http://www.cs.unh.edu/~ruml/cs758

1 handout: slides
Radix Sort
For \( n \) numbers in the range 0 to \( k \):

1. for \( x \) from 0 to \( k \)
2. \( \text{count}[x] \leftarrow 0 \)
3. for each input number \( x \)
4. increment \( \text{count}[x] \)
5. for \( x \) from 0 to \( k \)
6. do \( \text{count}[x] \) times
7. emit \( x \)
For $n$ numbers in the range 0 to $k$:

1. for $x$ from 0 to $k$ \quad $O(k)$
2. \quad \text{count}[x] \leftarrow 0
3. for each input number $x$ \quad $O(n)$
4. \quad \text{increment \text{count}[x]}
5. for $x$ from 0 to $k$ \quad $O(k)$ times around loop
6. \quad \text{do \text{count}[x] times} \quad \text{iterates $O(n)$ times total}
7. \quad \text{emit $x$} \quad \text{$O(1)$ each time}

\[ O(k + n + k + n) = O(2n + 2k) = O(n + k) \neq O(n \lg n) \]
\[ O(g(n)) = \{ f(n) : \text{there exist positive constants } c, n_0 \text{ such that } f(n) \leq cg(n) \text{ for all } n \geq n_0 \} \]

ignore constant factors
ignore ‘start-up costs’
upper bound

We can upper-bound \( f \) (except perhaps at start) by scaling \( g \) by a constant.

eg, running time of
\[ 10n^2 - 5n = O(n^2) \]
10n^2 + 5n = \Theta(n^2)

10n \lg \frac{n}{e} = O(n \lg n)
Stable Counting Sort

Input array contains $n$ records with keys in the range 0 to $k$. 

Radix Sort
- Counting Sort
- $O()$
- $O()$ Example
- **Stable Counting**
- Radix Sort

Analysis
Stable Counting Sort

Input array contains \( n \) records with keys in the range 0 to \( k \)

1. set \( \text{count}[x] \) to number of items with key \( = x \)
2. set \( \text{pos}[x] \) to total number of keys \( < x \)
3. for each input record \( r \) (in order)
4. write \( r \) in output array at position \( \text{pos}[\text{key of } r] \)
5. increment \( \text{pos}[\text{key of } r] \)

Complexity?
Invariants?
How to sort one million records?
Radix Sort

How to sort one million records?

How to sort one trillion 4-bit integers?
How to sort one million records?

How to sort one trillion 4-bit integers?

How to sort one billion 16-bit integers?
Radix Sort

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How to sort one billion 64-bit integers?
Radix Sort

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For \( n \) numbers with \( d \) digits (each digit has \( k \) values):
Radix Sort

Radix Sort
- Counting Sort
- $O()$
- $O()$ Example
- Stable Counting
- Radix Sort

Analysis

How to sort one million records?

How to sort one trillion 4-bit integers?

How to sort one billion 16-bit integers?

How to sort one billion 64-bit integers?

For $n$ numbers with $d$ digits (each digit has $k$ values):

1. for $i$ from 0 to $d$
2. stable sort on digit in place $i$ from right
Analysis
What’s the invariant in radix sort?
What’s the space complexity?
What’s the time complexity?
Why not implemented more?
- everyone receiving piazza notifications?
- books available?
- asst 1: agate, valgrind, submit, happy William
- probabilistic grading
- schedule: asst 2, asst 7
for $i$ from 2 to $n$
move $A[i]$ earlier until in place

worse case?
best case?
‘divide and conquer’: divide, conquer, combine

Mergesort\((A, i, j)\)
1. if \(i \geq j\), done
2. \(k \leftarrow (i + j)/2\)
3. Mergesort\((A, i, k)\)
4. Mergesort\((A, k + 1, j)\)
5. merge \(A[i..k]\) and \(A[k+1..j]\) into \(A[i..j]\)

how does merge work?
running time?
divide, conquer, combine?

**Quicksort**\((A, i, j)\)

1. choose pivot key \(x\)
2. partition \(A[i..j]\) into \(A[i..p - 1]\) and \(A[p + 1..j]\)
3. if \(p - 1 > i\) then Quicksort\((A, i, p - 1)\)
4. if \(j > p + 1\) then Quicksort\((A, p + 1, j)\)
Quicksort

divide, conquer, combine?

**Quicksort** \((A, i, j)\)

1. choose pivot key \(x\)
2. partition \(A[i..j]\) into \(A[i..p - 1]\) and \(A[p + 1..j]\)
3. if \(p - 1 > i\) then Quicksort\((A, i, p - 1)\)
4. if \(j > p + 1\) then Quicksort\((A, p + 1, j)\)

**+**: entirely in-place, no allocation
 often less copying than merge sort

**−**: expected \(O(n \lg n)\)
 needs tricks to avoid worst case
**Partition**

\[ \text{Partition}(A, i, j) \]

1. choose pivot key \( p \) and swap into \( A[j] \)
2. \( x = i \)
3. for \( y = i \) to \( j - 1 \)
4. if \( A[y] \leq p \)
5. swap \( A[x] \) and \( A[y] \)
6. \( x \leftarrow x + 1 \)
7. swap \( A[x] \) and \( A[j] \)

A: (i:) less (x:) greater (y:) unknown (j:) pivot
What is the minimum that a sorting algorithm must do?
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How many possible outputs are there for sorting $n$ items?
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How many possible outputs are there for sorting $n$ items?

binary tree with $n!$ leaves
What is the minimum that a sorting algorithm must do? How many possible outputs are there for sorting $n$ items? A binary tree with $n!$ leaves has height at least $\lg(n!)$.
What is the minimum that a sorting algorithm must do?

How many possible outputs are there for sorting \( n \) items?

A binary tree with \( n! \) leaves has height at least \( \lg(n!) \).

Stirling: \( n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n (1 + \Theta\left(\frac{1}{n}\right)) \)
What is the minimum that a sorting algorithm must do? How many possible outputs are there for sorting $n$ items?

A binary tree with $n!$ leaves has height at least $\lg(n!)$.

Stirling: 

$$n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n (1 + \Theta\left(\frac{1}{n}\right))$$

So:

$$\lg(n!) = \lg\left(\sqrt{2\pi n} \left(\frac{n}{e}\right)^n (1 + \Theta\left(\frac{1}{n}\right))\right)$$
What is the minimum that a sorting algorithm must do? How many possible outputs are there for sorting $n$ items? A binary tree with $n!$ leaves has height at least $\lg(n!)$.

Stirling: $n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n (1 + \Theta(\frac{1}{n}))$

So:

$$\lg(n!) = \lg(\sqrt{2\pi n} \left(\frac{n}{e}\right)^n (1 + \Theta(\frac{1}{n})))$$

$$= \lg \sqrt{2\pi} + \lg \sqrt{n} + \lg\left(\frac{n}{e}\right)^n + \lg(1 + \Theta(\frac{1}{n}))$$
What is the minimum that a sorting algorithm must do? How many possible outputs are there for sorting $n$ items?

A binary tree with $n!$ leaves has height at least $\lg(n!)$.

Stirling: $n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n (1 + \Theta\left(\frac{1}{n}\right))$

So:

$$\lg(n!) = \lg\left(\sqrt{2\pi n} \left(\frac{n}{e}\right)^n (1 + \Theta\left(\frac{1}{n}\right))\right)$$

$$= \lg\sqrt{2\pi} + \lg\sqrt{n} + \lg\left(\frac{n}{e}\right)^n + \lg(1 + \Theta\left(\frac{1}{n}\right))$$

$$= \Theta\left(\lg\sqrt{n} + n \lg\left(\frac{n}{e}\right)\right) + \lg(1 + \Theta\left(\frac{1}{n}\right))$$
What is the minimum that a sorting algorithm must do? How many possible outputs are there for sorting $n$ items?

Binary tree with $n!$ leaves has height at least $\lg(n!)$

Stirling: $n! = \sqrt{2\pi n} \left( \frac{n}{e} \right)^n \left( 1 + \Theta\left(\frac{1}{n}\right) \right)$

so:

$$
\lg(n!) = \lg(\sqrt{2\pi n} \left( \frac{n}{e} \right)^n \left( 1 + \Theta\left(\frac{1}{n}\right) \right))
$$

$$
= \lg \sqrt{2\pi} + \lg \sqrt{n} + \lg \left( \frac{n}{e} \right)^n + \lg(1 + \Theta\left(\frac{1}{n}\right))
$$

$$
= \Theta(\lg \sqrt{n} + n \lg \left( \frac{n}{e} \right)) + \lg(1 + \Theta\left(\frac{1}{n}\right))
$$

$$
= \Theta(n \lg n)
$$
What is the minimum that a sorting algorithm must do?
How many possible outputs are there for sorting $n$ items?

A binary tree with $n!$ leaves has height at least $\lg(n!)$

Stirling: $n! = \sqrt{2\pi n} \left(\frac{n}{e}\right)^n (1 + \Theta\left(\frac{1}{n}\right))$

so:

$$\lg(n!) = \lg(\sqrt{2\pi n} \left(\frac{n}{e}\right)^n (1 + \Theta\left(\frac{1}{n}\right)))$$

$$= \lg\sqrt{2\pi} + \lg\sqrt{n} + \lg\left(\frac{n}{e}\right)^n + \lg(1 + \Theta\left(\frac{1}{n}\right))$$

$$= \Theta(\lg\sqrt{n} + n \lg\left(\frac{n}{e}\right)) + \lg(1 + \Theta\left(\frac{1}{n}\right))$$

$$= \Theta(n \lg n)$$

so comparison-based sorting takes $\Omega(n \lg n)$ time
What’s still confusing?
What question didn’t you get to ask today?
What would you like to hear more about?

Please write down your most pressing question about algorithms and put it in the box on your way out.

*Thanks!*