

# Bounded Suboptimal Search: A Direct Approach Using Inadmissible Estimates

Jordan T. Thayer and Wheeler Ruml



UNIVERSITY *of* NEW HAMPSHIRE

`jtd7, ruml at cs.unh.edu`

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# Greedy Search

## Outline

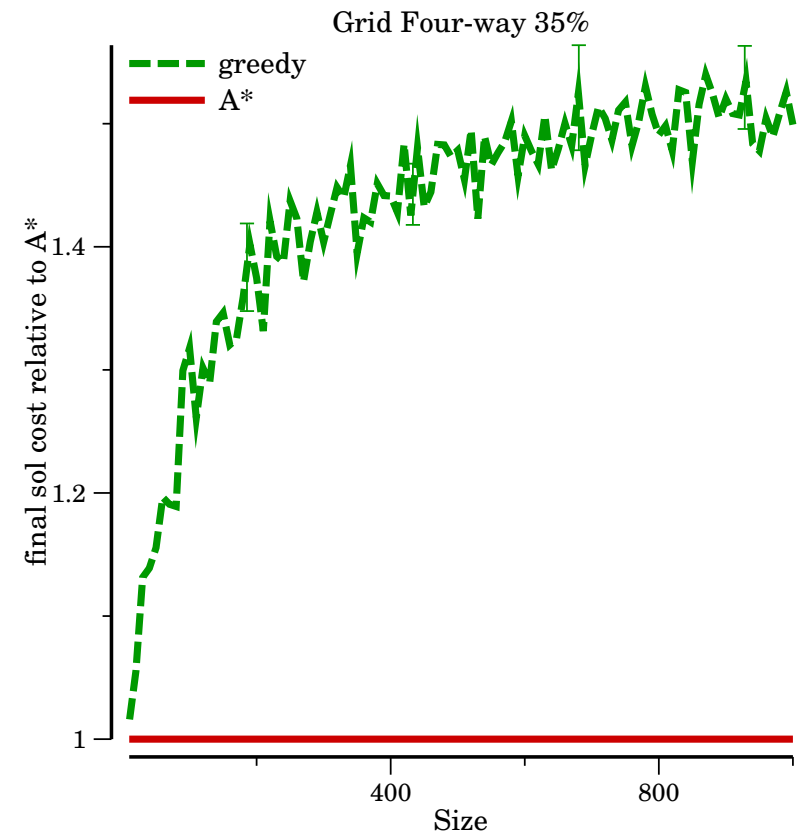
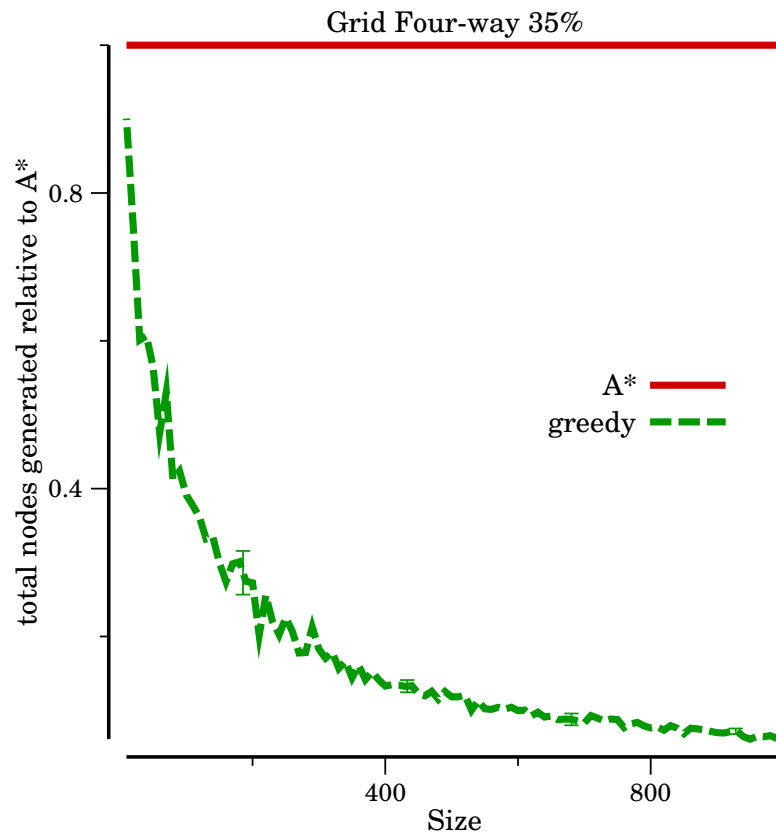
### Greedy Search

### Bounded Search

### Three Ideas

### EES

### Conclusion



- sacrificing optimality can speed search
- solutions could be arbitrarily bad

# Bounded Suboptimal Search: A Middle Ground

## Outline

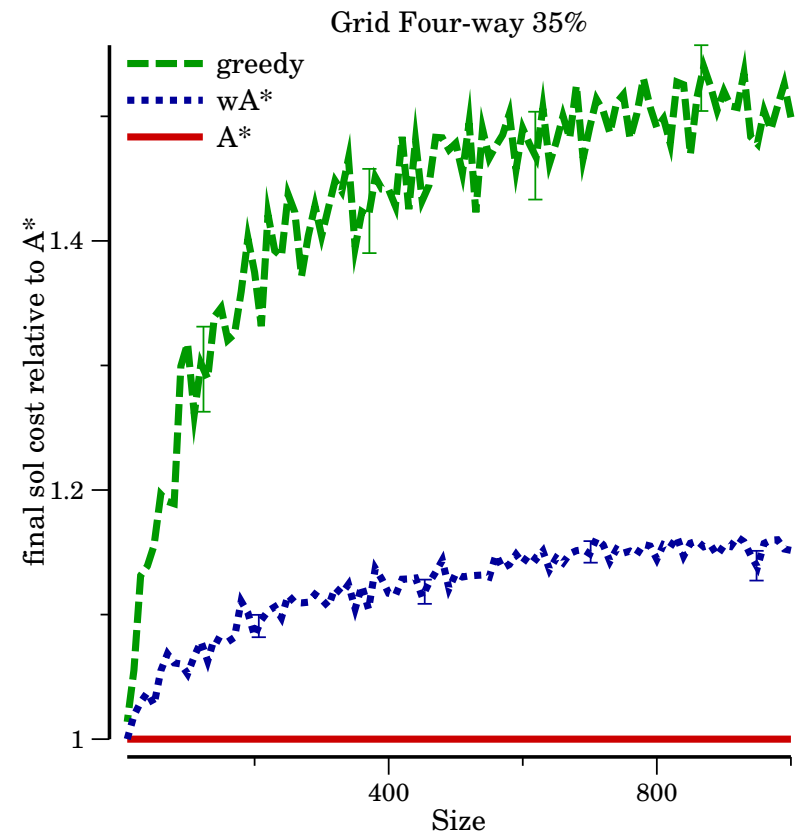
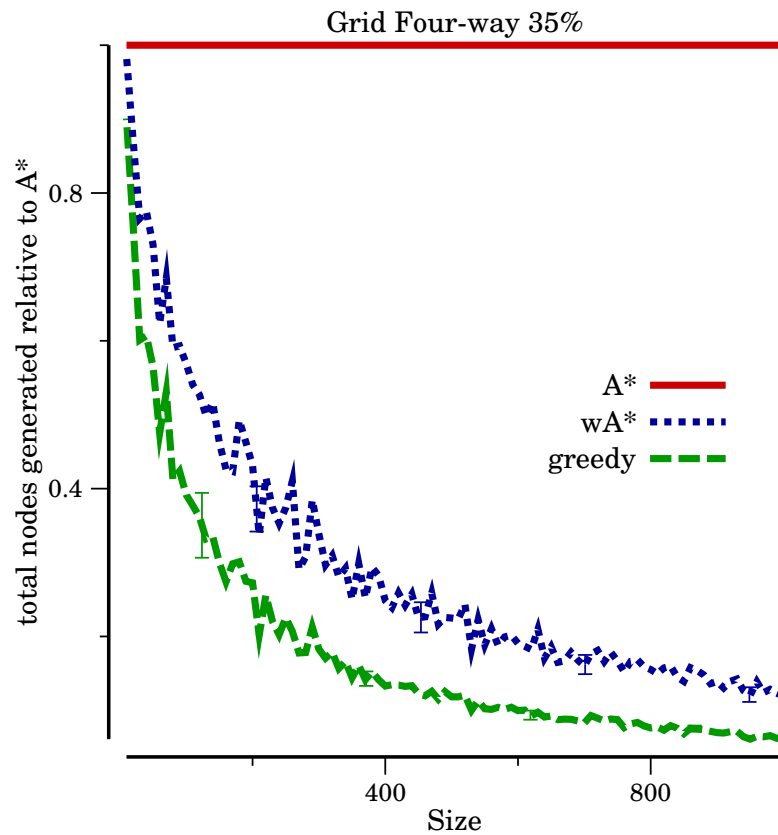
■ Greedy Search

■ Bounded Search

■ Three Ideas

EES

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given a suboptimality bound  $w$ ,  
find a solution with cost within a factor  $w$  of optimal  
as quickly as possible.

# Three Useful Ideas

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## Outline

- Greedy Search
- Bounded Search
- Three Ideas

## EES

## Conclusion

- finding solutions and proving bounds are separate tasks
- inadmissible cost estimates can be more informed
- searching on distance is faster than cost

# Three Useful Ideas

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- finding solutions and proving bounds are separate tasks
- inadmissible cost estimates can be more informed
- searching on distance is faster than cost
  
- $A_\epsilon^*$  Pearl and Kim, 1982
- Optimistic Search Thayer and Ruml, 2008
- Skeptical Search Thayer and Ruml, 2011

# Three Useful Ideas

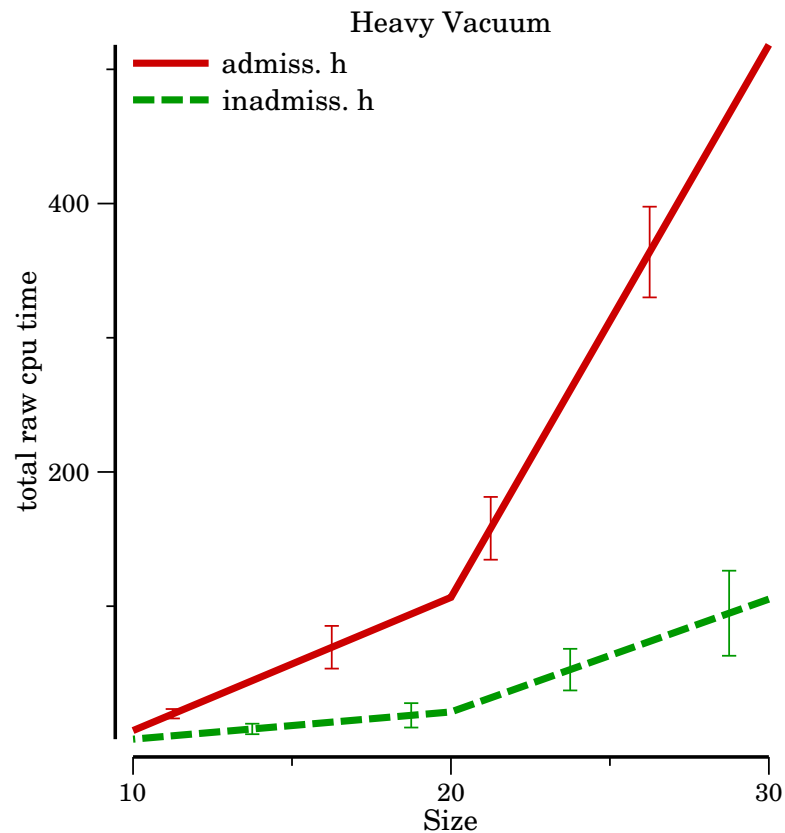
- finding solutions and proving bounds are separate tasks
- **inadmissible cost estimates can be more informed**
- searching on distance is faster than cost

## Outline

- Greedy Search
- Bounded Search
- **Three Ideas**

## EES

## Conclusion



# Three Useful Ideas

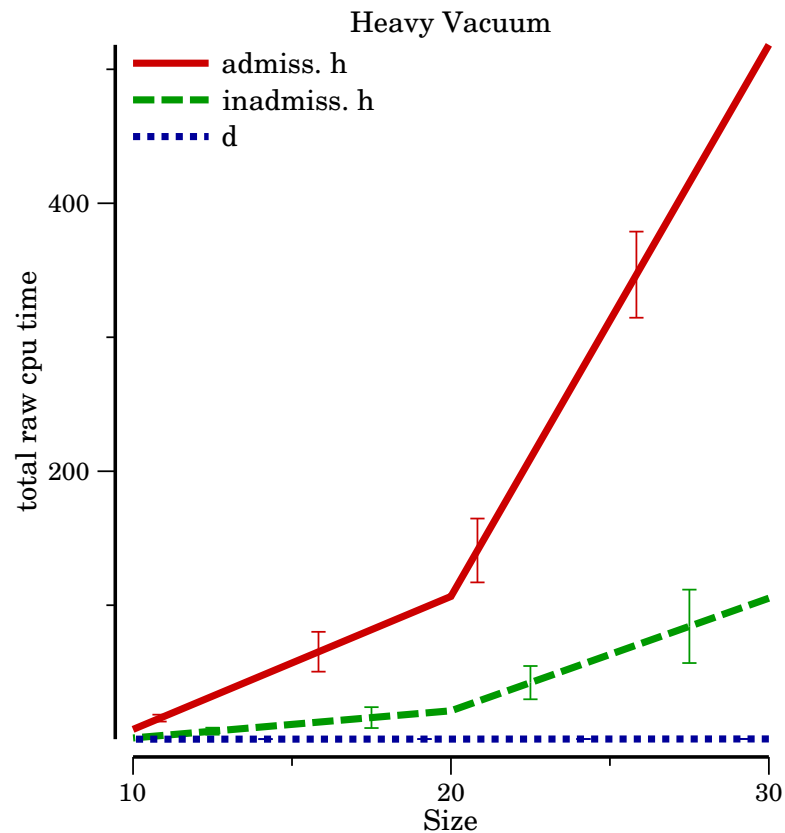
- finding solutions and proving bounds are separate tasks
- inadmissible cost estimates can be more informed
- **searching on distance is faster than cost**

## Outline

- Greedy Search
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# Three Useful Ideas

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## Outline

- Greedy Search
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## Conclusion

- finding solutions and proving bounds are separate tasks
- inadmissible cost estimates can be more informed
- searching on distance is faster than cost

Explicit Estimation Search (EES) combines these three ideas.



Outline

**EES**

- Direct Approach
- Three Heuristics
- EES

Conclusion

# Explicit Estimation Search

# A Direct Approach

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minimize solving time subject to suboptimality bound  $w$

Outline

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EES

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■ Direct Approach

■ Three Heuristics

■ EES

Conclusion

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# A Direct Approach

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Outline

EES

■ Direct Approach

■ Three Heuristics

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Conclusion

minimize solving time subject to suboptimality bound  $w$

weighted  $A^*$  ( $f'(n) = g(n) + w \cdot h(n)$ ) is **simple but ad hoc**  
(Pohl, AIJ vol 1, 1970)

# A Direct Approach

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EES

■ Direct Approach

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■ EES

Conclusion

minimize solving time subject to suboptimality bound  $w$

weighted  $A^*$  ( $f'(n) = g(n) + w \cdot h(n)$ ) is **simple but ad hoc**  
(Pohl, AIJ vol 1, 1970)

expand the node closest to a solution within the bound

$best_{\hat{d}}$ : node estimated within bound closest to a goal

# Three Heuristic Sources Of Information

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Outline

EES

■ Direct Approach

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■ EES

Conclusion

1.  $h$ : an admissible estimate of cost-to-go

$$f(n) = g(n) + h(n)$$

finding solutions and proving bounds are separate tasks

# Three Heuristic Sources Of Information

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■ EES

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1.  $h$ : an admissible estimate of cost-to-go

$$f(n) = g(n) + h(n)$$

finding solutions and proving bounds are separate tasks

2.  $\hat{h}$ : a potentially inadmissible estimate of cost-to-go

inadmissible cost estimates can be more informed

$$\hat{f}(n) = g(n) + \hat{h}(n)$$

(Thayer and Ruml, ICAPS-11)

# Three Heuristic Sources Of Information

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Conclusion

1.  $h$ : an admissible estimate of cost-to-go

$$f(n) = g(n) + h(n)$$

finding solutions and proving bounds are separate tasks

2.  $\hat{h}$ : a potentially inadmissible estimate of cost-to-go

inadmissible cost estimates can be more informed

$$\hat{f}(n) = g(n) + \hat{h}(n)$$

(Thayer and Ruml, ICAPS-11)

3.  $\hat{d}$ : a potentially inadmissible estimate of distance-to-go

searching on distance is faster than cost

(Pearl and Kim, IEEE PAMI 1982,

Thayer et al, ICAPS-09)

# Finding $best_{\hat{d}}$

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Outline

EES

■ Direct Approach

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■ EES

Conclusion

$best_f$ : open node with minimum  $f$

$$\operatorname{argmin}_{n \in open} f(n)$$



# Finding $best_{\hat{d}}$

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$best_f$ : open node with minimum  $f$

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$best_{\hat{f}}$ : open node with minimum  $\hat{f}$

$$\operatorname{argmin}_{n \in open} \hat{f}(n)$$

# Finding $best_{\hat{d}}$

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Conclusion

$best_f$ : open node with minimum  $f$

$$\operatorname{argmin}_{n \in open} f(n)$$

$best_{\hat{f}}$ : open node with minimum  $\hat{f}$

$$\operatorname{argmin}_{n \in open} \hat{f}(n)$$

pursuing the shortest solution within the bound  
should be fast

$best_{\hat{d}}$ : estimated  $w$ -admissible node with minimum  $\hat{d}$

$$\operatorname{argmin}_{n \in open \wedge \hat{f}(n) \leq w \cdot \hat{f}(best_{\hat{f}})} \hat{d}(n)$$

# EES Expansion Order

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Outline

EES

■ Direct Approach

■ Three Heuristics

■ EES

Conclusion

$best_f$ : open node with minimum  $f$

$best_{\hat{f}}$ : open node with minimum  $\hat{f}$

$best_{\hat{d}}$ : estimated  $w$ -admissible node with minimum  $\hat{d}$

node to expand next:

1. pursue the shortest solution that is within the bound.
- 2.
- 3.

in other words:

1.  $best_{\hat{d}}$
- 2.
- 3.

# EES Expansion Order

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$best_f$ : open node with minimum  $f$

$best_{\hat{f}}$ : open node with minimum  $\hat{f}$

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node to expand next:

1. pursue the shortest solution that is within the bound.
- 2.
- 3.

in other words:

1. **if**  $\hat{f}(best_{\hat{d}}) \leq w \cdot f(best_f)$  **then**  $best_{\hat{d}}$
- 2.
- 3.

note that  $f(best_f) \leq f(opt)$  and  $f(n) \leq \hat{f}(n)$

# EES Expansion Order

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$best_f$ : open node with minimum  $f$

$best_{\hat{f}}$ : open node with minimum  $\hat{f}$

$best_{\hat{d}}$ : estimated  $w$ -admissible node with minimum  $\hat{d}$

node to expand next:

1. pursue the shortest solution that is within the bound.
2. pursue the optimal solution.
- 3.

in other words:

1. **if**  $\hat{f}(best_{\hat{d}}) \leq w \cdot f(best_f)$  **then**  $best_{\hat{d}}$
2. **else if**  $\hat{f}(best_{\hat{f}}) \leq w \cdot f(best_f)$  **then**  $best_{\hat{f}}$
- 3.

# EES Expansion Order

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Outline

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Conclusion

$best_f$ : open node with minimum  $f$

$best_{\hat{f}}$ : open node with minimum  $\hat{f}$

$best_{\hat{d}}$ : estimated  $w$ -admissible node with minimum  $\hat{d}$

node to expand next:

1. pursue the shortest solution that is within the bound.
2. pursue the optimal solution.
3. raise the lower bound on optimal solution cost.

in other words:

1. **if**  $\hat{f}(best_{\hat{d}}) \leq w \cdot f(best_f)$  **then**  $best_{\hat{d}}$
2. **else if**  $\hat{f}(best_{\hat{f}}) \leq w \cdot f(best_f)$  **then**  $best_{\hat{f}}$
3. **else**  $best_f$

see paper for further justification

# EES Results

Outline

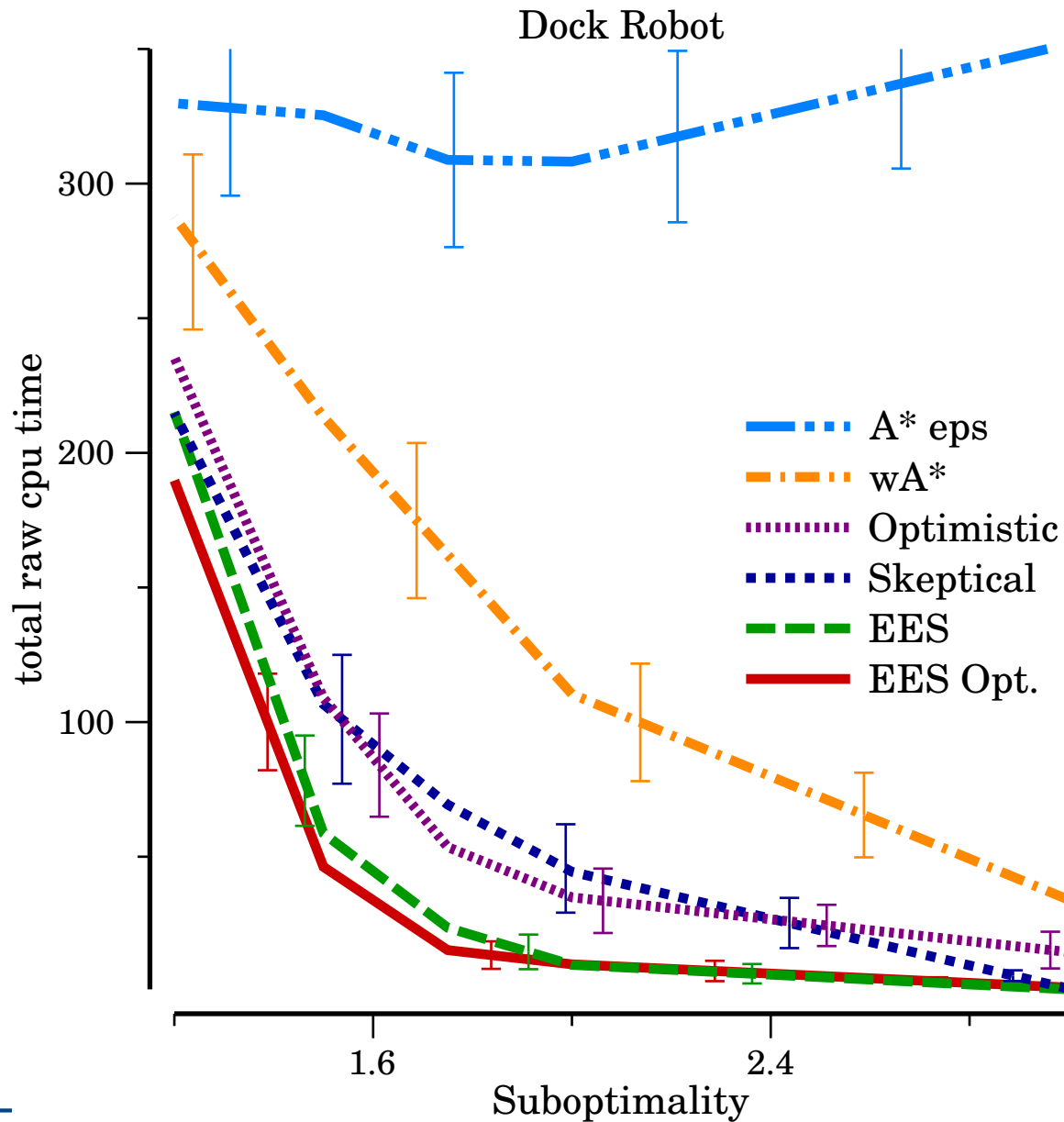
EES

■ Direct Approach

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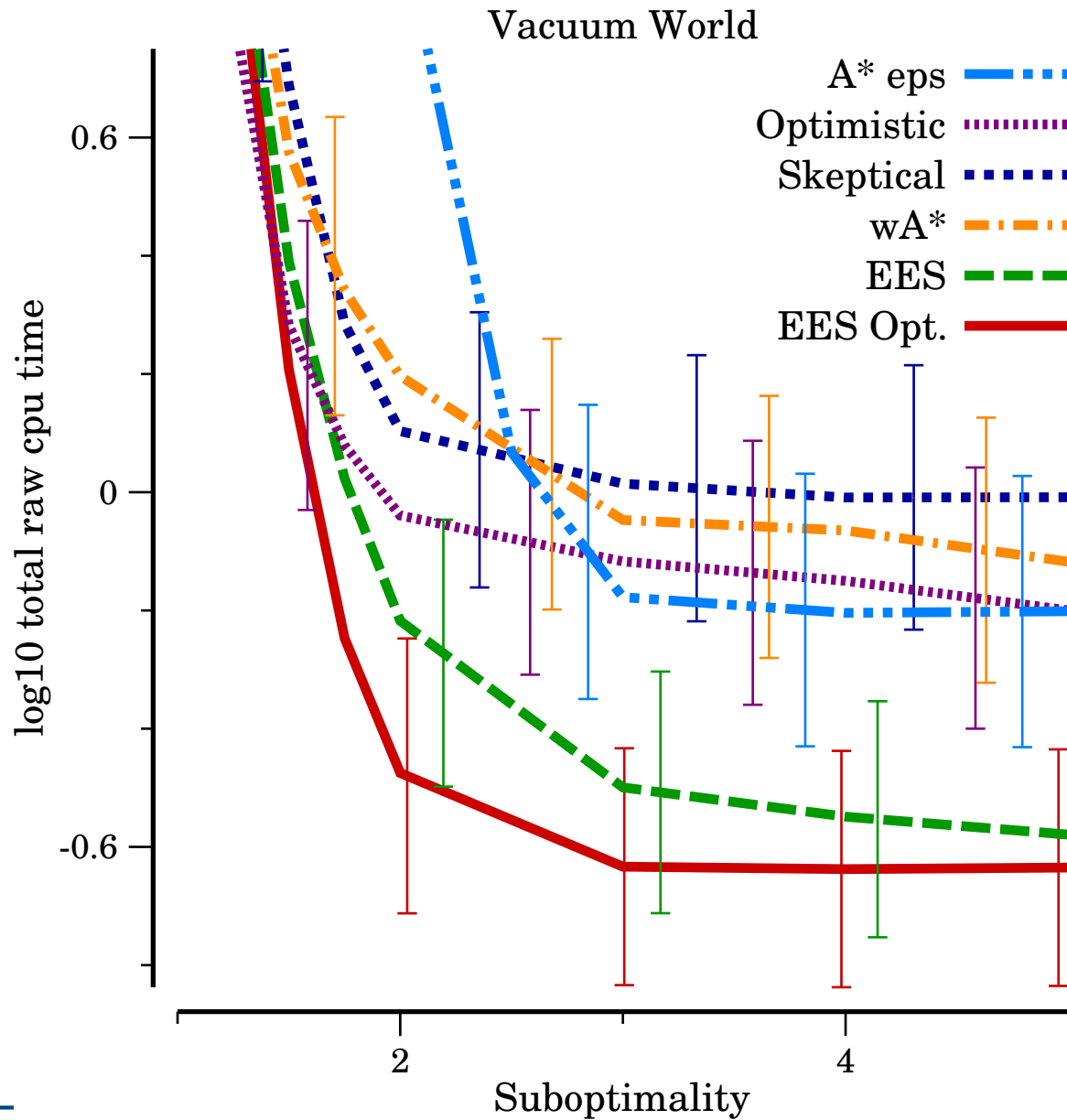
■ EES

Conclusion



# EES Results

- Outline
- EES
  - Direct Approach
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  - EES**
- Conclusion





# EES Results

Outline

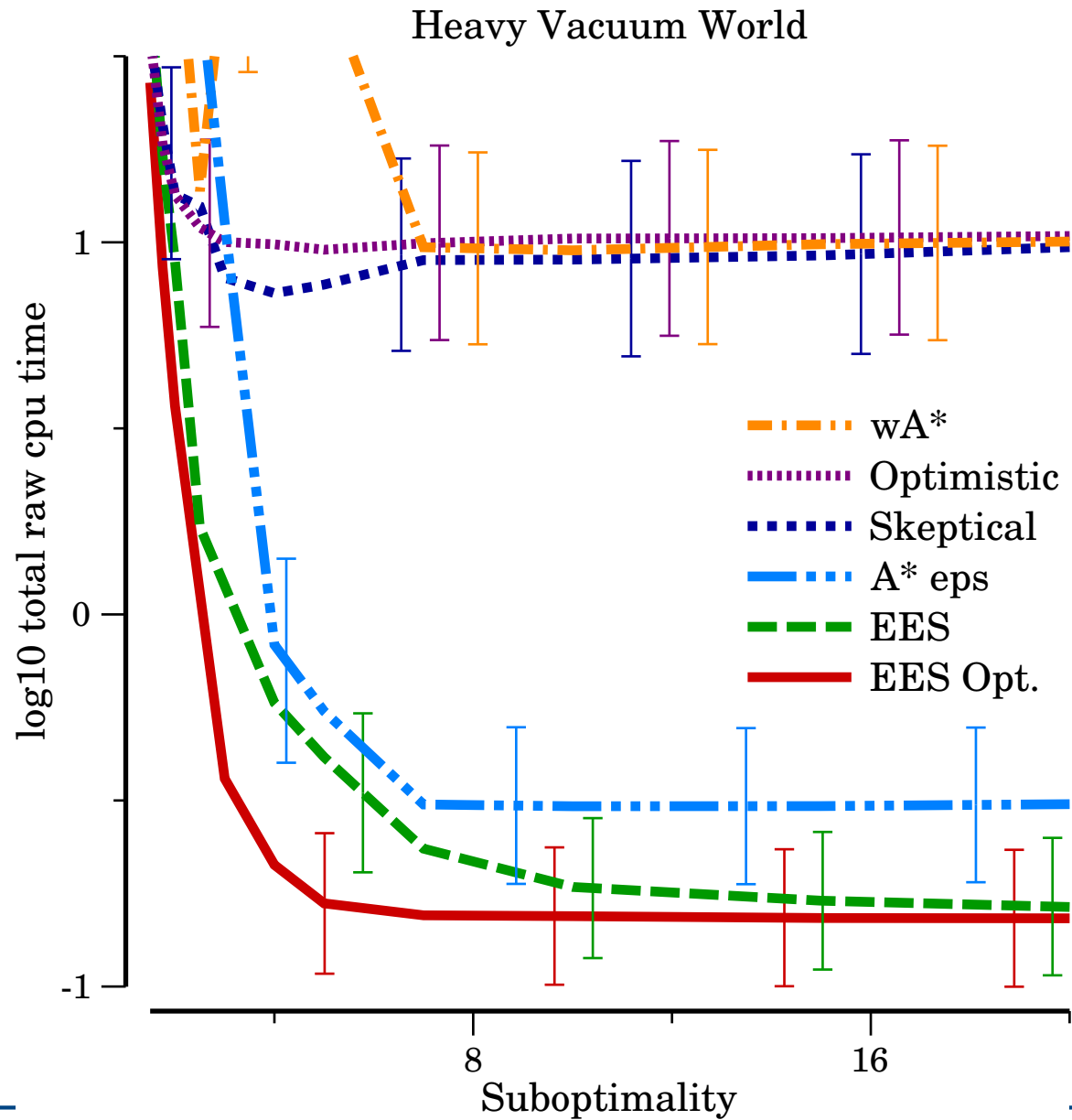
EES

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## Explicit Estimation Search (EES)

- follows directly from the objectives of bounded suboptimal search
- state of the art search bounded suboptimal search
- use inadmissible heuristics without losing bounds
- robust, works best in domains with action costs

# The University of New Hampshire

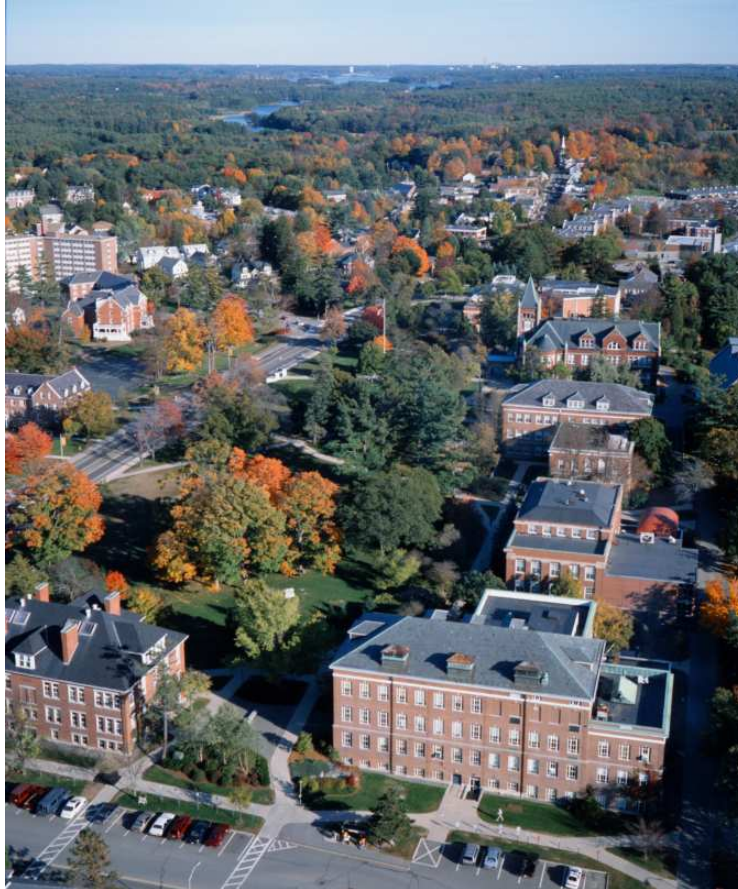
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Outline

EES

Conclusion

tell your students to apply to grad school in cs at UNH!



- friendly faculty
- funding
- individual attention
- beautiful campus
- low cost of living
- easy access to Boston, White Mountains
- strong in AI, infoviz, networking, systems, bioinformatics

Outline

EES

Conclusion

Backup Slides

■ EES Nodes

■ Bound

■ Overhead

■  $A_\epsilon^*$

■  $A_\epsilon^*$  Failure

$$best_{\hat{f}} = \operatorname{argmin}_{n \in open} \hat{f}(n)$$

$$best_{\hat{d}} = \operatorname{argmin}_{n \in open \wedge \hat{f}(n) \leq w \cdot \hat{f}(best_{\hat{f}})} \hat{d}(n)$$

$$best_f = \operatorname{argmin}_{n \in open} f(n)$$

# EES Respects a Bound

Outline

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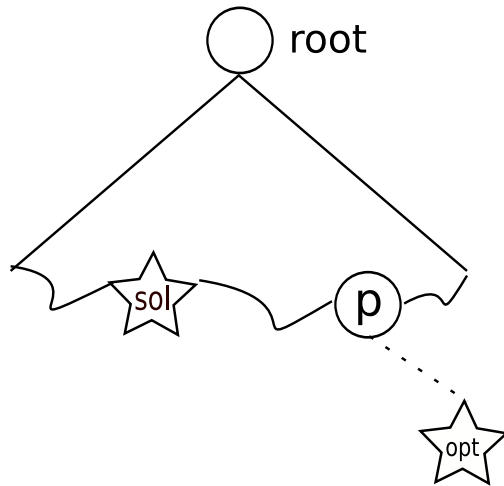
■ EES Nodes

■ Bound

■ Overhead

■  $A_\epsilon^*$

■  $A_\epsilon^*$  Failure



- $p$  is the deepest node on an optimal path to  $opt$ .
- $best_f$  is the node with the smallest  $f$  value.

$$f(best_f) \leq f(p) \leq f(opt)$$

$best_f$  provides a lower bound on solution cost.

determine  $best_f$  by priority queue sorted on  $f$

# Why Doesn't $A_\epsilon^*$ Work Well?

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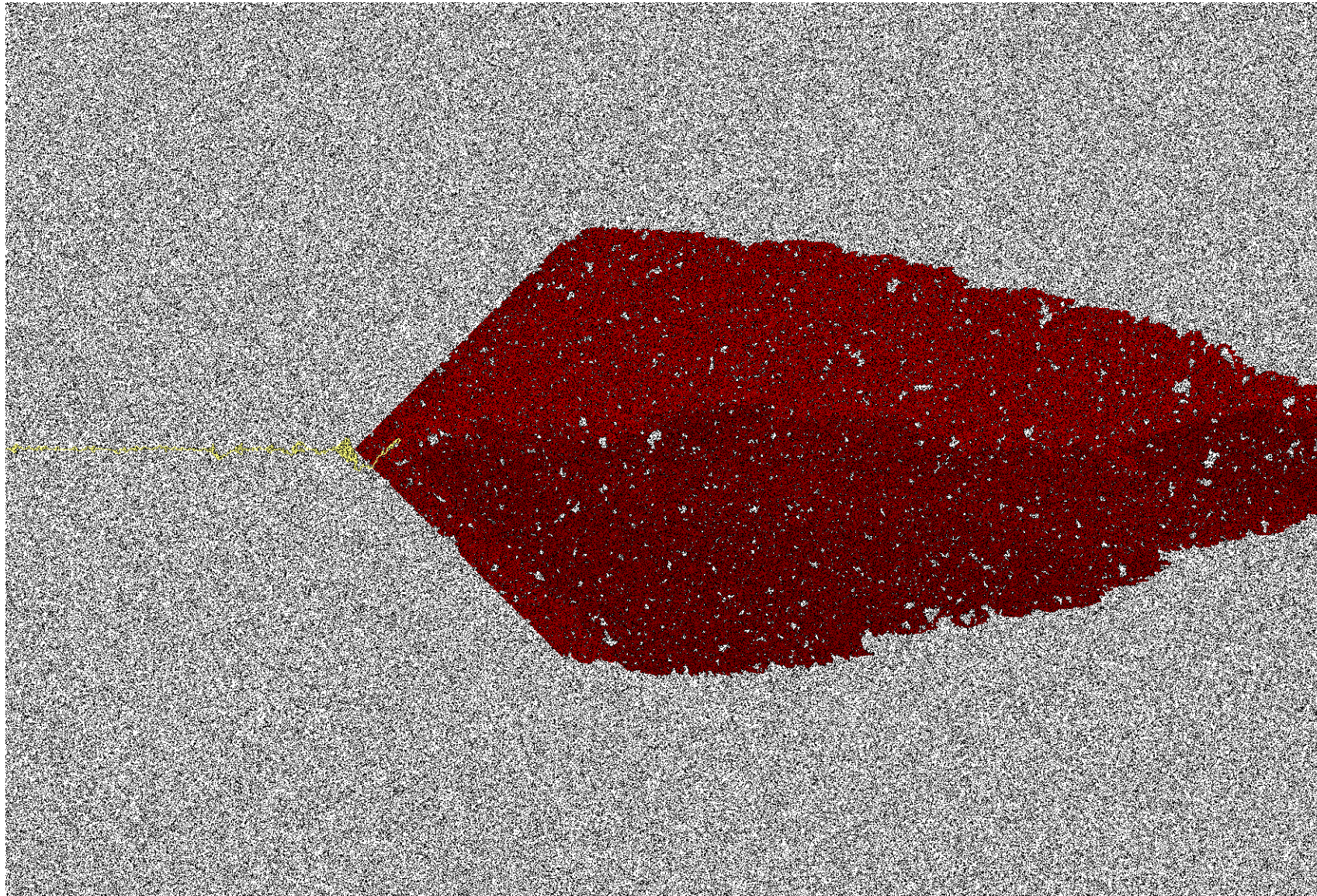
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# EES Overhead

Outline

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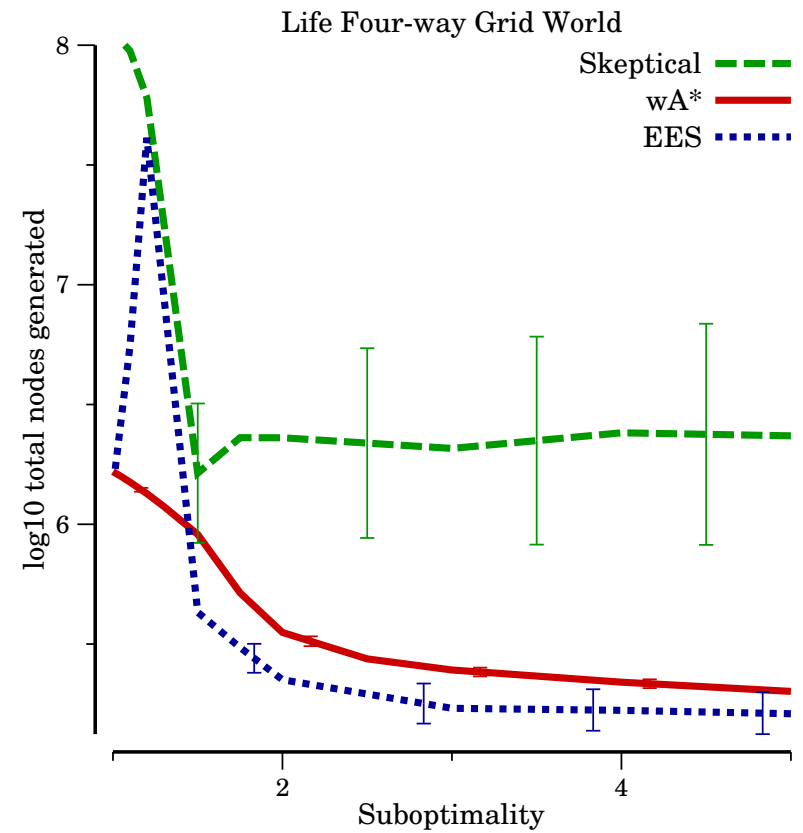
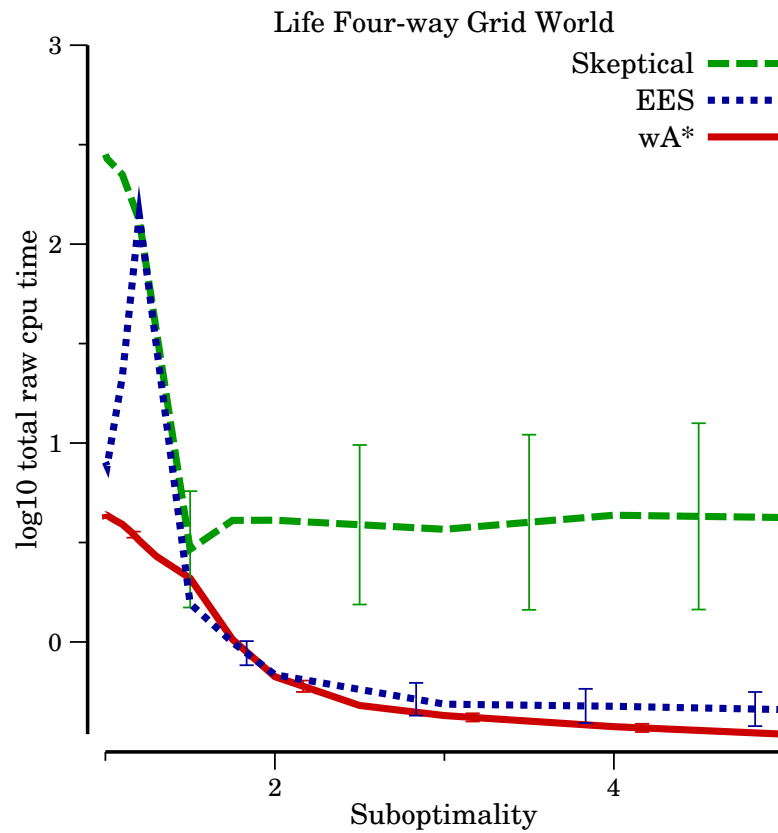
■ EES Nodes

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■  $A_\epsilon^*$  Failure



Outline

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■ EES Nodes

■ Bound

■ Overhead

■ A\*  
ε

■ A\*  
ε Failure

intuition: of all solutions within the bound, the nearest should be the fastest to find.

$$f(n) = g(n) + h(n)$$

$best_f$ : generated but unexpanded node with minimum  $f$

best-first search on two lists:

*open*: all generated but unexpanded nodes, sorted on  $f(n)$

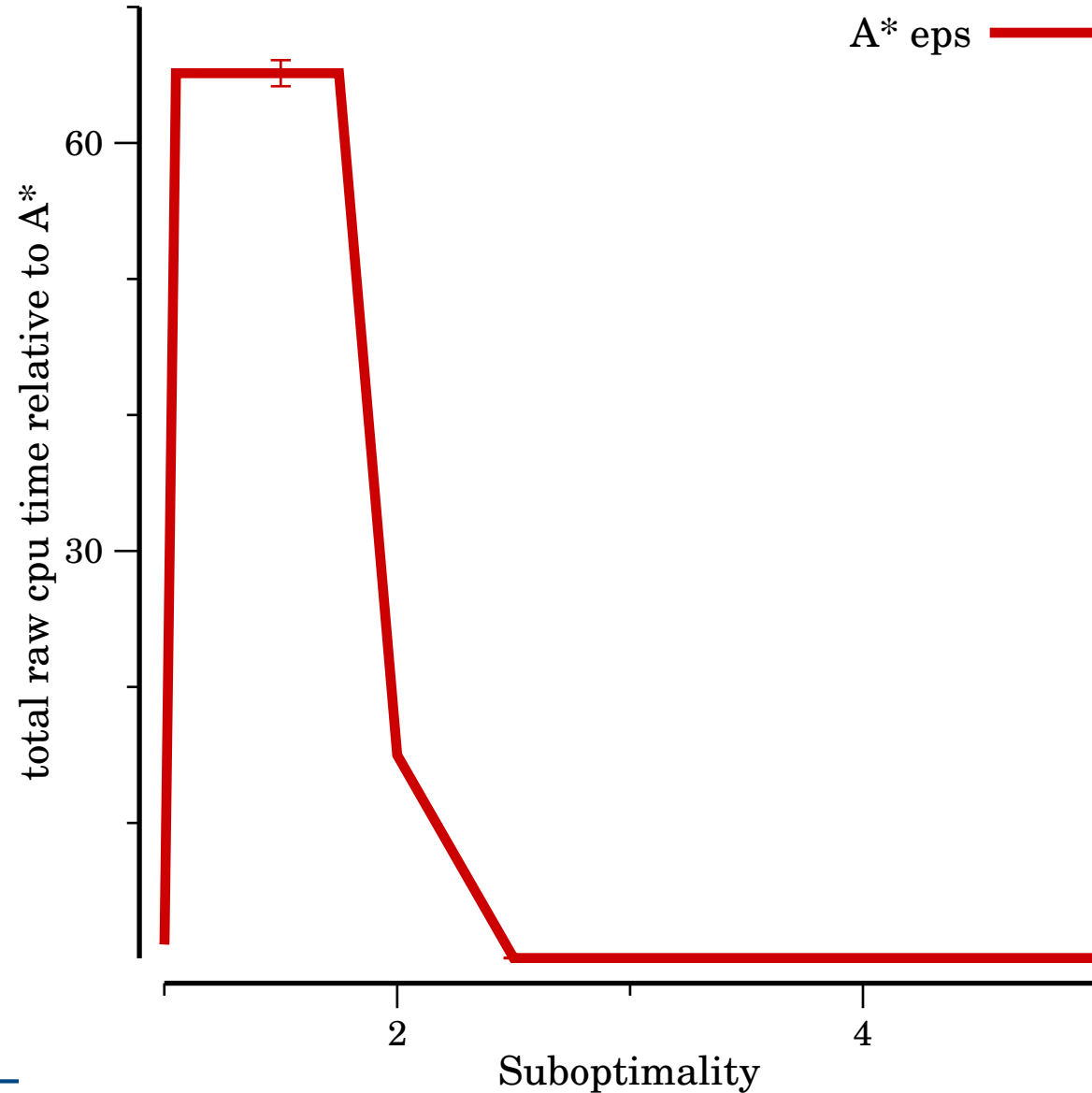
*focal*: all nodes where  $f(n) \leq w \cdot f(best_f)$  sorted on  $\hat{d}(n)$

expand the best node from *focal*



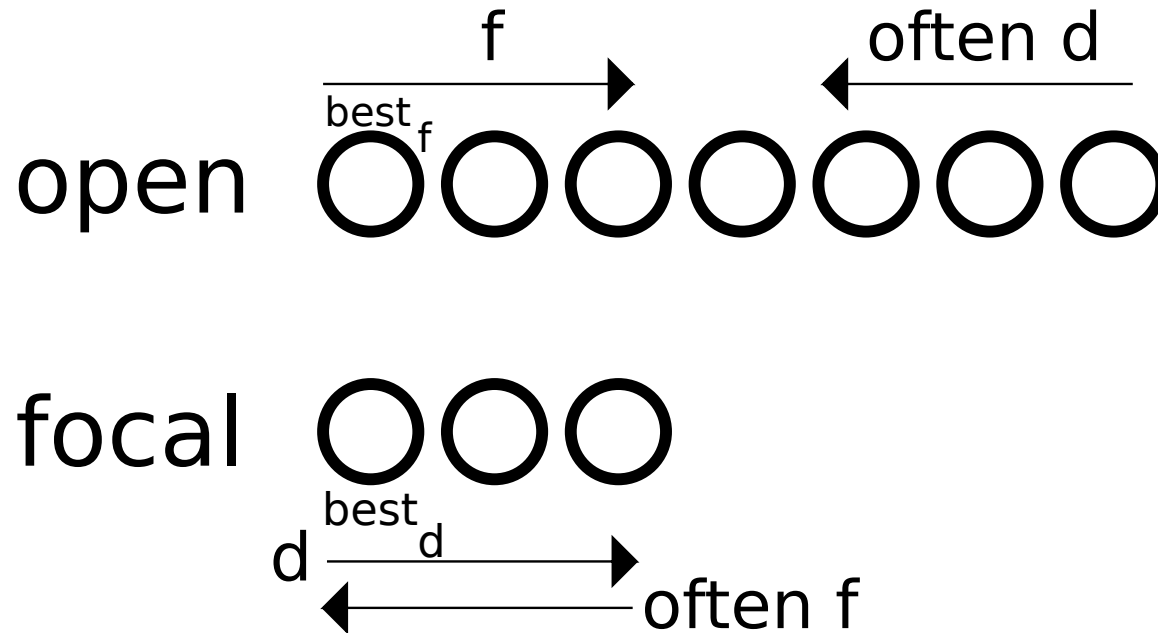
- Outline
- EES
- Conclusion
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  - $A^*_\epsilon$  Failure

### Life Four-way Grid World



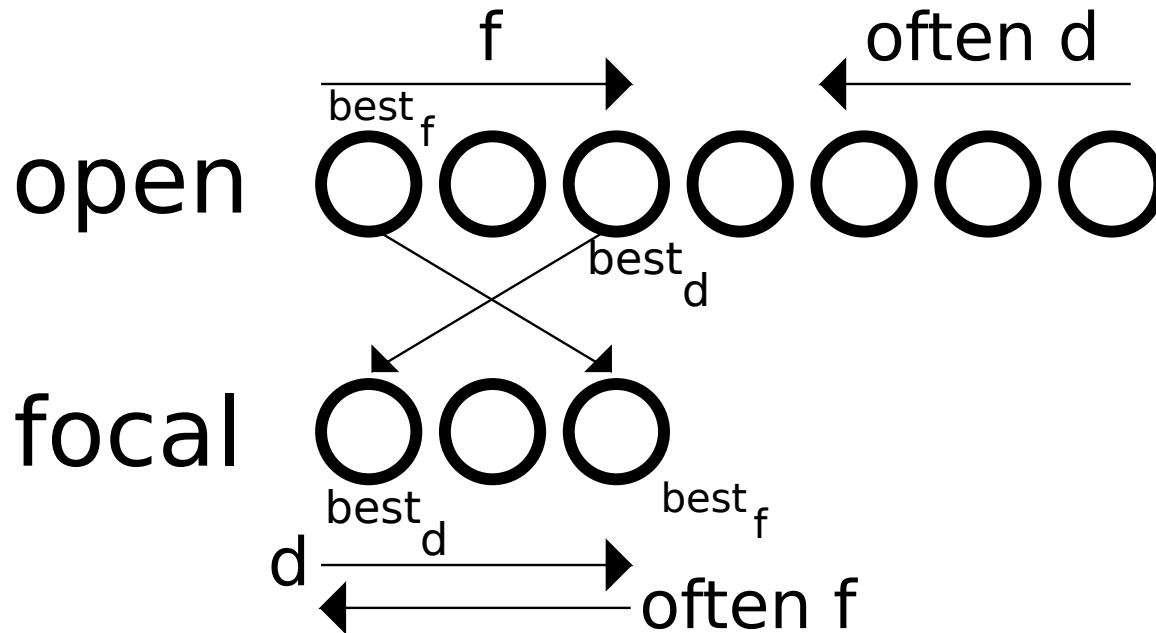
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- EES
- Conclusion
- Backup Slides
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*open*: all generated but unexpanded nodes, sorted on  $f(n)$   
*focal*: all nodes where  $f(n) \leq w \cdot f(best_f)$  sorted on  $\hat{d}(n)$



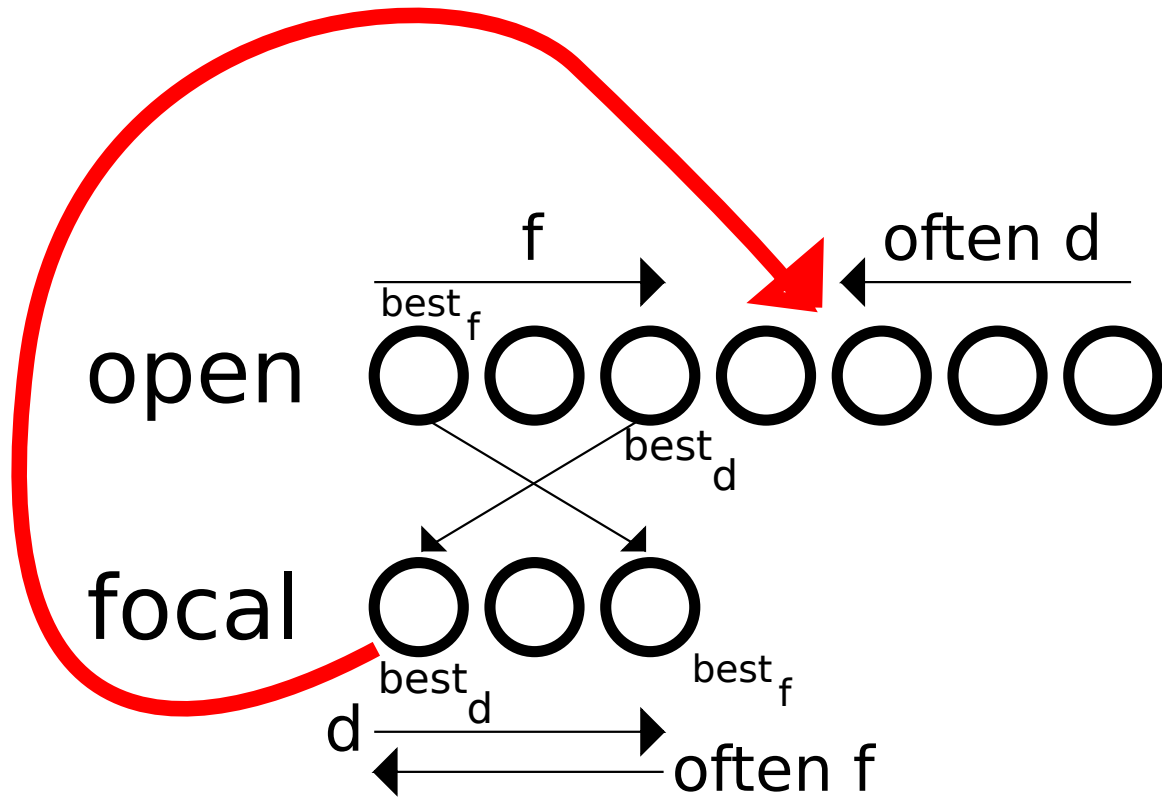
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*f* rises as search progresses (*h* is admissible)  
*best<sub>d</sub>*'s children won't remain on focal