http://www.cs.unh.edu/~ruml/cs758
Dictionaries

### Dictionaries

- ‘associative array’
- ‘map’
- ‘look-up table’
- ‘set’

$n$ items, key length $k$

<table>
<thead>
<tr>
<th>Structure</th>
<th>Find</th>
<th>Insert</th>
<th>Delete</th>
</tr>
</thead>
<tbody>
<tr>
<td>List (unsorted)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>List (sorted)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Array (unsorted)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Array (sorted)</td>
<td></td>
<td></td>
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<tr>
<td>Heap</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hash table</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Binary tree (unbalanced)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Binary tree (balanced)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Hash Tables
applications:

1. dictionaries
2. object method tables
3. string matching
4. set operations: $\cup, \cap, -$ 

first methods:

1. direct-address tables: small key range. eg, bit vectors.
2. chaining: deletion?
Time Complexity

$n$ items in $m$ buckets

time complexity of search =
Time Complexity

- Searching
- Hash Tables
- Time Complexity
- More Collisions
- Open Addressing
- Break

Hash Functions

$n$ items in $m$ buckets

time complexity of search = number of items per bucket

assume nice hash: $P(h(i) = x) = 1/m$
$n$ items in $m$ buckets

time complexity of search = number of items per bucket

assume nice hash: $P(h(i) = x) = 1/m$

let $X_i$ be 1 iff $h(i) = x$, 0 otherwise

$$E\left[\sum_{i=1}^{n} X_i \right] = \sum_{i=1}^{n} E[X_i]$$

$$= \sum_{i=1}^{n} 1/m$$

$$= n/m$$

let $\alpha = \frac{n}{m}$ ‘load factor’

expected number of items per bucket is $\alpha$

expected time is $\Theta(1 + \alpha)$
More Collisions

Probability that \( k \) of \( n \) elements land in same of \( m \) bins:

Let \( \alpha = \frac{n}{m} \) ‘load factor’

\[
\begin{align*}
\binom{n}{k} \left( \frac{1}{m} \right)^k \left( 1 - \frac{1}{m} \right)^{n-k} & \approx \frac{\alpha^k}{e^{\alpha k} k!} \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>( k )</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.37</td>
</tr>
<tr>
<td>1</td>
<td>0.37</td>
</tr>
<tr>
<td>2</td>
<td>0.18</td>
</tr>
<tr>
<td>3</td>
<td>0.06</td>
</tr>
<tr>
<td>4</td>
<td>0.015</td>
</tr>
<tr>
<td>5</td>
<td>0.003</td>
</tr>
<tr>
<td>&gt; 5</td>
<td>0.002 total</td>
</tr>
</tbody>
</table>

If \( n = m \), \( \approx \frac{1}{e k!} \):
### Open Addressing

#### Searching

**Hash Tables**
- Hash Tables
- Time Complexity
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**Hash Functions**

1. **linear probing:** \( h(k, i) = (h_1(k) + i) \mod m \) for increasing \( i \)
   - the runs

2. **double hashing:** \( h(k, i) = (h_1(k) + ih_2(k)) \mod m \) for increasing \( i \)
   - requires: \( h_2 \neq 0 \), \( h_2(k) \) and \( m \) relatively prime
   - eg, \( m \) prime and \( h_2(k) < m \)
   - or, \( m = 2^x \) and \( h_2(k) \) odd

3. **cuckoo hashing:** lookups \( O(n) \), insertions amortized expected \( O(1) \)
   - moral: low load factor

deletion?
Searching
Hash Tables
- Hash Tables
- Time Complexity
- More Collisions
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Hash Functions

- asst 1
- asst 2
- asst 3
- schedule
Hash Functions
Hash Functions

\[ h : \text{key} \rightarrow 0..m - 1 \]

1. mediocre is easy, good takes effort
2. want time (at most) linear in key size
3. perfect hashing is possible (and efficient) if keys known
   - linear time to construct, linear space to store
4. minimal perfect hashing is possible!
$h : key \rightarrow 0..m - 1$

1. mediocre is easy, good takes effort
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4. minimal perfect hashing is possible!

bad news:

- if $|keys| \geq m$, there must be collisions
- if $|keys| \geq n \cdot m$, then $\exists$ set of $n$ that map to same bin
Desiderata:

- make collisions unlikely
  - spread keys across all hashes
  - for each key, each hash equally likely
- similar keys get different hashes
  - all bits of key affect the hash
  - every bit of key affects every bit of hash
- no input always gives worst-case behavior
- fast to compute
- low memory requirement
- easy to implement
Basic Hash

1. $hash \leftarrow 0$
2. for each byte of key
3. $hash \leftarrow (hash \times multiplier) + \text{byte}$
5. return $hash$ mod $table-size$

want $multiplier$ to smear bits, not shift them (to avoid interaction with table size)

$multiplier = 31 \text{ or } 127$
assume we have a table of 256 random integers

1. \( \text{hash} \leftarrow 0 \)
2. for each byte of key
3. rotate the bits in \( \text{hash} \) by 1
4. \( \text{hash} \leftarrow \text{hash} \oplus \text{table}[\text{byte}] \)
5. return \( \text{hash} \mod \text{table-size} \)

each byte affects all bits
rotate makes order matter

**universal** class of hash functions: for randomly chosen keys, randomly chosen function from class has \( P(\text{collision}) = 1/m \)

good on average case (over inputs) \( \neq \) good average case on any input
What’s still confusing?
What question didn’t you get to ask today?
What would you like to hear more about?

Please write down your most pressing question about algorithms and put it in the box on your way out.

Thanks!