1 handout: slides

*You think you know when you can learn, are more sure when you can write, even more when you can teach, but certain when you can program.*
Adversarial Search
Adversarial Search

- Problems
- Different!
- Minimax
- Tic-tac-toe
- Improvements
- Break
- $\alpha$-$\beta$ Pruning
- $\alpha$-$\beta$ Pseudo-code
- Why $\alpha$-$\beta$?
- Progress
- EOLQs

Adversarial Search
Planning Problems

Observability: complete, partial, hidden
State: discrete, continuous
Actions: deterministic, stochastic, discrete, continuous
Nature: static, deterministic, stochastic
Interaction: one decision, sequential
Time: static/off-line, on-line, discrete, continuous
Percepts: discrete, continuous, uncertain
Others: solo, cooperative, competitive
Multi-agent is Different

- Shortest-path (M&C, vacuum, tile puzzle)
  - want least-cost path to goal at unknown depth
- Decisions with an adversary (chess, tic-tac-toe)
  - adversary might prevent path to best goal
  - want best assured outcome assuring rational opponent
Each *ply* corresponds to half a *move*. Terminal states are labeled with value.

Incorrect version by Zermelo (1912) full treatment by von Neumann and Morgenstern (1944)

Can also bound depth and use a *static evaluation function* on non-terminal states.
A 3-length is a complete row, column, or diagonal.

\[
\begin{align*}
\text{value of position} & = \infty \text{ if win for me}, \\
& \text{or } -\infty \text{ if a win for you}, \\
\text{otherwise} & = \# \text{ 3-lengths open for me} - \\
& \# \text{ 3-lengths open for you}
\end{align*}
\]
Tic-tac-toe: two-ply search

Fig. 3.8 Minimax applied to tic-tac-toe (stage 1).
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**Fig. 3.9 Minimax applied to tic-tac-toe (stage 2).**
Tic-tac-toe: third move

Fig. 3.10 Minimax applied to tic-tac-toe (stage 3).
Improving the Search

- partial expansion, SEF
- symmetry (‘transposition tables’)
- search more ply as we have time (De Groot figure)
- avoid unnecessary evaluations
asst 3
asst 4
projects! proposal due in 3 weeks.
Which Values are Necessary?

Adversarial Search
- Problems
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- Improvements

Break
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- $\alpha$-$\beta$ Pseudo-code
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\( \alpha \) best outcome Max can force at previous decision on this path (init to \(-\infty\))

\( \beta \) best outcome Min can force at previous decision on this path (init to \(\infty\))

\( \alpha \) and \( \beta \) values are copied down the tree (but not up). Minmax values are passed up the tree, as usual.

Max-value (state, $\alpha$, $\beta$):
  when depth-cutoff (state), return SEF(state)
  for each child of state
    $\alpha \leftarrow \max(\alpha, \text{Min-value (child, } \alpha, \beta))$
    when $\alpha \geq \beta$, return $\alpha$
  return $\alpha$

Min-value (state, $\alpha$, $\beta$):
  when depth-cutoff (state), return SEF(state)
  for each child of state
    $\beta \leftarrow \min(\beta, \text{Max-value (child, } \alpha, \beta))$
    when $\beta \leq \alpha$, return $\beta$
  return $\beta$
Fig. 3.12 An example illustrating the alpha-beta search procedure.
Time complexity of $\alpha-\beta$ is about $O(b^{d/2})$
Computers best: chess, checkers, backgammon, Scrabble, Jeopardy, Go
Computers competitive: bridge, crosswords, poker
Computers amateur: soccer?
Please write down the most pressing question you have about the course material covered so far and put it in the box on your way out.

Thanks!