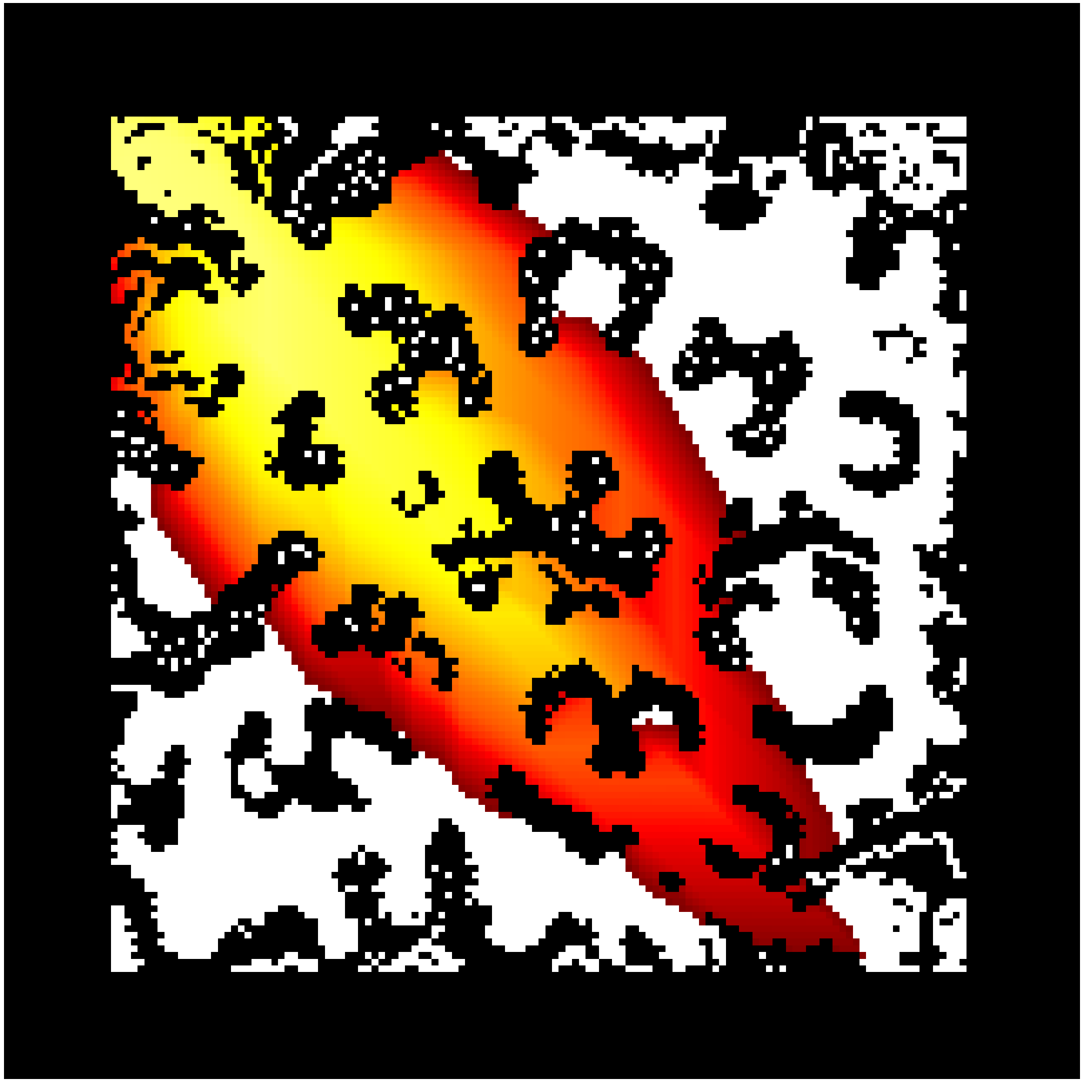

## Quick Note: A Lower Bound on Solution Cost



$$f(n) = g(n) + h(n)$$

$g(n)$ is actual cost-so-far, so
when $h(n)$ is a lower bound on cost-to-go,
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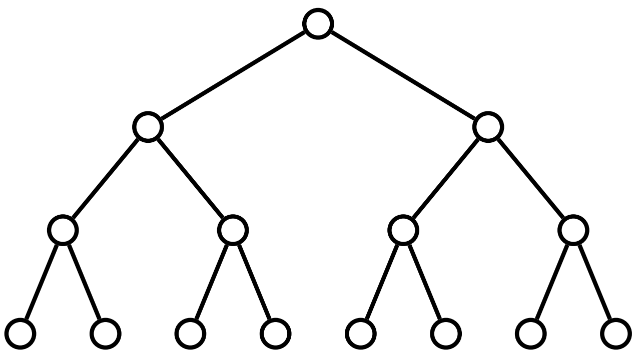
lowest $f(n)$ on frontier gives lower bound for entire problem!

$$best_f = \underset{n \in open}{\operatorname{argmin}} f(n)$$



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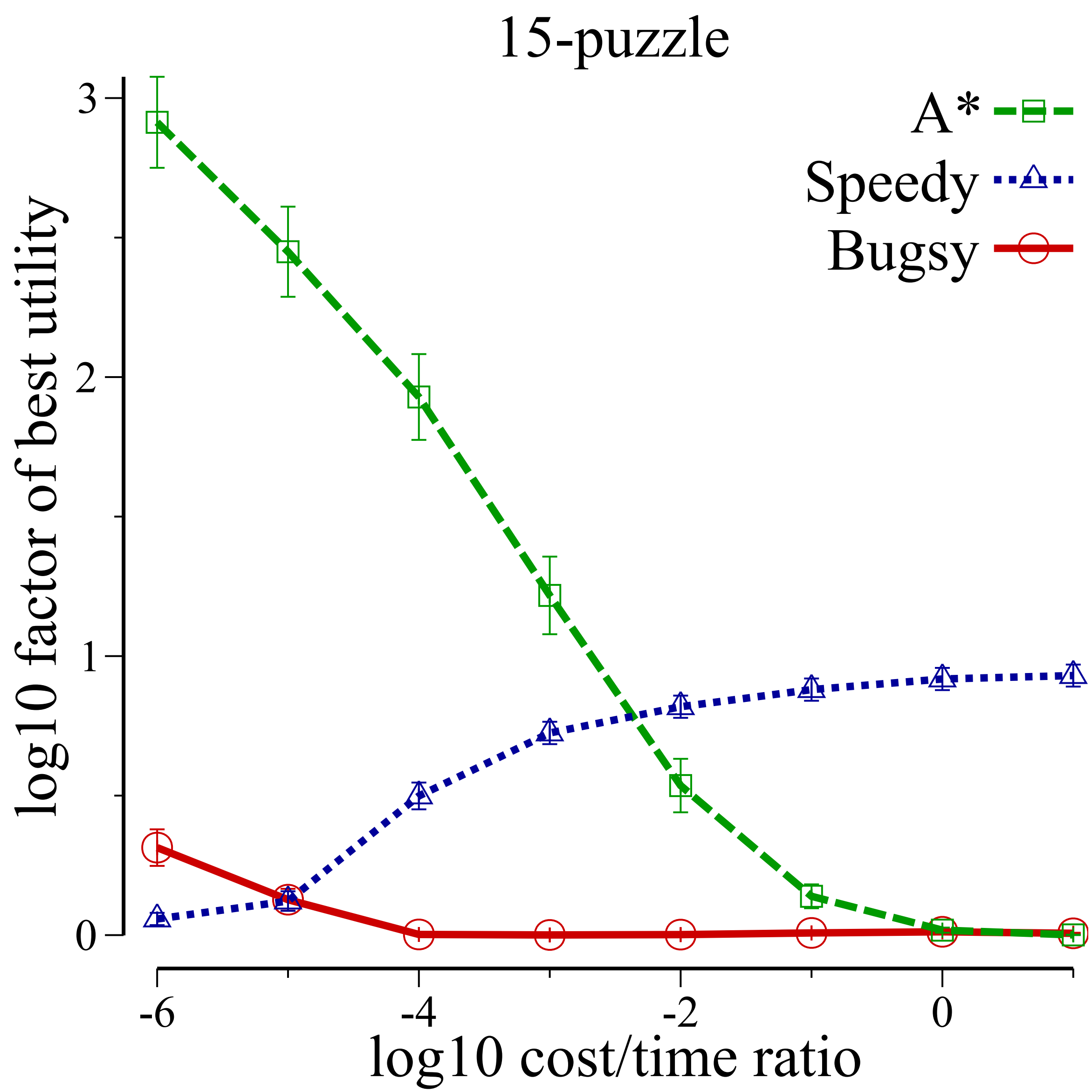
We must trade cost for time.

# Optimizing Utility of Cost + Time





# A New Generation of Problem Settings



**optimal:** minimize solution cost

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**bounded cost:** minimize time subject to absolute cost bound

**contract:** minimize cost subject to absolute time bound

**utility function:** maximize utility function of cost and time
eg, goal achievement time =
plan makespan + search time

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# Bounded Suboptimal Search

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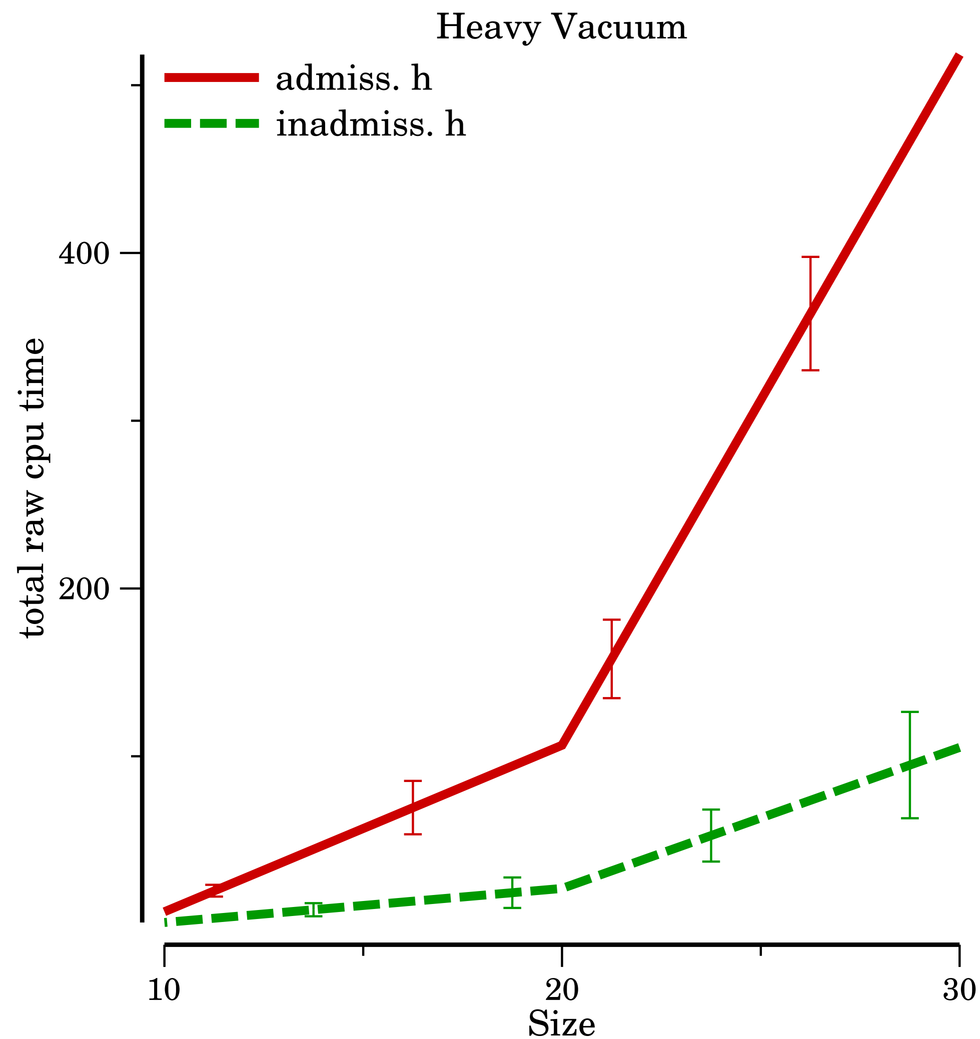

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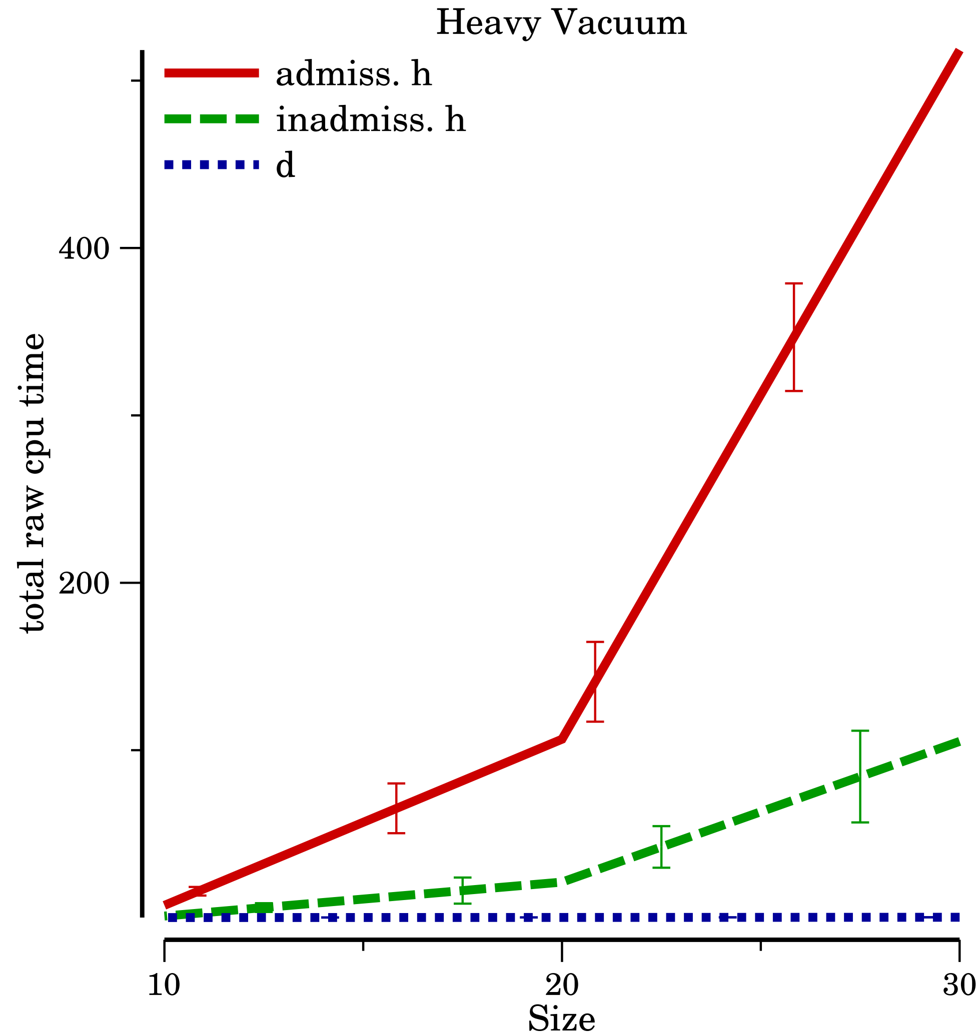

# Explicit Estimation Search (Thayer and Ruml, 2011)



- unbiased estimates can be more informed than lower bounds
- nearest goal is the easiest to find

minimize solving time subject to cost $\leq w \cdot$optimal:

pursue nearest goal estimated to lie within bound

need more information than just lower bound on cost ($h(n)$)!

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   (Thayer and Ruml, ICAPS-11)

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3. $\widehat{d}$: an estimate of distance-to-go

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   (Pearl and Kim, IEEE PAMI 1982,
   Thayer et al, ICAPS-09)

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# Search Strategy: Which Node to Expand?



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pursue nearest goal estimated to lie within bound

$best_{\widehat{d}}$: estimated $w$-suboptimal node with minimum $\widehat{d}$

$$\operatorname*{argmin}_{n \in open \wedge \widehat{f}(n) \leq w \cdot \widehat{f}(best_{\widehat{f}})} \widehat{d}(n)$$

# EES Expansion Order



$best_f$: open node giving lower bound on cost
$best_{\widehat{f}}$: open node giving estimated optimal cost
$best_{\widehat{d}}$: estimated $w$-suboptimal node with minimum $\widehat{d}$

node to expand next:

1. pursue the nearest goal estimated to lie within the bound
2.
3.

in other words:

1. $best_{\widehat{d}}$
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1. **if** $\widehat{f}(best_{\widehat{d}}) \leq w \cdot f(best_f)$ **then** $best_{\widehat{d}}$
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node to expand next:

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2. pursue the estimated optimal solution
3. raise the lower bound on optimal solution cost

in other words:

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2. **else if** $\widehat{f}(best_{\widehat{f}}) \leq w \cdot f(best_f)$ **then** $best_{\widehat{f}}$
3. **else** $best_f$

see paper for further justification. Note: no magic numbers!

# EES Respects the Suboptimality Bound



how does $\widehat{f}(n) \leq w \cdot f(best_f)$ ensure the suboptimality bound?

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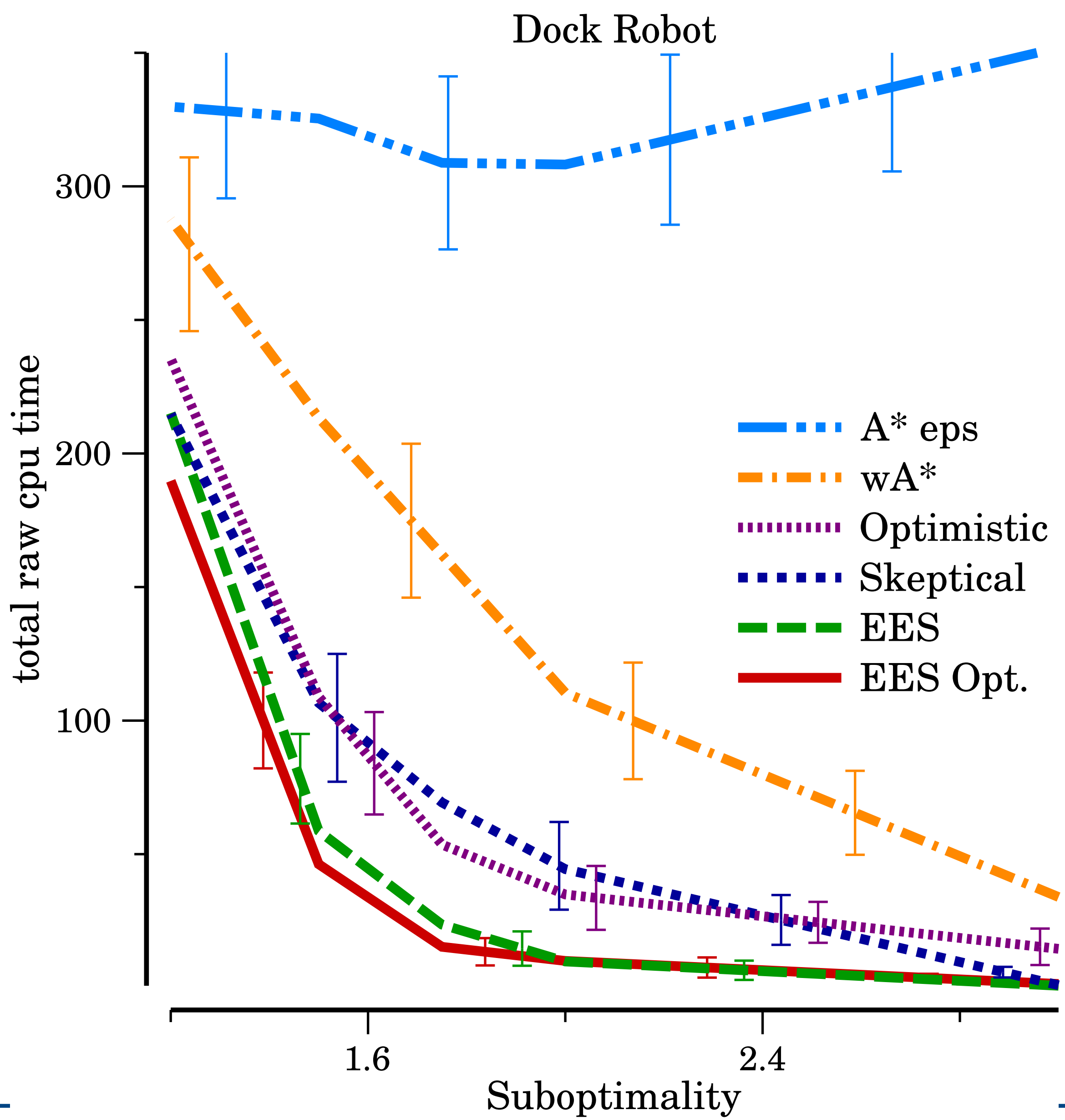
$$
\begin{array}{rcll}
f(n) & \leq & \widehat{f}(n) & f(n) \text{ is a lower bound for } n \\
\widehat{f}(n) & \leq & w \cdot f(best_f) & \text{expansion criterion} \\
w \cdot f(best_f) & \leq & w \cdot f^*(opt) & \text{because } f(best_f) \text{ is a lower} \\
 & & & \quad \text{bound for the entire problem} \\
\hline
f(n) & \leq & w \cdot f^*(opt) & \text{suboptimality bound}
\end{array}
$$

**bounded suboptimal search**:
minimize time subject to
relative cost bound (factor of optimal)

# EES Performance





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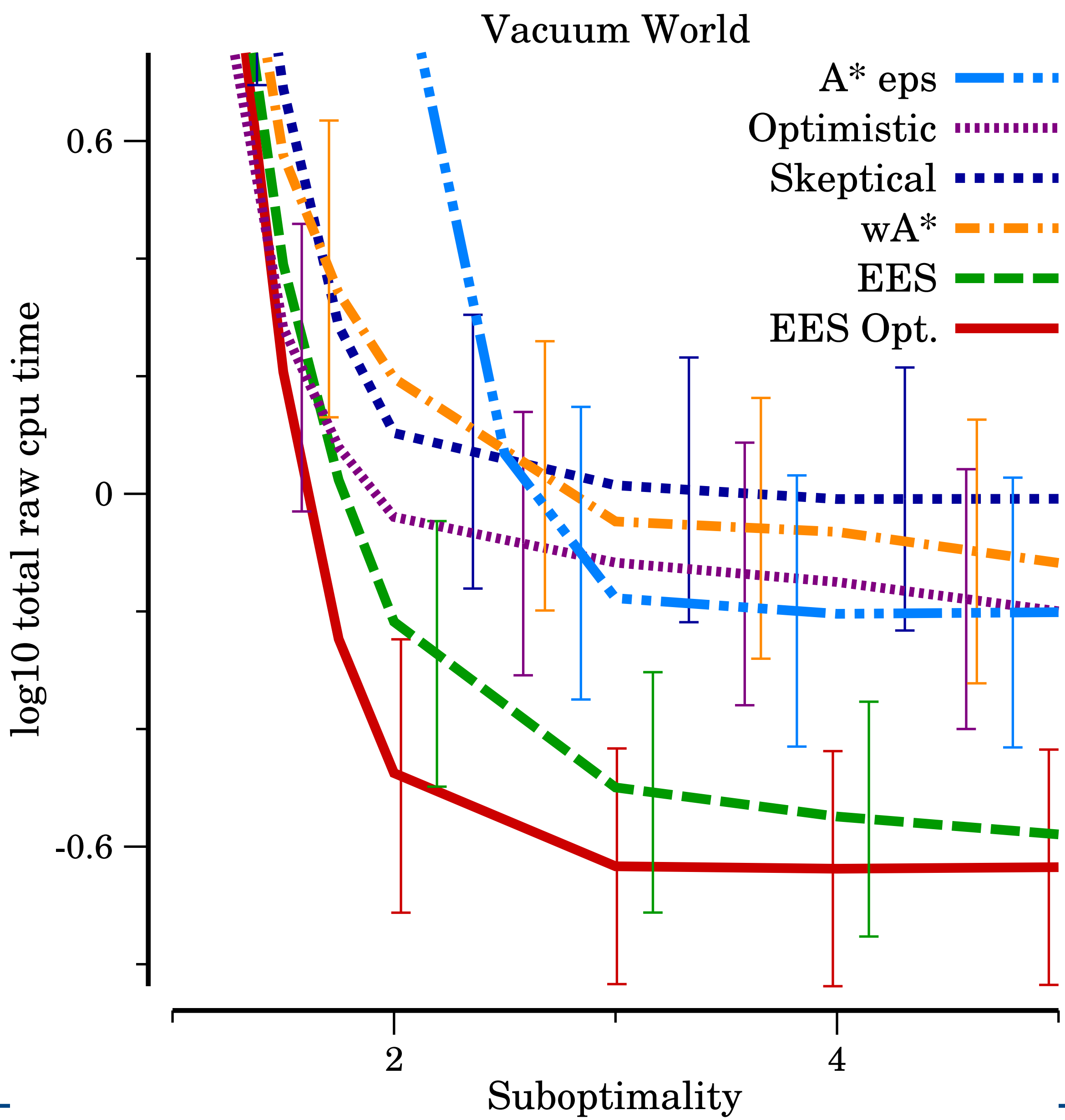

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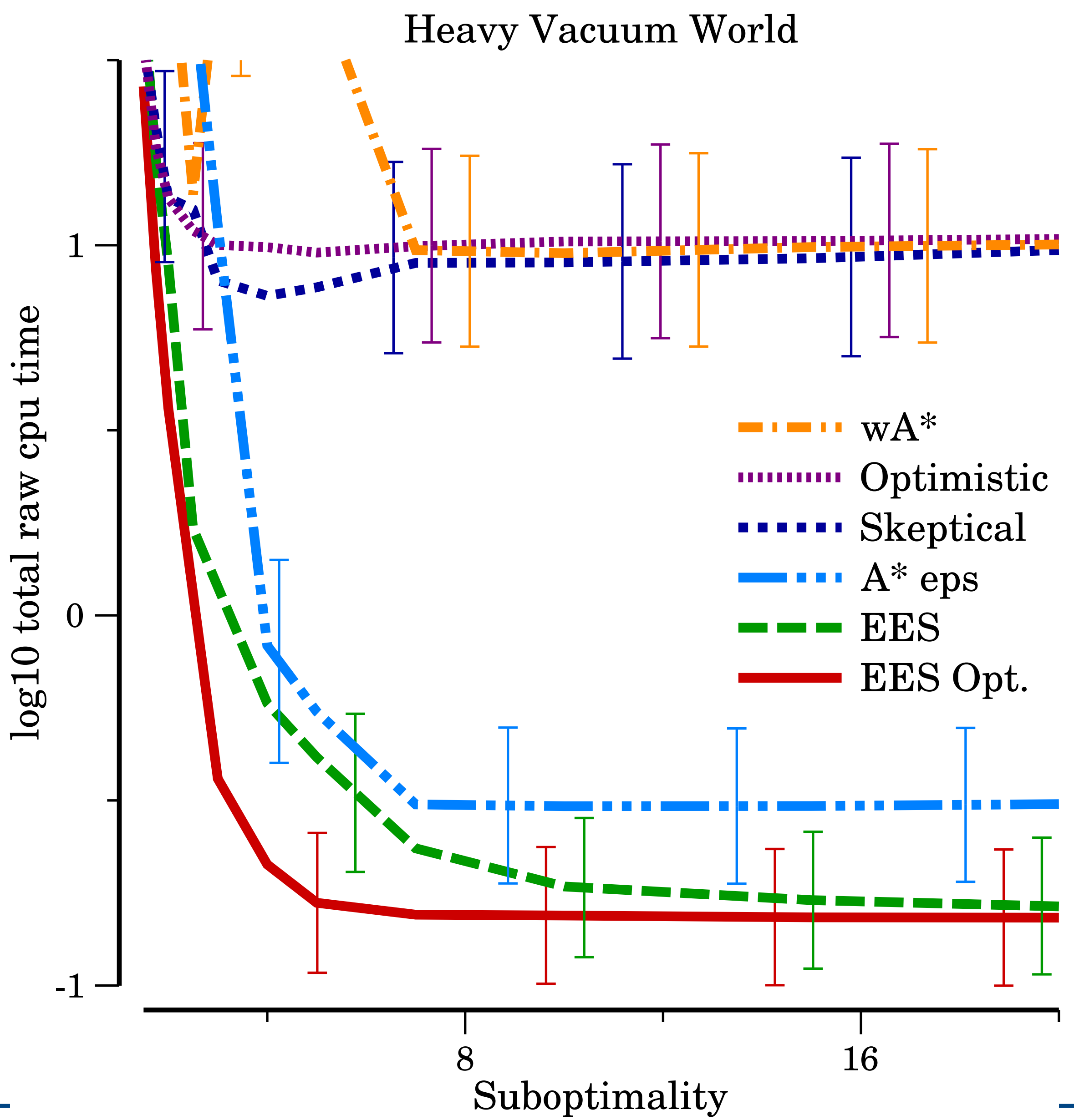


# Conclusion

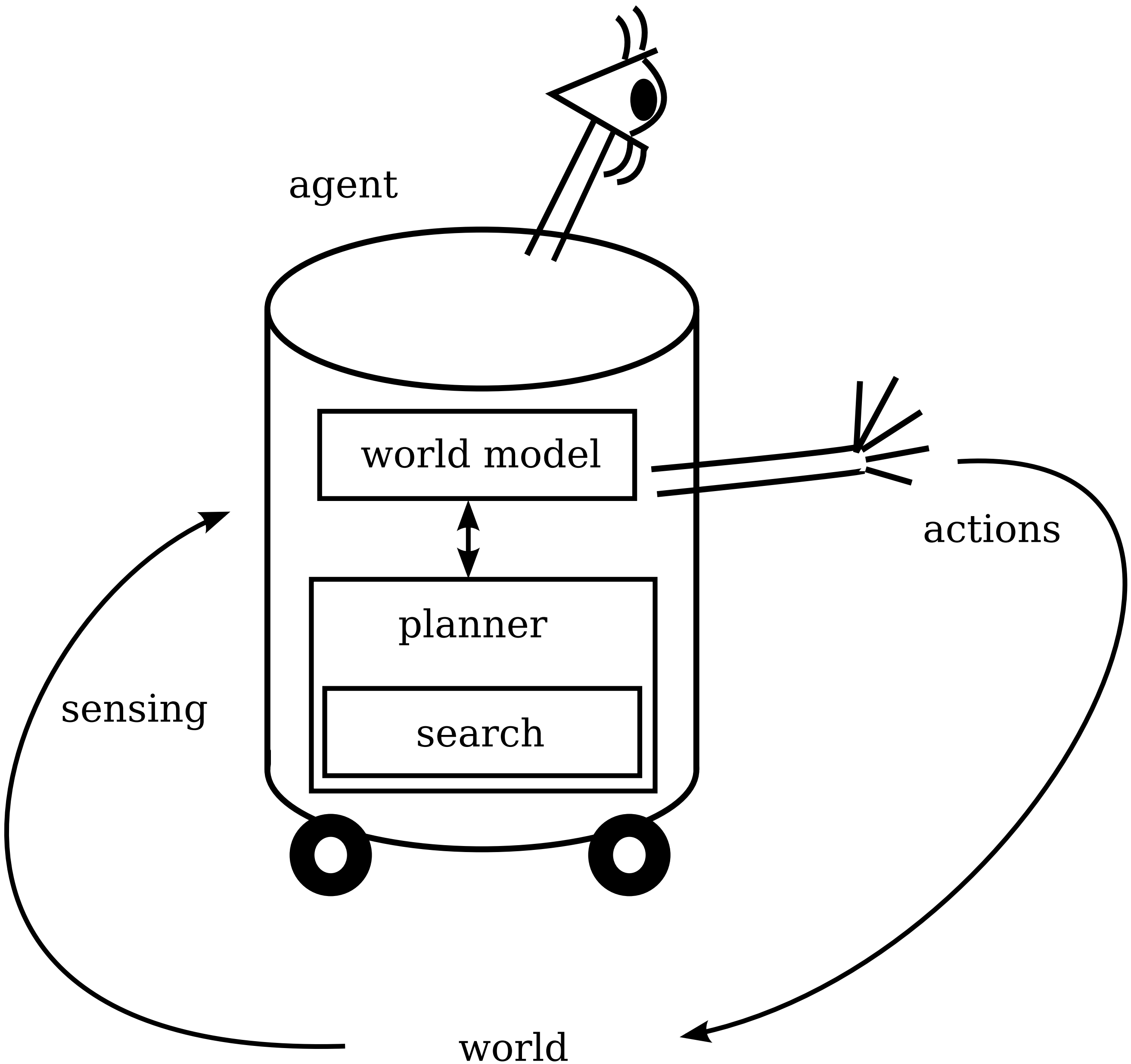
# A Science of Suboptimal Search



- what are the problem settings?
  - guarantees beyond optimal search
  - eg, bounded suboptimal search
  - utility-based optimization
- what are the sources of information?
  - where do inadmissible heuristics come from?
  - estimates instead of lower bounds
  - distance in addition to cost
- how to exploit and combine information?
  - new generation of suboptimal algorithms
  - meta-reasoning

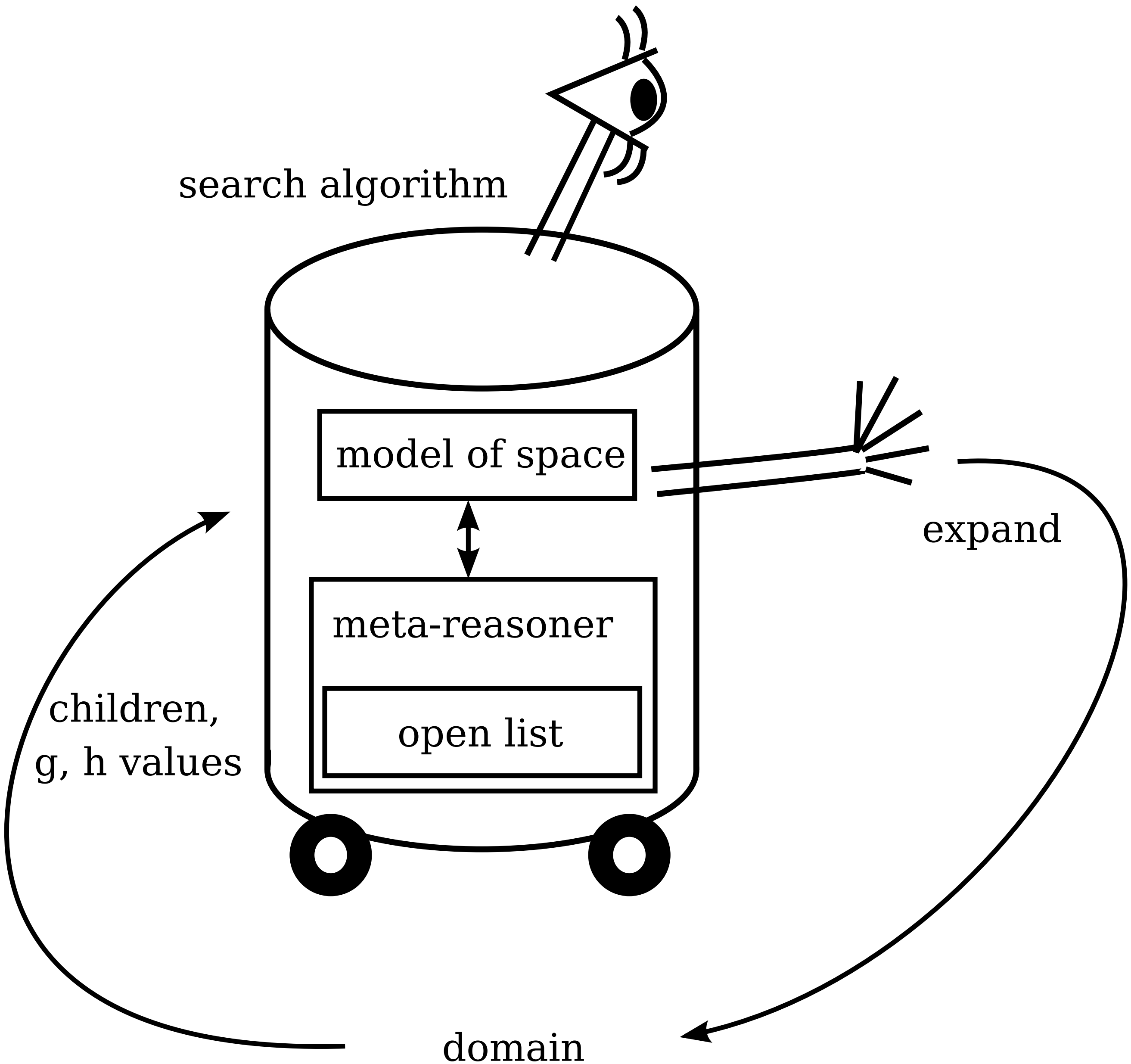
# The AI Vision





# Search Algorithms as Agents





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