http://www.cs.unh.edu/~ruml/cs758
Applications of Cuts and Flows
value of a flow = flow across any cut

any flow value $\leq$ capacity of cut

Theorem: these are the same:

1. $f$ is a maximum flow
2. the residual network $G_f$ contains no augmenting paths
3. there exists a cut whose capacity is the value of $f$

1+2: FF is correct; 1+3: FF also finds minimum cuts
an image as a graph!

maximize

\[
\sum_{i \in A} a_i + \sum_{i \in B} b_i - \sum_{e \text{ cut by } A} p_{i,j}
\]

minimize

\[
\sum_{i \in A} b_i + \sum_{i \in B} a_i + \sum_{e \text{ cut by } A} p_{i,j}
\]

cut crosses three types of edges: \(s_i, t_i, \) and \(p_{i,j}\)
Maximum Matching

bipartite graphs: jobs/machines, classes/instructors, ...
Maximum Matching

bipartite graphs: jobs/machines, classes/instructors, ...

unit capacities

flow = matching

FF guarantees integer flow

running time? (hint: bound |f*|)
asst 11
does a feasible schedule exist using only 3 machines (allowing preemption)?

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<thead>
<tr>
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<th>2</th>
<th>3</th>
<th>4</th>
</tr>
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<tbody>
<tr>
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<td>1.5</td>
<td>1.25</td>
<td>2.1</td>
<td>3.6</td>
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<tr>
<td>release date</td>
<td>3</td>
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<tr>
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arcs from $s$ to jobs labeled with job size

arcs from job to feasible intervals labeled with length of interval

arcs from interval to $t$ labeled with total achievable work (num machines times length of interval)
selection

Multicommodity flow is NP-hard for integer flows. Use LP for fractional flows.
For example:

- What’s still confusing?
- What question didn’t you get to ask today?
- What would you like to hear more about?

Please write down your most pressing question about algorithms and put it in the box on your way out.

*Thanks!*