http://www.cs.unh.edu/~ruml/cs758

1 handout: slides
Red-Black Trees

- Red-Black Trees
- BST Deletion
- Single Child
- Immed. Succ.
- Deep Succ.
- Break

Deletion Fixup

Red-Black Trees
Red-Black Trees

node: data, left, right, parent, color

1. every node is either red or black
2. the root is black
3. (consider nil to be black)
4. both children of a red node are black
5. from any node, all paths to leaves have the same ‘black height’
Plain Binary Tree Deletion

4 cases of \text{delete}(z):

1. no left child, or no kids: substitute right subtree at parent.
2. no right child: substitute left subtree at parent.
3. successor $y$ is $z$’s right child:
   (a) substitute $y$ for $z$
   (b) attach $z$’s left subtree as $y$’s left subtree
4. successor $y$ is deeper:
   (a) substitute $y$’s right subtree for $y$
   (b) attach $z$’s right subtree as $y$’s right subtree
   (c) as above, substitute $y$ for $z$
   (d) as above, attach $z$’s left subtree as $y$’s left subtree

What if it’s a red-black tree?
Cases 1 and 2: Single Child

1. every node is either red or black
2. the root is black
3. (consider nil to be black)
4. both children of a red node are black
5. from any node, all paths to leaves have the same ‘black height’

deleting $z$ with single child $x$

1. $x$ takes $z$’s place
Cases 1 and 2: Single Child

1. every node is either red or black
2. the root is black
3. (consider nil to be black)
4. both children of a red node are black
5. from any node, all paths to leaves have the same ‘black height’

deleting \( z \) with single child \( x \)

1. \( x \) takes \( z \)’s place
2. book uses \( y \) for \( z \) for short code
3. if \( y = z \) was black, we have ‘extra black’ at \( x \), so call fixup routine at \( x \)
Case 3: Two Children, Successor is Child

1. every node is either red or black
2. the root is black
3. (consider nil to be black)
4. both children of a red node are black
5. from any node, all paths to leaves have the same ‘black height’

deleting $z$, successor $y$ is right child

1. $y$ takes $z$’s place and color
2. attach $z$’s left subtree as $y$’s left subtree
Case 3: Two Children, Successor is Child

1. every node is either red or black
2. the root is black
3. (consider nil to be black)
4. both children of a red node are black
5. from any node, all paths to leaves have the same ‘black height’

deleting $z$, successor $y$ is right child

1. $y$ takes $z$’s place and color
2. attach $z$’s left subtree as $y$’s left subtree
3. if $y$ was black, we need ‘extra black’ at $y$’s right child $x$, so call fixup routine at $x$
Case 4: Two Children, Successor is Deeper

deleting $z$, successor $y$ is deep down

1. substitute $y$’s right child $x$ for $y$
2. attach $z$’s right subtree as $y$’s right subtree
   as in simpler case:
3. $y$ takes $z$’s place and color
4. attach $z$’s left subtree as $y$’s left subtree
5. if $y$ was black, we need ‘extra black’ at $x$, so call fixup routine at $x$
asst 4: write verifier
asst 3? recitation?
Red-Black Trees

Deletion Fixup
- Fix-up Loop
- Case 1
- Case 2
- Case 3
- Case 4
- Complexity
- EOLQs

Red-Black Tree Deletion Fixup
need to find a red node to make black when $x$ red or root, color black and terminate

$x$ is non-root black node.

assume $x$ is a left child (other cases symmetric).

Must have sibling $w$, since $x$ holds ‘extra blackness’.

4 cases:

1. $w$ is red
2. $w$ and both its children are black
3. $w$ is black, its right child is black, its left child is red
4. $w$ is black, its right child is red

fix-up loop invariant: all properties hold if ‘extra black’ at $x$ is considered, heights of fringe (greek) nodes preserved
**Case 1**

case 1: $w$ is red. so parent and children must be black.

solution:

1. rotate and recolor to get black sibling (moves red horizontally)
2. fall through to case 2, 3, or 4
Case 2

case 2: \( w \) and both its children are black

solution:

1. color \( w \) red. subtree at parent now ‘black-balanced’.
2. move \( x \)'s blackness (and \( w \)'s) up the tree
3. recur at parent

if from case 1, \( x \) now red, so will terminate
case 3: \( w \) is black, its right is black, left is red

solution:
1. rotate right and move red over to right child
2. fall through to case 4
case 4: $w$ is black, its right child is red

solution:

1. rotate and recolor to annihilate red with $x$’s black
2. set $x$ to root to force termination
finding successor is $O(\lg n)$

one fixup iteration is constant time

fixup loops only when moving up, so is $O(\lg n)$

how many rotations are performed?
For example:

- What’s still confusing?
- What question didn’t you get to ask today?
- What would you like to hear more about?

Please write down your most pressing question about algorithms and put it in the box on your way out.

Thanks!