http://www.cs.unh.edu/~ruml/cs758
Graph Traversal

Graphs
Breadth-first
The Algorithm
Factoids
Proof
Break
Depth-first Search
Edges
EOLQs

Graph Traversal
directed, arc/edge, weighted, labeled. drawings
representation
relations → edges
cycle, DAG, tree, planar
traversal: graph $\rightarrow$ forest
The Algorithm

1. foreach vertex, label it undiscovered and $v.d \leftarrow \infty$
2. $\text{start}'s\ label \leftarrow \text{discovered}, \ d \leftarrow 0, \ \pi \leftarrow \text{nil}$
3. $Q \leftarrow \{\text{start}\}$
4. while $Q$ not empty
5. \hspace{1em} $u \leftarrow \text{dequeue}(Q)$
6. foreach neighbor $v$ of $u$
7. \hspace{1em} if $v$ is undiscovered
8. \hspace{2em} label $v$ discovered, $v.d \leftarrow u.d + 1, \ v.\pi \leftarrow u$
9. \hspace{1em} enqueue $v$ in $Q$
10. label $u$ finished

Which vertices does $Q$ hold (at line 4)?
Do we really need all the labels?
What’s the time complexity?
1. Distances we assign always stay the same or go down.

2. \( v.d \geq \delta(s, v) \)
   Proof: Show \( v.d \geq \delta(s, v) \forall v \) via induction over iterations:
   Holds at start.
   \( v.d \) is updated to \( u.d + 1 \geq \delta(s, u) + 1 \geq \delta(s, v) \).

3. \( d \) values in queue are nondecreasing and last in queue exceeds first by at most 1.
   Proof: By induction. True when queue is \( s \).
   Preserved by dequeue.
   Enqueue: \( \text{new}.d = \text{removed}.d + 1 \leq \text{first}.d + 1 \) and \( \text{last}.d \leq \text{removed}.d + 1 = \text{new}.d \).

4. At termination, \( v.d = \delta(s, v) = \) shortest path length
Claim: at termination, \( v.d = \delta(s, v) = \) shortest path length

Consider \( v \) with minimum incorrect distance, and \( u \) that is before it on a shortest path. \( v.d > \delta(u) + 1 = u.d + 1 \). When \( u \) is dequeued:

if \( v \) is undiscovered, it would then be correct, contradiction.
if \( v \) is already finished, then \( v.d \leq d.u \), contradiction.
if \( v \) is discovered, let \( w \) be predecessor. \( v.d = w.d + 1 \) and \( w.d \leq u.d \) so \( v.d \leq u.d + 1 \), contradiction.
asst 8
midterm
wildcard vote in one month
Depth-first Search

1. forall vertices, label ← undiscovered
2. DFS-visit(start)

DFS-visit(u)
3. label u discovered
4. foreach neighbor v of u
5. if v is undiscovered
6. v.π ← u
7. DFS-visit(v)
8. label u finished

What’s the time complexity?
Discovery and finish times are parenthesized
Vs breadth-first?
Edges

- **tree:** in depth-first tree
- **back:** connects to ancestor in tree
- **forward:** non-tree edge connecting to descendant in tree
- **cross:** others: non-ancestors/non-descendants or different DFS tree

when edge is explored, label of arc dest gives type
For example:

- What’s still confusing?
- What question didn’t you get to ask today?
- What would you like to hear more about?

Please write down your most pressing question about algorithms and put it in the box on your way out.

*Thanks!*