

CS 730/730W/830: Intro AI

Propositional Logic

First-Order Logic

1 handout: slides

Propositional Logic

- Logic
- The PSSH
- Semantics
- Reasoning
- Refutation
- CNF
- Break

First-Order Logic

Propositional Logic

Propositional Logic

■ Logic

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First-Order Logic

A logic is a formal system:

- syntax: defines sentences
- semantics: relation to world
- inference rules: reaching new conclusions

three layers: proof, models, reality

soundness, completeness

flexible, general, principled (Advice Taker, 1958)

Empirical Philosophy = Science

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First-Order Logic

The Physical Symbol System Hypothesis: A physical symbol system has the necessary and sufficient means for general intelligent action. (Newell and Simon)

where a

Symbol is a designating pattern that can be combined with others to form another designating pattern

and

Designation means standing in for something in the world

Semantics

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First-Order Logic

Interpretation: possible world = state of affairs = truth value for each proposition

Model: interpretation in which sentence is true

Meaning: values across all models

Entailment (\models): α true in all models of KB

x	y	$(x \wedge \neg y)$	z	$(x \wedge \neg y) \rightarrow z$
T	T	F	T	T
T	T	F	F	T
T	F	T	T	T
T	F	T	F	F
F	T	F	T	T
F	T	F	F	T
F	F	F	T	T
F	F	F	F	T

Propositional Reasoning

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First-Order Logic

computing entailment
soundness, completeness
modus ponens, resolution

$\alpha \models \beta$ iff $\alpha \leftarrow \beta$ is valid

determining validity/tautology is co-NP-complete (easy to test
proof of no)

therefore, verification that α is not entailed is polytime

$\alpha \models \beta$ iff $\alpha \wedge \neg\beta$ is unsatisfiable

determining satisfiability is NP-complete (easy to test proof of
yes)

Resolution Refutation Proofs

Given KB, is α entailed?

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Resolution Refutation Proofs

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First-Order Logic

Given KB, is α entailed?
(Is it true in all models of the KB?)

Resolution Refutation Proofs

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First-Order Logic

Given KB, is α entailed?

(Is it true in all models of the KB?)

Is $KB \wedge \neg\alpha$ satisfiable?

Resolution Refutation Proofs

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First-Order Logic

Given KB, is α entailed?

(Is it true in all models of the KB?)

Is $KB \wedge \neg\alpha$ satisfiable?

Resolution is refutation complete.

Conversion to Conjunctive Normal Form

Propositional Logic

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■ CNF

- Break

First-Order Logic

1. eliminate \leftrightarrow
2. eliminate \rightarrow
3. move \neg inward: $\neg\neg x$, $\neg(x \wedge y)$, $\neg(x \vee y)$
4. distribute \vee : $x \vee (y \wedge z)$

Break

Propositional Logic

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■ Break

First-Order Logic

- asst 2
- office hours: Mon or Wed?

Propositional Logic

First-Order Logic

■ First-Order Logic

■ EOLQs

First-Order Logic

First-Order Logic

Propositional Logic

First-Order Logic

■ First-Order Logic

■ EOLQs

Gottlob Frege (1848-1925)

PhD at 25

Begriffsschrift, 1879 (concept script)

"a formula language, modelled on that of arithmetic, of pure thought."



First-Order Logic

Propositional Logic

First-Order Logic

■ First-Order Logic

■ EOLQs

$$\forall person \text{ ItIsRaining}() \rightarrow \text{IsWet}(person)$$

1. Things:
 - constants: *John, Chair23*
 - functions (thing \rightarrow thing): *MotherOf(John), SumOf(1,2)*
2. Relations:
 - predicates (objects \rightarrow T/F): *IsWet(John), IsSittingOn(MotherOf(John), Chair23)*
3. Complex sentences:
 - connectives: *IsWet(John) \vee IsSittingOn(MotherOf(John), Chair23)*
 - quantifiers and variables: *$\forall person \text{IsWet}(person) \dots, \exists person \dots$*

First-Order Logic

Propositional Logic

First-Order Logic

■ First-Order Logic

■ EOLQs

1. constants: objects
2. predicates: relations between objects
3. variables
4. quantifiers
5. functions
6. connectives

More First-Order Logic

Propositional Logic

First-Order Logic

■ First-Order Logic

■ EOLQs

$$\forall person \quad (ItIsRaining() \wedge \neg \exists umbrella \text{ Holding}(person, umbrella)) \\ \text{IsWet}(person)$$

John loves Mary.

All crows are black.

Dolphin are mammals that live in the water.

Mary likes the color of one of John's ties.

Propositional Logic

First-Order Logic

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■ EOLQs

Please write down the most pressing question you have about the course material covered so far and put it in the box on your way out.

Thanks!