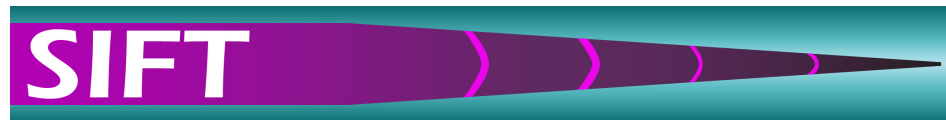


Faster Bounded-Cost Search Using Inadmissible Estimates

Jordan Thayer, Roni Stern, Ariel Felner, Wheeler Ruml

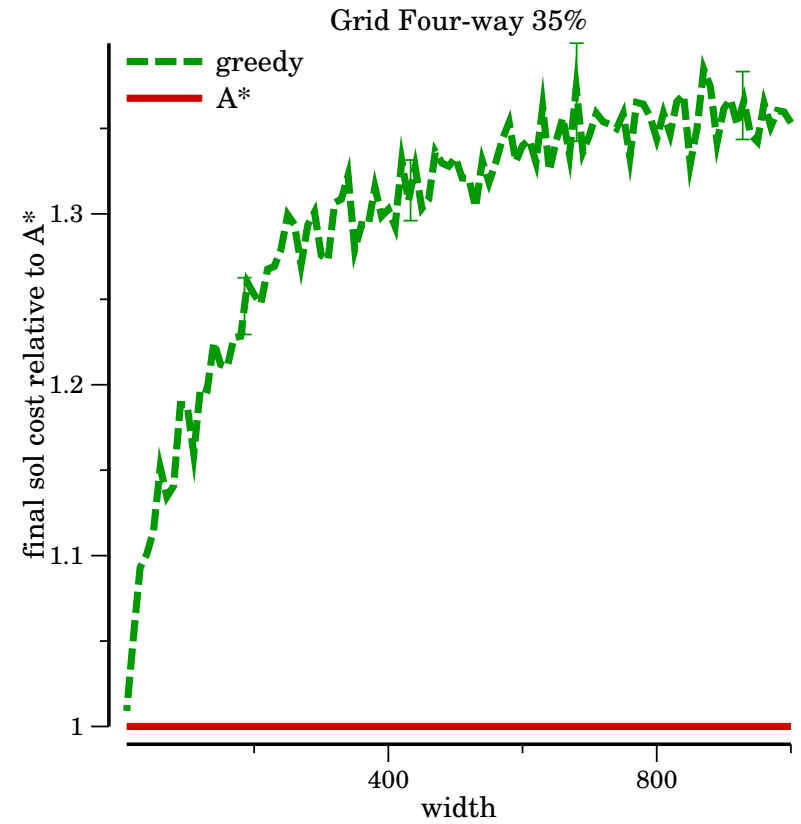
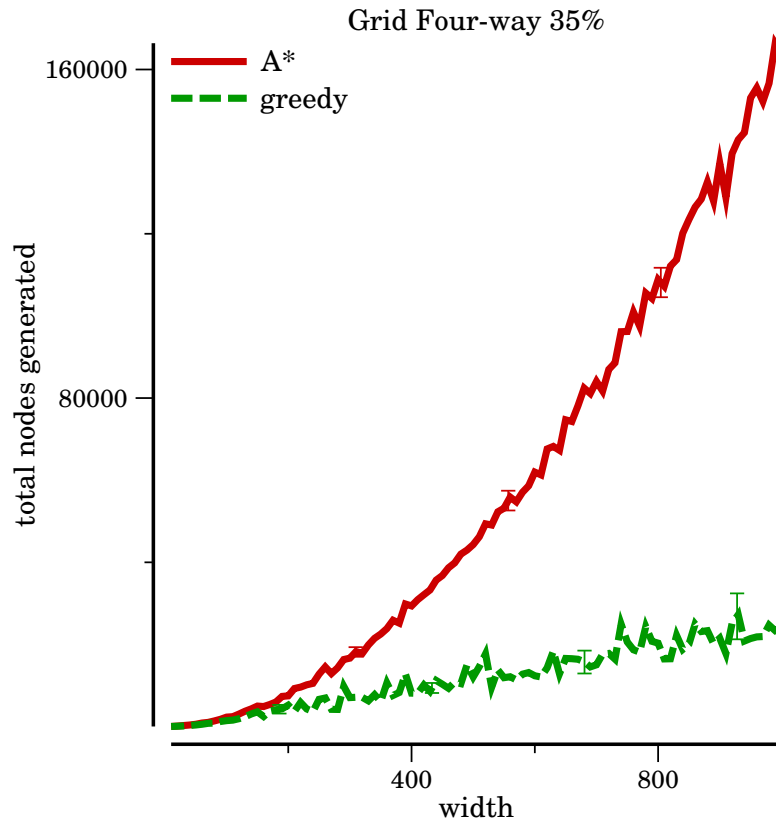


UNIVERSITY of NEW HAMPSHIRE

`jthayer@sift.net` `roni.stern@gmail.com` `felner@bgu.ac.il` `ruml@cs.unh.edu`

Motivation: Optimal Search Hard, Greedy Solutions Expensive

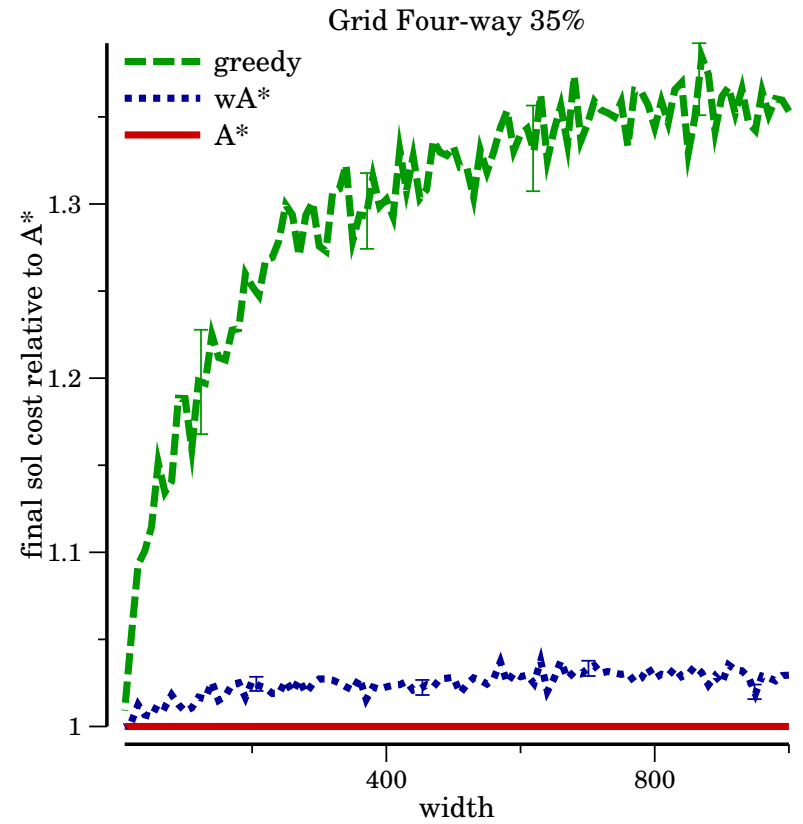
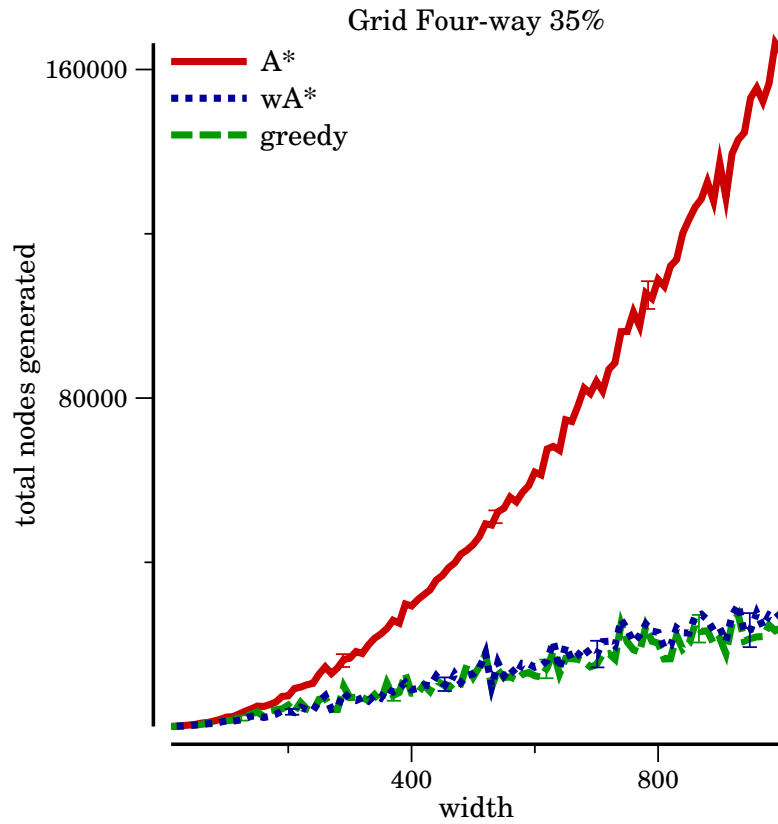
- Introduction
- Motivation**
- Outline
- Potential Search
- BEES
- Conclusion
- BEEPS, $PTS-\hat{h}$



optimal search won't scale
greedy solutions too expensive

Motivation: Bounded Suboptimal Provides Middle Ground

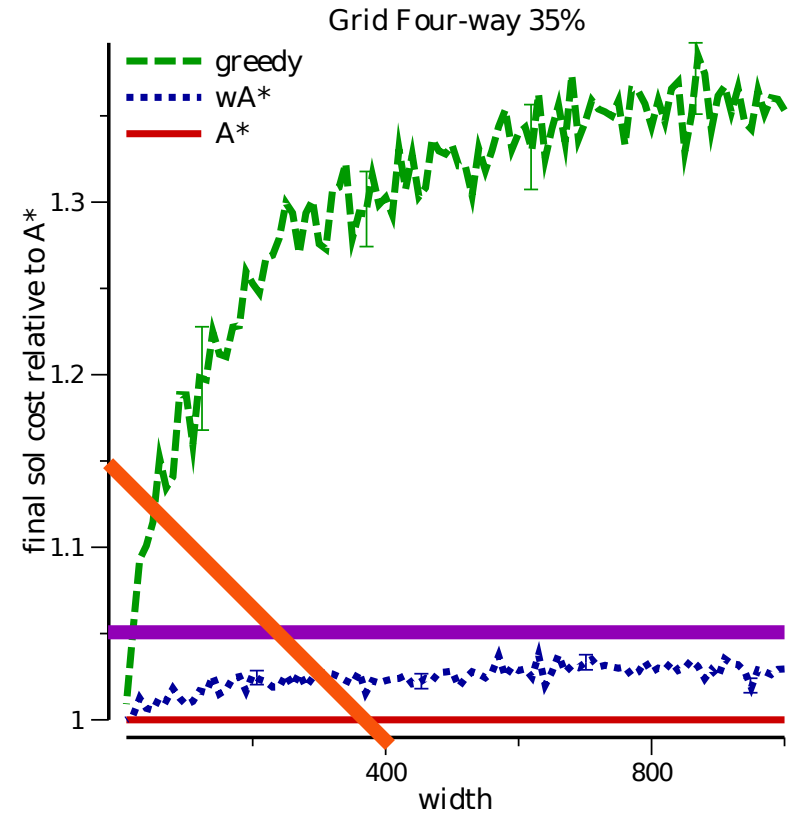
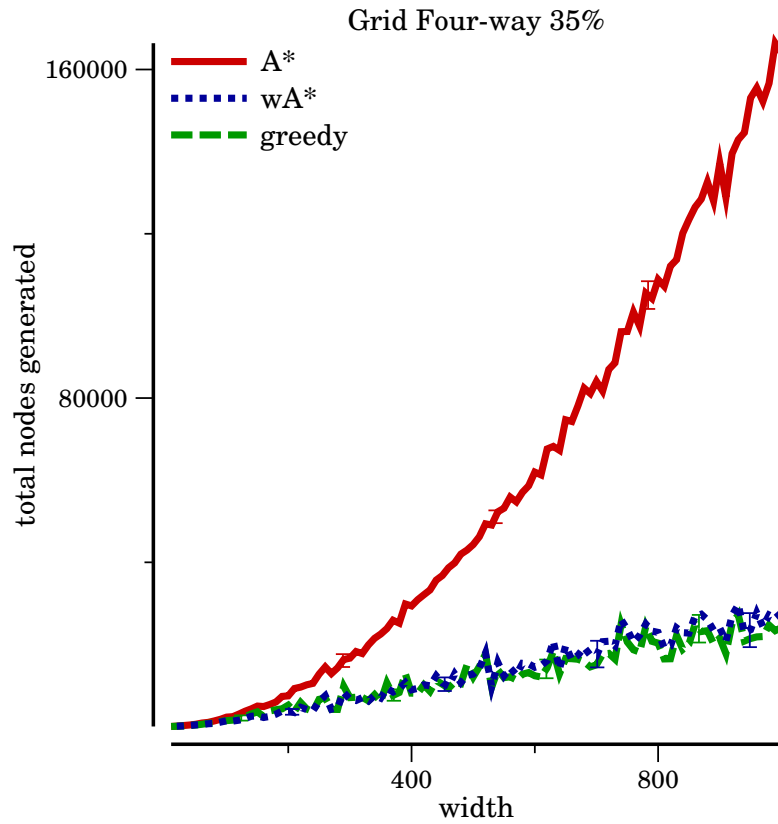
- Introduction
- Motivation**
- Outline
- Potential Search
- BEES
- Conclusion
- BEEPS, $\widehat{PTS-h}$



impose bounds to limit cost (relative to optimal)

Motivation: What if I have a Budget instead of w ?

- Introduction
- Motivation**
- Outline
- Potential Search
- BEES
- Conclusion
- BEEPS, $PTS-\hat{h}$



What if we have a budget instead of a relative bound?

Bounded Cost Search: find a solution of cost no more than C

Outline

Introduction

■ Motivation

■ Outline

Potential Search

BEES

Conclusion

BEEPS, PTS- \hat{h}

- motivation
- previous approach: potential search
- problem with potential search
- BEES
- bigger picture
- PTS on inadmissible heuristics, BEEPS

Introduction

Potential Search

■ PTS

■ Performance

■ Shortcoming

BEES

Conclusion

BEEPS, $PTS-\hat{h}$

best-first search in order of chance of satisfying cost bound C

[Introduction](#)

[Potential Search](#)

■ **PTS**

■ Performance

■ Shortcoming

[BEES](#)

[Conclusion](#)

[BEEPS, PTS- \$\hat{h}\$](#)

best-first search in order of chance of satisfying cost bound C

$$PT_C(n) = Pr(g(n) + h^*(n) \leq C)$$

[Introduction](#)[Potential Search](#)[■ PTS](#)[■ Performance](#)[■ Shortcoming](#)[BEES](#)[Conclusion](#)[BEEPS, PTS- \$\hat{h}\$](#)

best-first search in order of chance of satisfying cost bound C

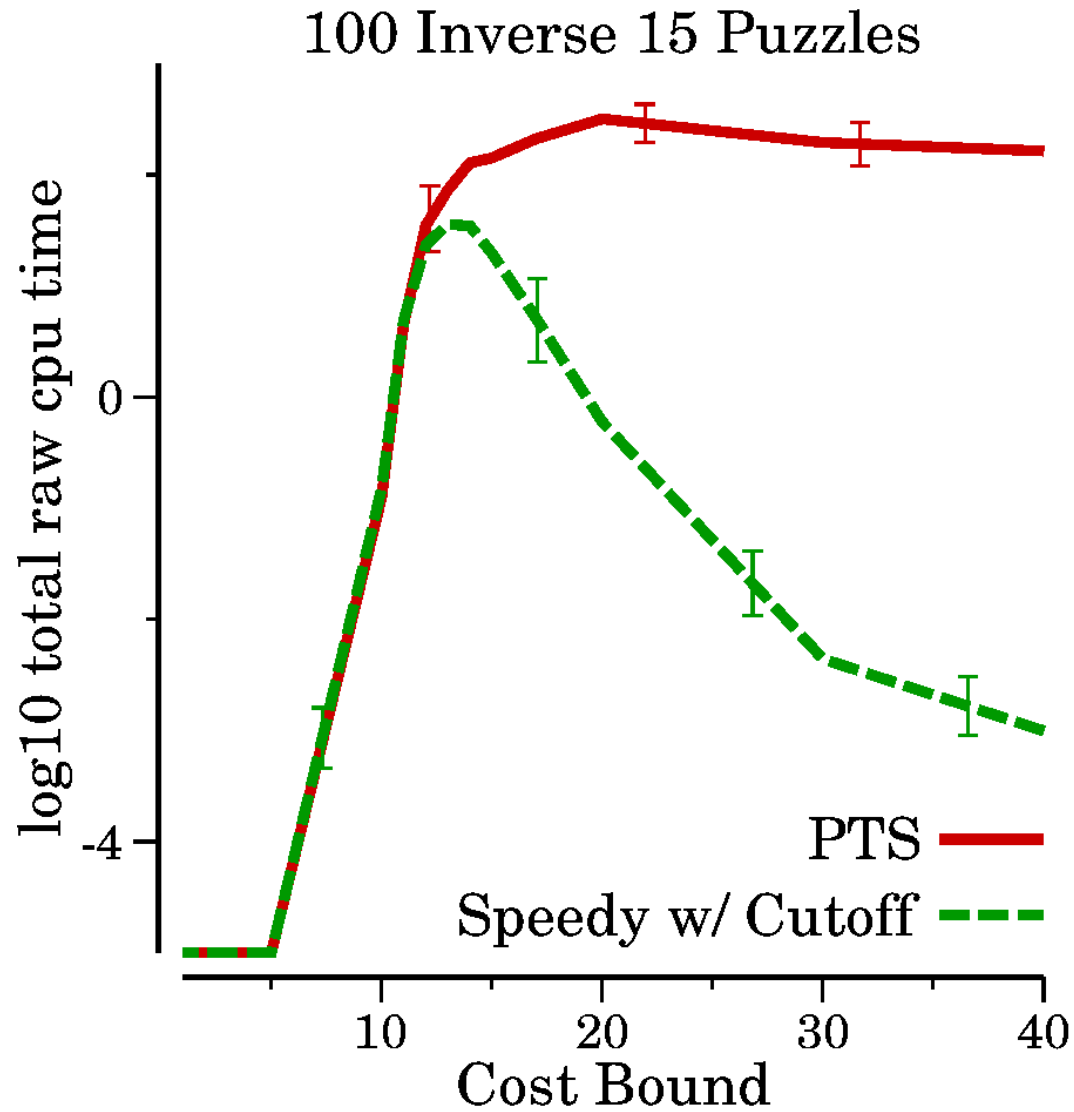
$$PT_C(n) = Pr(g(n) + h^*(n) \leq C)$$

unfortunately, we may not be able to compute that

$f_{lnr}(n) = \frac{h(n)}{C-g(n)}$ produces an equivalent order
under certain assumptions

PTS Performance: Non-Unit Cost Performance is Bad!

- Introduction
- Potential Search
 - PTS
 - Performance
 - Shortcoming
- BEES
- Conclusion
- BEEPS, $PTS-\hat{h}$



Potential Search Ignores Solution Length

Introduction

Potential Search

■ PTS

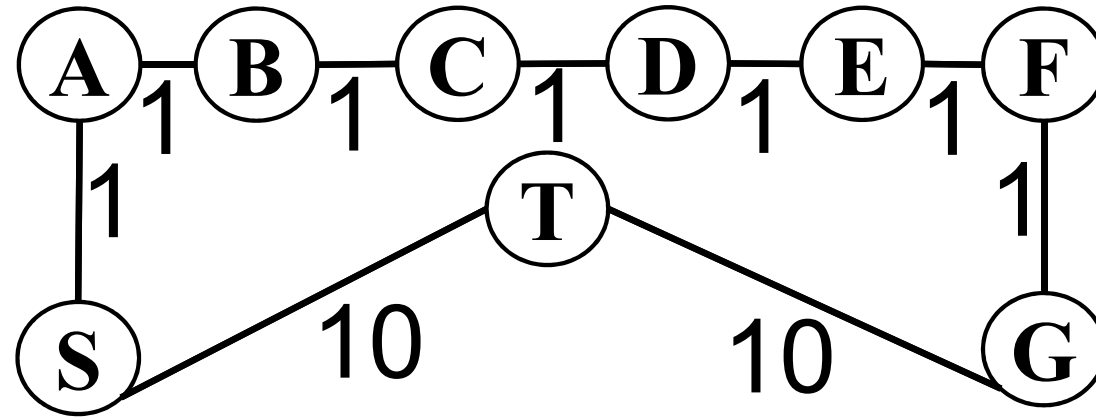
■ Performance

■ Shortcoming

BEES

Conclusion

BEEPS, $PTS-\hat{h}$



$$d(A) = 6, h(A) = 6$$

$$d(T) = 1, h(T) = 10$$

$$C = 20$$

Potential Search Ignores Solution Length

Introduction

Potential Search

■ PTS

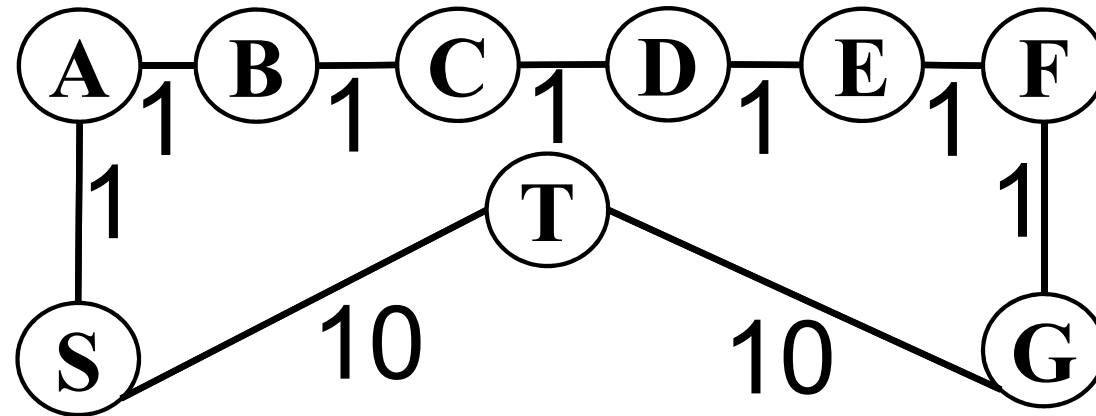
■ Performance

■ Shortcoming

BEES

Conclusion

BEEPS, PTS- \hat{h}



$$d(A) = 6, h(A) = 6$$

$$d(T) = 1, h(T) = 10$$

$$C = 20$$

$$f_{lnr}(A) = \frac{6}{20-1}$$

$$f_{lnr}(T) = \frac{10}{20-10}$$

Potential Search Ignores Solution Length

Introduction

Potential Search

■ PTS

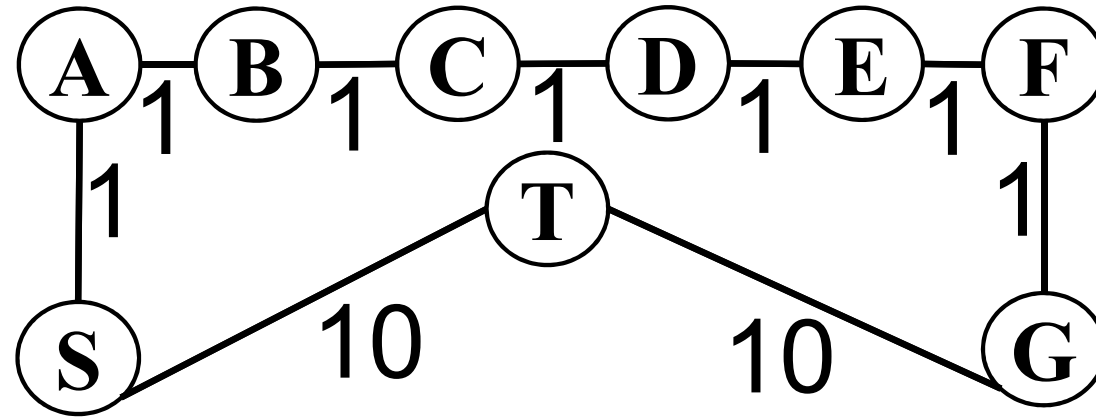
■ Performance

■ Shortcoming

BEES

Conclusion

BEEPS, PTS- \hat{h}



$$d(A) = 6, h(A) = 6$$

$$d(T) = 1, h(T) = 10$$

$$C = 20$$

$$f_{lnr}(A) = \frac{6}{20-1}$$

$$f_{lnr}(T) = \frac{10}{20-10}$$

PTS prefers A to T

Introduction

Potential Search

BEES

- Goals
- Algorithm
- Defining Focal
- Pseudo-Code
- Example
- Performance

Conclusion

BEEPS, $PTS-\hat{h}$

Bounded Cost Explicit Estimation Search

Goals for New Bounded Cost Search

[Introduction](#)

[Potential Search](#)

[BEES](#)

■ [Goals](#)

■ [Algorithm](#)

■ [Defining Focal](#)

■ [Pseudo-Code](#)

■ [Example](#)

■ [Performance](#)

[Conclusion](#)

[BEEPS, PTS- \$\hat{h}\$](#)

1. avoid reliance on error models (ie $f_{l_{nr}}$)
2. improve performance on non-unit cost domains
without losing performance in unit cost domains

Bounded-Cost Explicit Estimation Search (BEES)

[Introduction](#)

[Potential Search](#)

[BEES](#)

■ Goals

■ **Algorithm**

■ Defining Focal

■ Pseudo-Code

■ Example

■ Performance

[Conclusion](#)

[BEEPS, PTS- \$\hat{h}\$](#)

1. estimate which nodes are within cost bound
2. best-first search of these on estimated actions-to-go

Step 1: Estimate which nodes are within cost bound

Introduction

Potential Search

BEES

■ Goals

■ Algorithm

■ Defining Focal

■ Pseudo-Code

■ Example

■ Performance

Conclusion

BEEPS, PTS- \hat{h}

$h(n)$: an admissible cost-to-go estimate

$$f(n) = g(n) + h(n)$$

$f(n) \leq C$: could lead to a solution in bound

$$\text{focal} = \{n \in \text{open} \mid f(n) \leq C\}$$

Step 1: Estimate which nodes are within cost bound

Introduction

Potential Search

BEES

■ Goals

■ Algorithm

■ Defining Focal

■ Pseudo-Code

■ Example

■ Performance

Conclusion

BEEPS, PTS- \hat{h}

$h(n)$: an admissible cost-to-go estimate

$\hat{h}(n)$: best-guess estimate of cost-to-go

$$f(n) = g(n) + h(n)$$

$$\hat{f}(n) = g(n) + \hat{h}(n)$$

$f(n) \leq C$: could lead to a solution in bound

$\hat{f}(n) \leq C$: probably leads to a solution in bound

$$\text{focal} = \{n \in \text{open} \mid \hat{f}(n) \leq C\}$$

What If No Nodes Appear to Be Within Bound?

Introduction

Potential Search

BEES

■ Goals

■ Algorithm

■ Defining Focal

■ Pseudo-Code

■ Example

■ Performance

Conclusion

BEEPS, PTS- \hat{h}

$$\hat{f}(n) = g(n) + \hat{h}(n)$$

$$f(n) = g(n) + h(n)$$

$$\hat{f}(n) \geq f(n)$$

$$\text{focal} = \{n \in \text{open} \mid \hat{f}(n) \leq C\}$$

$$\text{open} = \{n \mid f(n) \leq C\}$$

A* provides an efficient way to prove no solution in C

What If No Nodes Appear to Be Within Bound?

Introduction

Potential Search

BEES

■ Goals

■ Algorithm

■ Defining Focal

■ Pseudo-Code

■ Example

■ Performance

Conclusion

BEEPS, PTS- \hat{h}

$$\hat{f}(n) = g(n) + \hat{h}(n)$$

$$f(n) = g(n) + h(n)$$

$$\hat{f}(n) \geq f(n)$$

$$\text{focal} = \{n \in \text{open} \mid \hat{f}(n) \leq C\}$$

$$\text{open} = \{n \mid f(n) \leq C\}$$

A^* provides an efficient way to prove no solution in C

1. Estimate which nodes are within cost bound
2. Best-first search of these on estimated actions-to-go
3. A^* search if we think no solution exists within C

Pseudo-Code for BEES

[Introduction](#)

[Potential Search](#)

[BEES](#)

■ Goals

■ Algorithm

■ Defining Focal

■ **Pseudo-Code**

■ Example

■ Performance

[Conclusion](#)

[BEEPS, PTS- \$\hat{h}\$](#)

BEES is a best first search on the following rule

$$\text{open} = \{n \mid f(n) \leq C\}$$

$$\text{focal} = \{n \in \text{open} \mid \hat{f}(n) \leq C\}$$

selectNode

1. **if** focal $\neq \{\}$
2. **then** return $n \in \text{focal}$ estimated nearest to a goal
3. **else** return $n \in \text{open}$ with minimum f

PTS vs. BEES on Explicit Graph

Introduction

Potential Search

BEES

■ Goals

■ Algorithm

■ Defining Focal

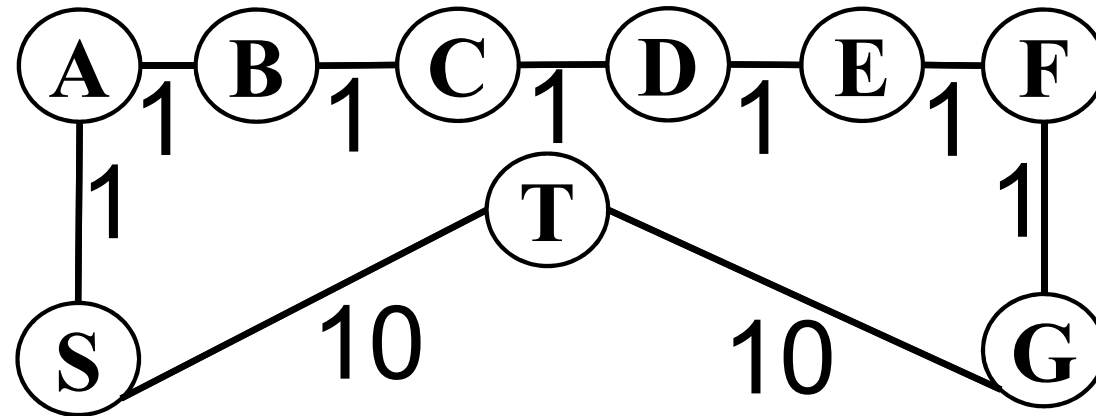
■ Pseudo-Code

■ Example

■ Performance

Conclusion

BEEPS, PTS- \hat{h}



$$d(A) = 6, h(A) = 6, \hat{f}(A) = 7$$

$$d(T) = 1, h(T) = 10, \hat{f}(T) = 20$$

$$C = 20 \quad f_{lnr}(A) = \frac{6}{20-1}$$

$$f_{lnr}(T) = \frac{10}{20-10}$$

BEES prefers T , PTS prefers A

Empirical Evaluation

[Introduction](#)

[Potential Search](#)

[BEES](#)

■ Goals

■ Algorithm

■ Defining Focal

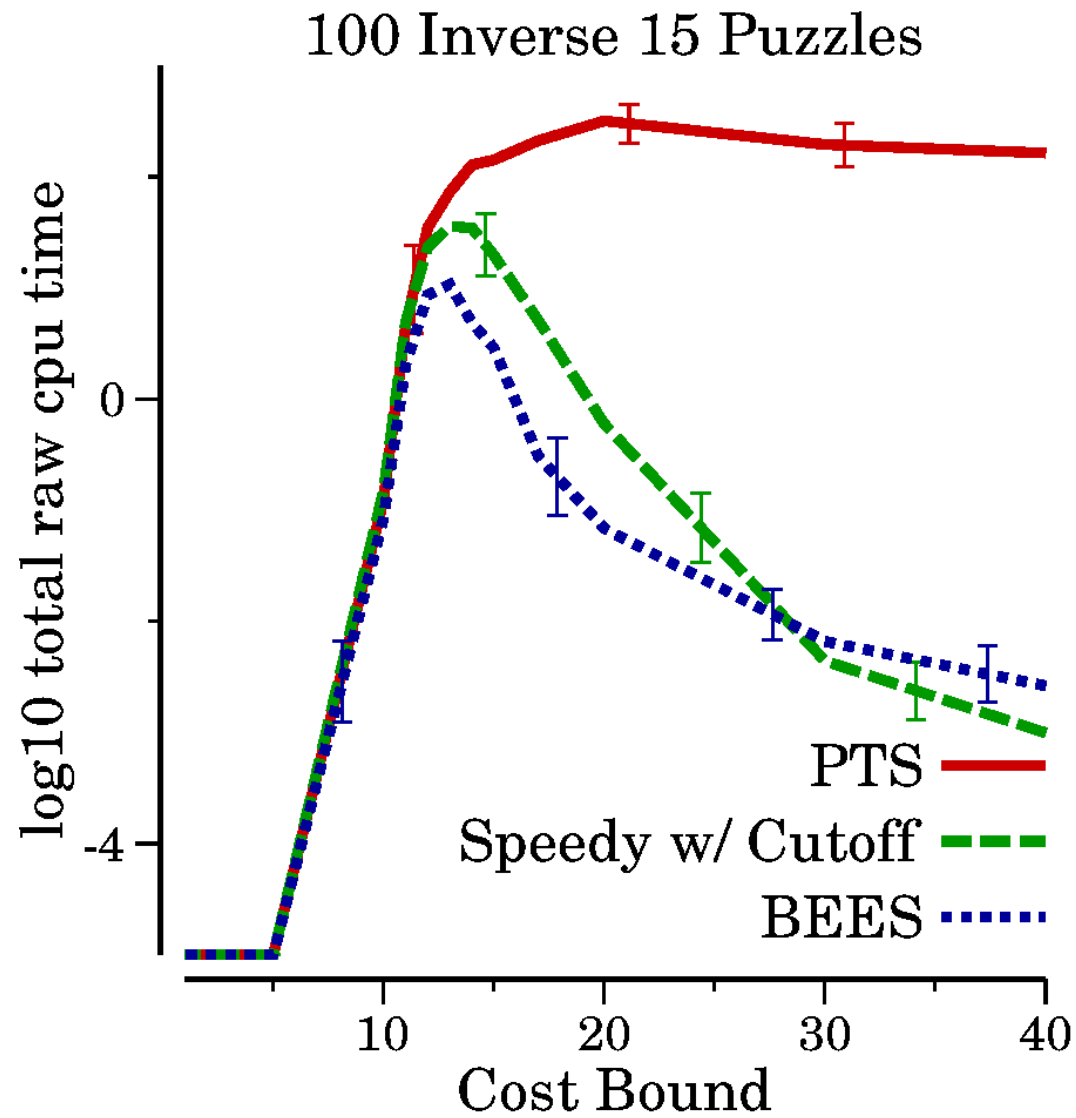
■ Pseudo-Code

■ Example

■ Performance

[Conclusion](#)

[BEEPS, PTS- \$\hat{h}\$](#)



Empirical Evaluation

Introduction

Potential Search

BEES

■ Goals

■ Algorithm

■ Defining Focal

■ Pseudo-Code

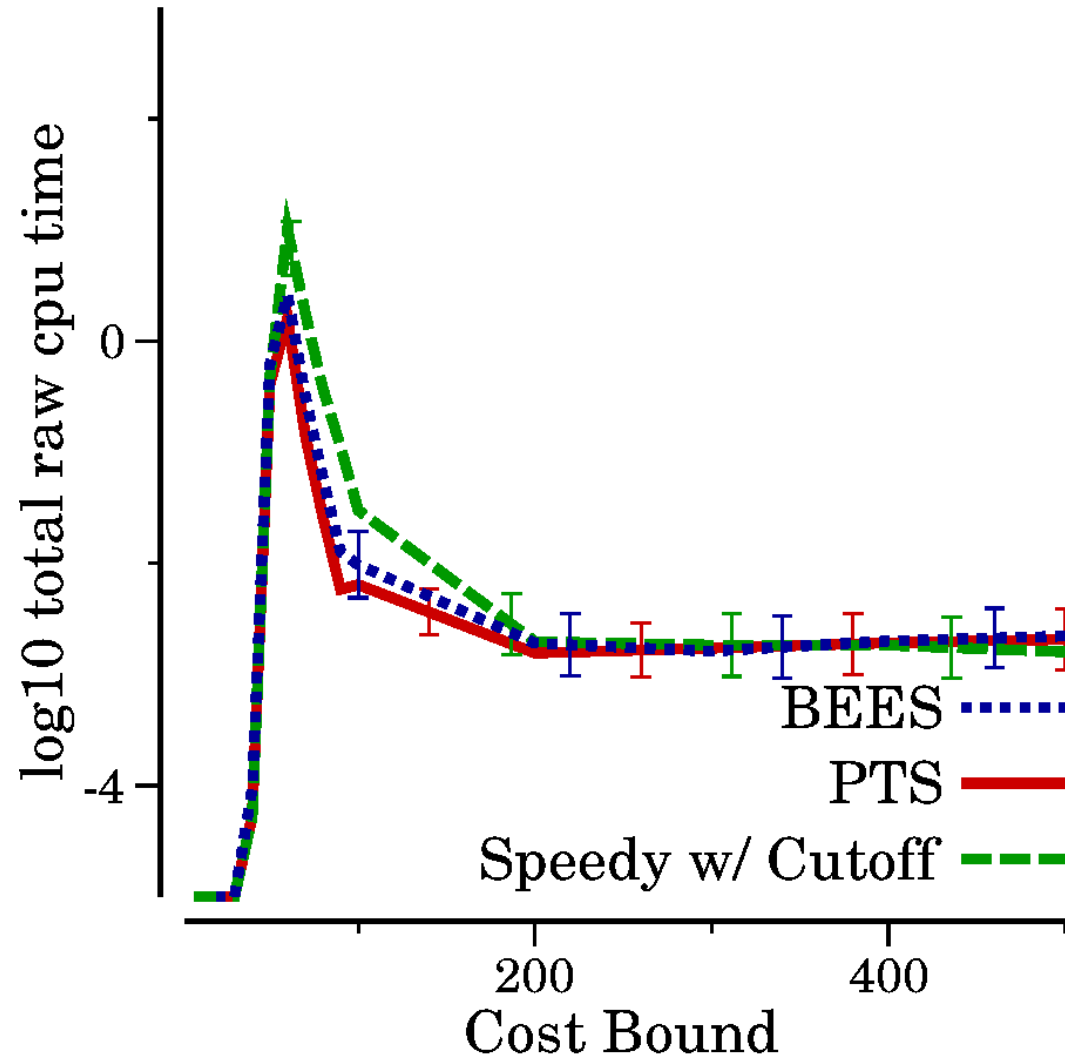
■ Example

■ Performance

Conclusion

BEEPS, $PTS-\hat{h}$

Korf's 100 15 Puzzles



Empirical Evaluation

[Introduction](#)

[Potential Search](#)

[BEES](#)

■ Goals

■ Algorithm

■ Defining Focal

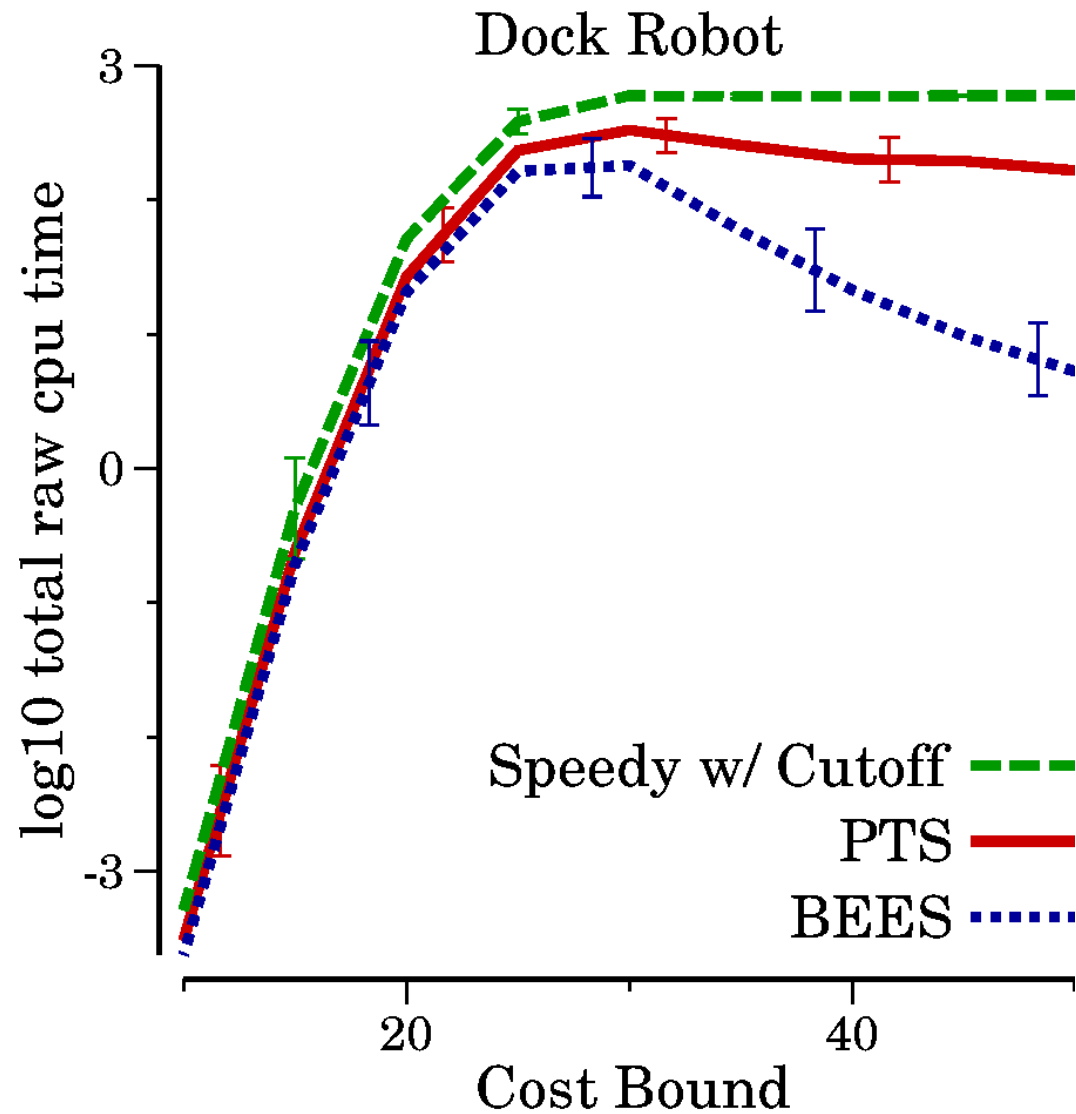
■ Pseudo-Code

■ Example

■ Performance

[Conclusion](#)

[BEEPS, \$\widehat{PTS-h}\$](#)



Summary

[Introduction](#)

[Potential Search](#)

[BEES](#)

[Conclusion](#)

■ [Summary](#)

[BEEPS, PTS- \$\hat{h}\$](#)

- BEES outperforms previous state-of-the-art
- when action costs differ, take advantage of d
- inadmissible heuristics can speed up search
- finding solutions, making proofs are different

Finding Solutions, Proving Bounds Different Tasks

Introduction

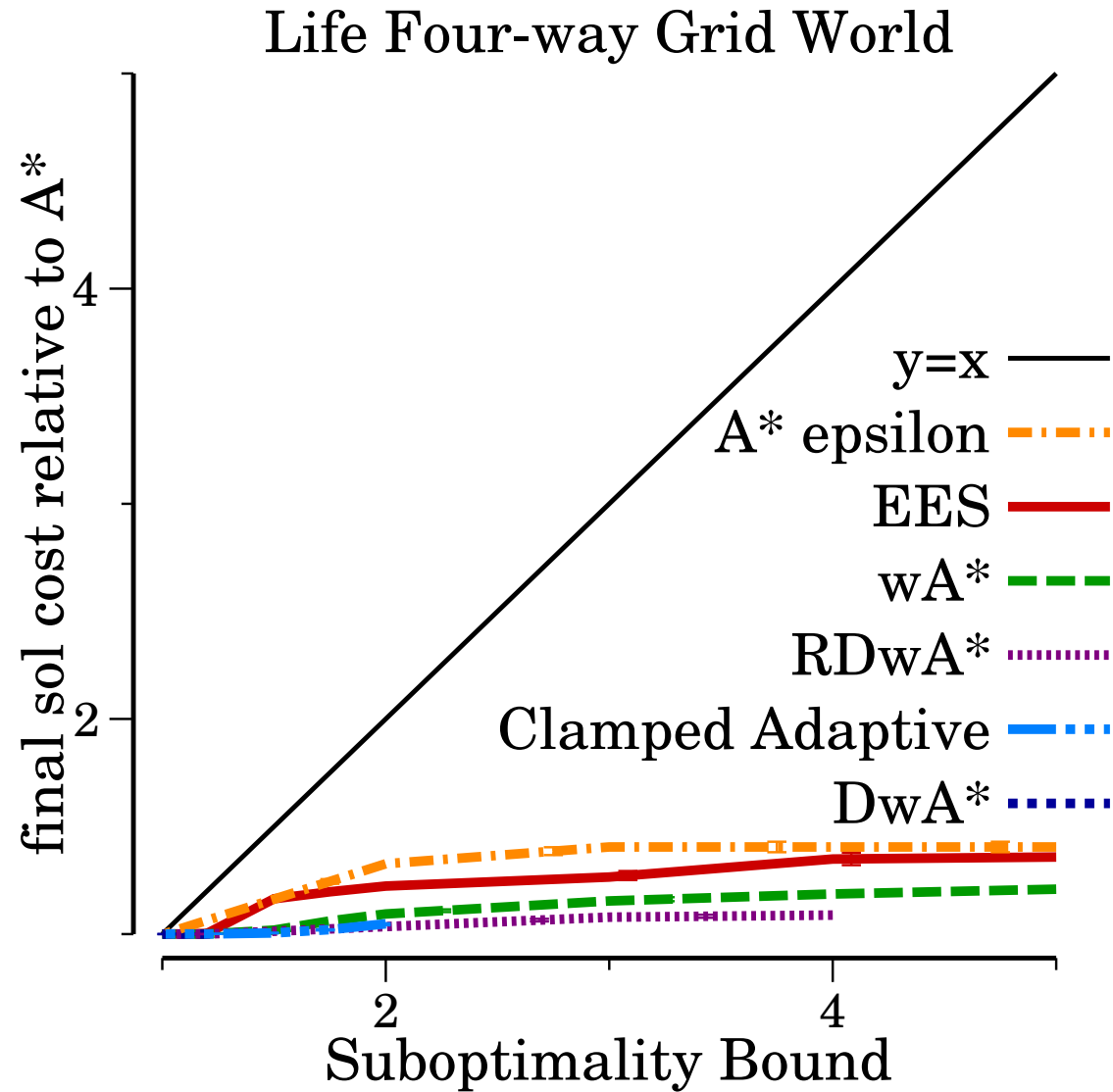
Potential Search

BEES

Conclusion

■ Summary

BEEPS, $PTS-\hat{h}$



Inadmissible Heuristics Speed Search

[Introduction](#)

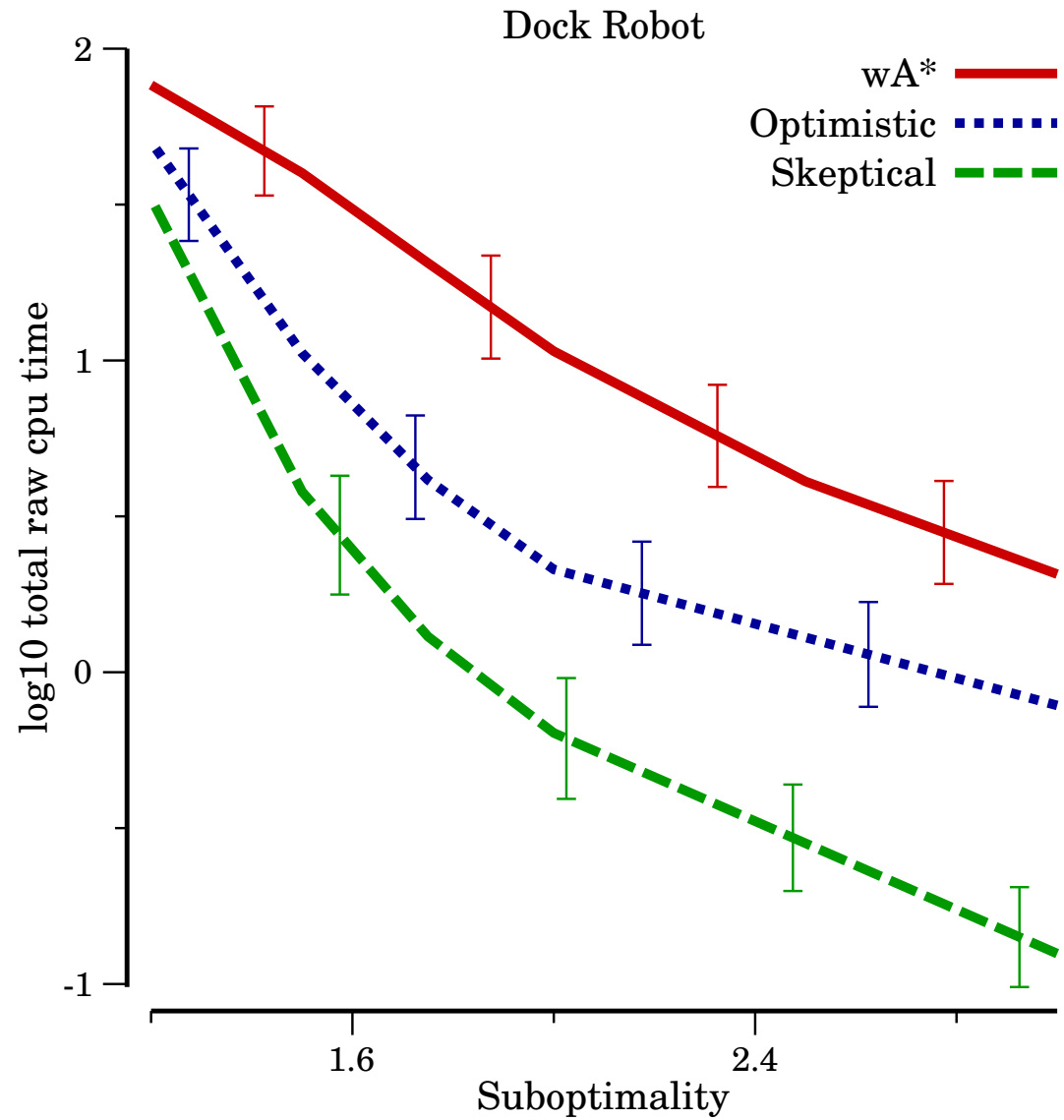
[Potential Search](#)

[BEES](#)

[Conclusion](#)

Summary

[BEEPS, \$\widehat{PTS-h}\$](#)



Use d to Go Fast

[Introduction](#)

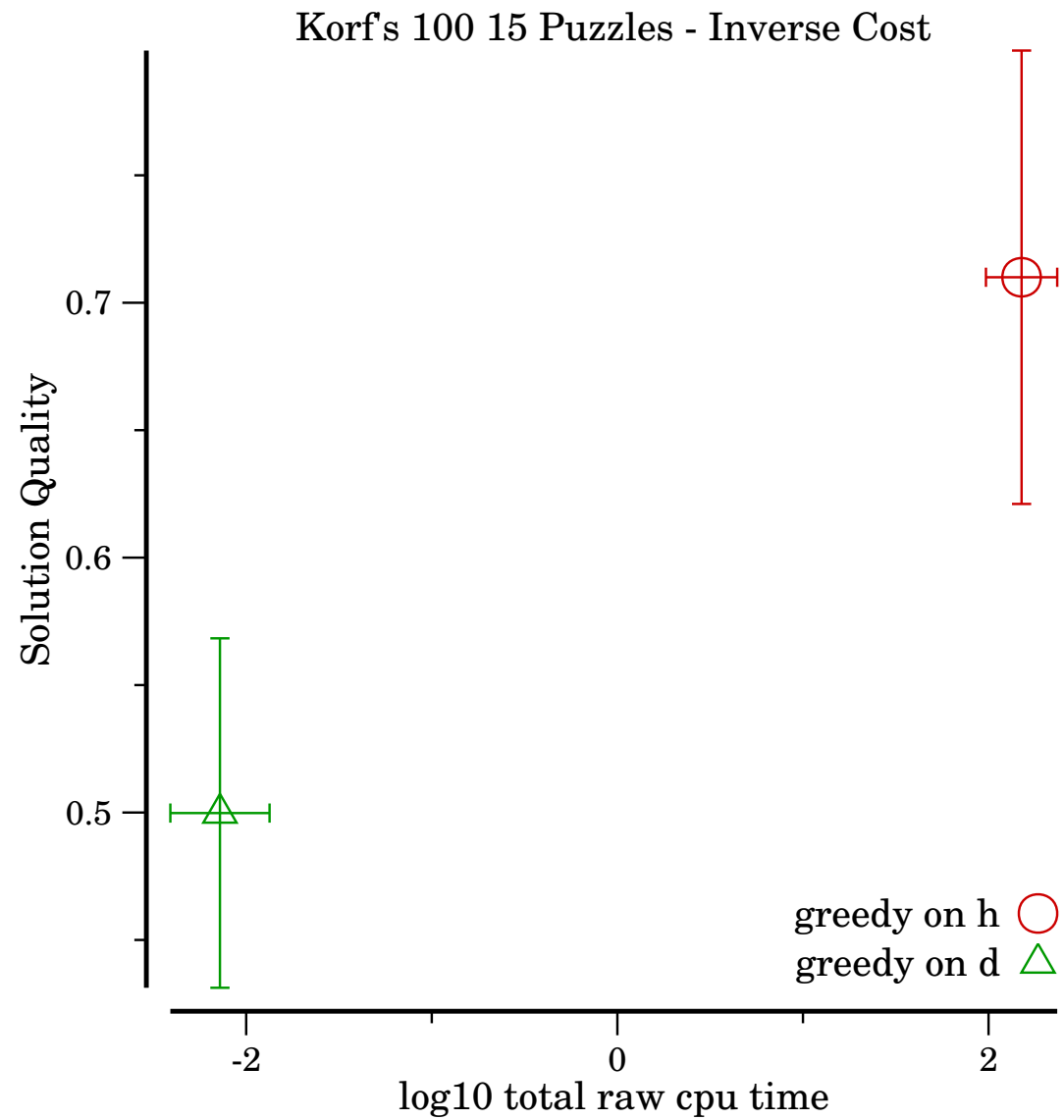
[Potential Search](#)

[BEES](#)

[Conclusion](#)

■ [Summary](#)

[BEEPS, \$PTS_{\hat{h}}\$](#)



Summary

[Introduction](#)

[Potential Search](#)

[BEES](#)

[Conclusion](#)

■ [Summary](#)

[BEEPS, PTS- \$\hat{h}\$](#)

- BEES outperforms previous state-of-the-art
- when action costs differ, take advantage of d
- inadmissible heuristics can speed up search
- finding solutions, making proofs are different

Using \hat{h} in PTS

Introduction

Potential Search

BEES

Conclusion

BEEPS, PTS- \hat{h}

■ Using \hat{h} in PTS

■ BEEPS - BEES
with Potential
Measurements

■ Performance
■ EES as BC

$$f_{lnr}(n) = \frac{h(n)}{C-g(n)}$$

$$f_{lnr}(n) = \frac{\hat{h}(n)}{C-g(n)}$$

BEEPS - BEES with Potential Measurements

Introduction

Potential Search

BEES

Conclusion

BEEPS, PTS- \hat{h}

■ Using \hat{h} in PTS

■ BEEPS - BEES
with Potential
Measurements

■ Performance

■ EES as BC

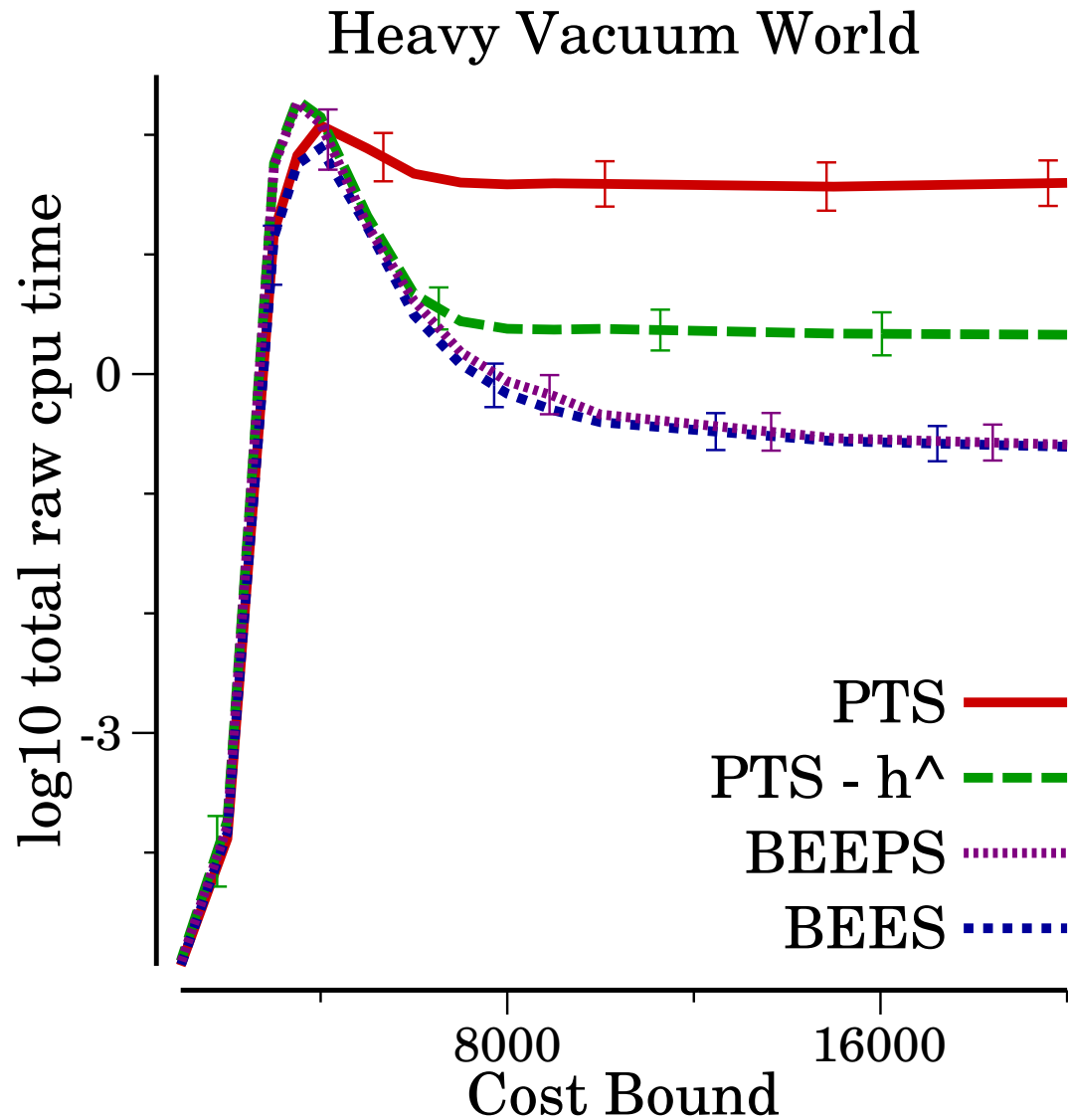
BEES is a best first search on the following rule

selectNode

1. **if** focal $\neq \{\}$
2. **then** return $n \in$ focal with minimum \hat{d}
3. **else** return $n \in$ open with minimum f_{lnr}

Empirical Results

- Introduction
- Potential Search
- BEES
- Conclusion
- BEEPS, PTS- \hat{h}
- Using \hat{h} in PTS
- BEEPS - BEES with Potential Measurements
- Performance
- EES as BC



Empirical Results

Introduction

Potential Search

BEES

Conclusion

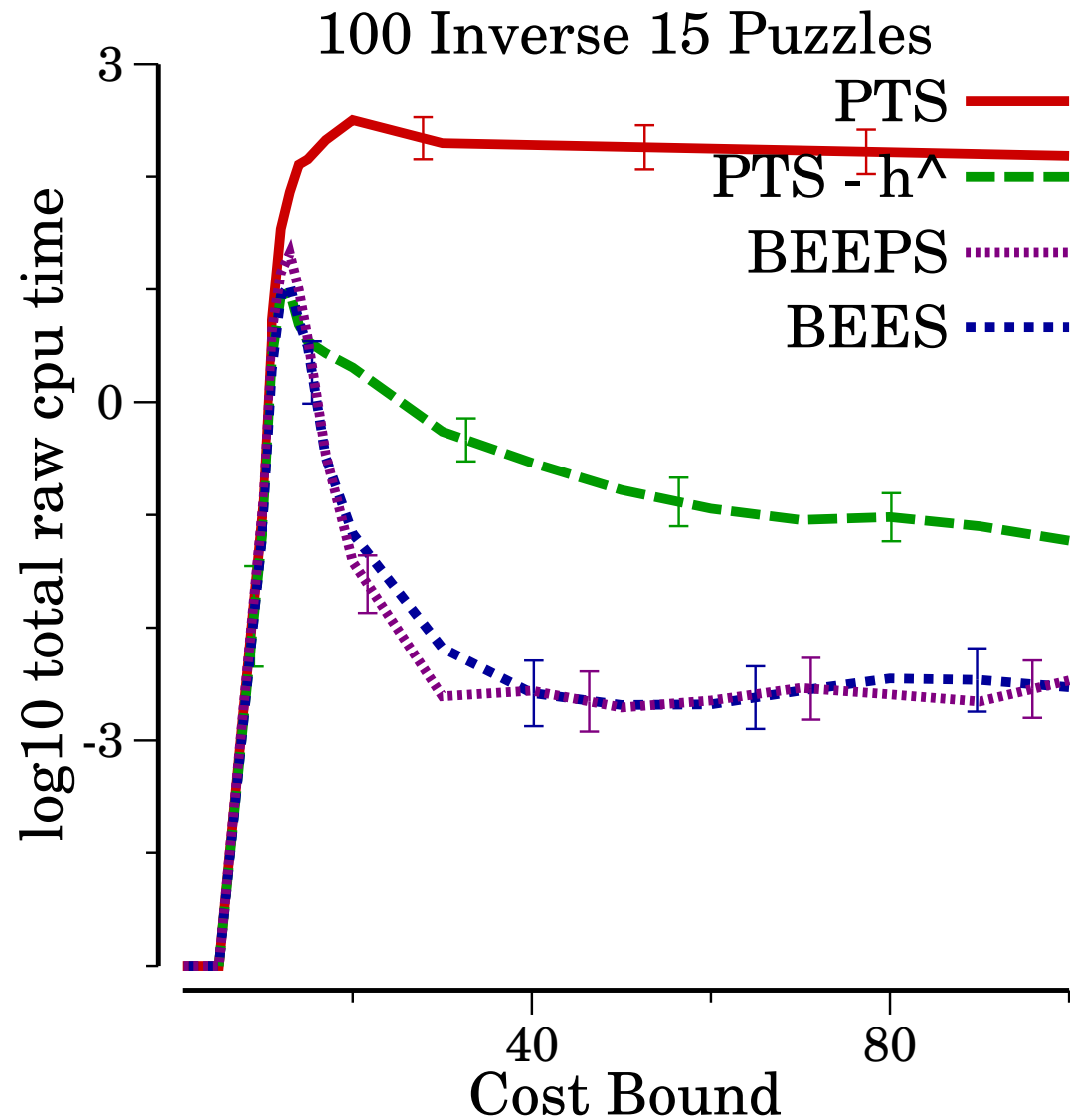
BEEPS, PTS- \hat{h}

■ Using \hat{h} in PTS

■ BEEPS - BEES with Potential Measurements

■ Performance

■ EES as BC



Empirical Results

Introduction

Potential Search

BEES

Conclusion

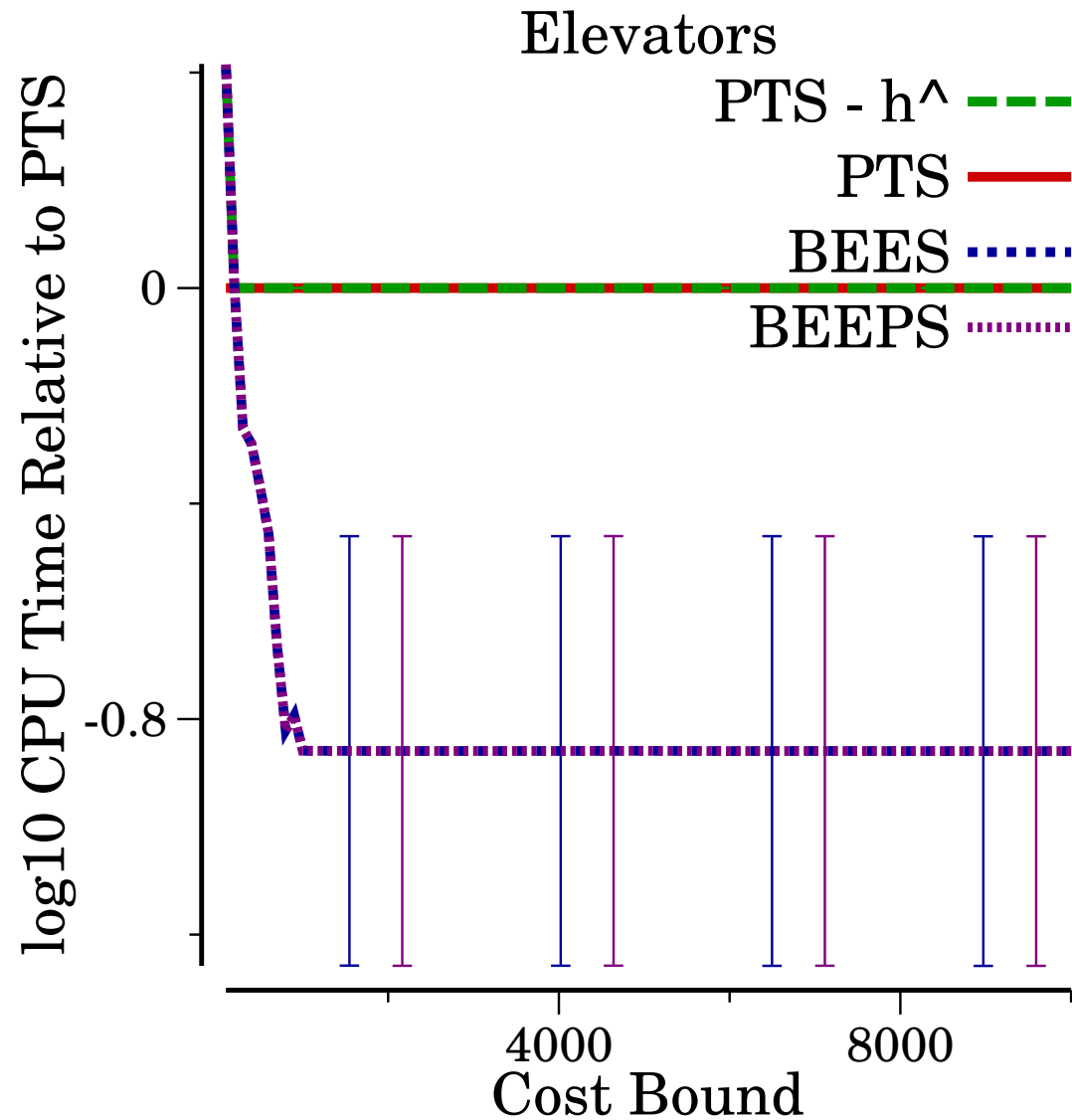
BEEPS, PTS- \hat{h}

■ Using \hat{h} in PTS

■ BEEPS - BEES with Potential Measurements

■ Performance

■ EES as BC



EES Doesn't Work as Bounded Cost Algorithm

- Introduction
- Potential Search
- BEES
- Conclusion
- BEEPS, PTS- \hat{h}
- Using \hat{h} in PTS
- BEEPS - BEES with Potential Measurements
- Performance
- EES as BC

