http://www.cs.unh.edu/~ruml/cs758
Backtracking
Hardness

NPC: SAT, vertex cover, clique, subset sum, ...

greedy: local choice is optimal
DP: poly number of options to track
search: exponential number of options, often combinations
A tree representation of alternatives in a small combinatorial problem.
depth-first search
child ordering
lower bounds
branch-and-bound
duplicate detection: transposition table
**Depth-first Search**

DFS \((node)\)
1. If is-leaf\((node)\)
2. Visit\((node)\)
3. else
4. For \(i\) from 0 to \(num\text{-}children\)
5. DFS\(\text{child}(node, i)\)
Wheeler Ruml (UNH)
Improved Discrepancy Search

ILDS \((node, allowance, remaining)\)

1. If is-leaf\((node)\)
2. Visit\((node)\)
3. else
4.   If allowance > 0
5.      ILDS(child\((node, 1), allowance - 1, remaining - 1)\)
6.   If remaining > allowance
7.      ILDS(child\((node, 0), allowance, remaining - 1)\)

start with ILDS(root, iteration, max-depth)
The second pass of ILDS visits all leaves with one discrepancy in their path from the root.
Break

- asst 14
- recitation: last year’s final
- final exam: Wed Dec 12, 3:30-5:30pm, N101
Local Search
A graph representing an improvement-based search.
Local Search

- hill climbing
- simulated annealing
- large neighborhood search
- genetic algorithms
- particle swarm optimization

Backtracking

Local Search

- Local Search
- Max Cut
- Suboptimality
- EOLQs
maximize weight of edges crossing the cut $w(A, B)$

decision version is NP-complete

simple local search:

move vertex $u$ from $A$ to $B$ iff

$$\sum_{v \neq u \in A} w_{uv} > \sum_{v \in B} w_{uv}$$

it’s possible to bound suboptimality of local minima under this neighborhood!
Suboptimality of Local Search

for any $u$ in $A$,

$$\sum_{v \neq u \in A} w_{uv} \leq \sum_{v \in B} w_{uv}$$

summing over all $u$ in $A$,

$$2 \sum_{(u,v) \in A} w_{uv} \leq \sum_{u \in A, v \in B} w_{uv} = w(A, B)$$

same from perspective of $B$:

$$2 \sum_{(u,v) \in B} w_{uv} \leq \sum_{u \in A, v \in B} w_{uv} = w(A, B)$$

add:

$$2 \sum_{(u,v) \in A} w_{uv} + 2 \sum_{(u,v) \in B} w_{uv} \leq 2w(A, B)$$
Suboptimality of Local Search

divide by 2:

\[ \sum_{(u,v) \in A} w_{uv} + \sum_{(u,v) \in B} w_{uv} \leq w(A, B) \]

eg, more weight crossing than within partitions

let \( W \) be sum of all weight in graph.

add crossing weight to both sides:

\[ W \leq 2w(A, B) \]

\[ W/2 \leq w(A, B) \]

note optimal is at most \( W \)
For example:

- What’s still confusing?
- What question didn’t you get to ask today?
- What would you like to hear more about?

Please write down your most pressing question about algorithms and put it in the box on your way out.

*Thanks!*