

Recap

Heuristic Search

1 handout: slides
730W journal entries were due

Recap

Heuristic Search

Recap

- Problem Solving
- Comparison
- Are We Done?

Heuristic Search

Recap

Formalizing Problem Solving

Recap

■ Problem Solving

■ Comparison

■ Are We Done?

Heuristic Search

State: hypothetical world state

Operators: actions that modify world

Goal: desired state or test



(Herbert Simon and Allen Newell, “Computer simulation of human thinking and problem solving”, 1961)

Comparison

Recap

■ Problem Solving

■ Comparison

■ Are We Done?

Heuristic Search

Algorithm	Time	Space	Complete	Admissible
Depth-first	b^m	bm	If $m \geq d$	No
Breadth-first	b^d	b^d	Yes	If ops cost 1
Uniform-cost	b^d	b^d	Yes	Yes
Iterative deepening	b^d	bd	Yes	Yes

branching factor b
maximum depth m
solution depth d

Are We Done?

Recap

- Problem Solving
- Comparison
- Are We Done?

Heuristic Search



Recap

Heuristic Search

- Dijkstra Behavior
- Terminology
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- EOLQs

Heuristic Search

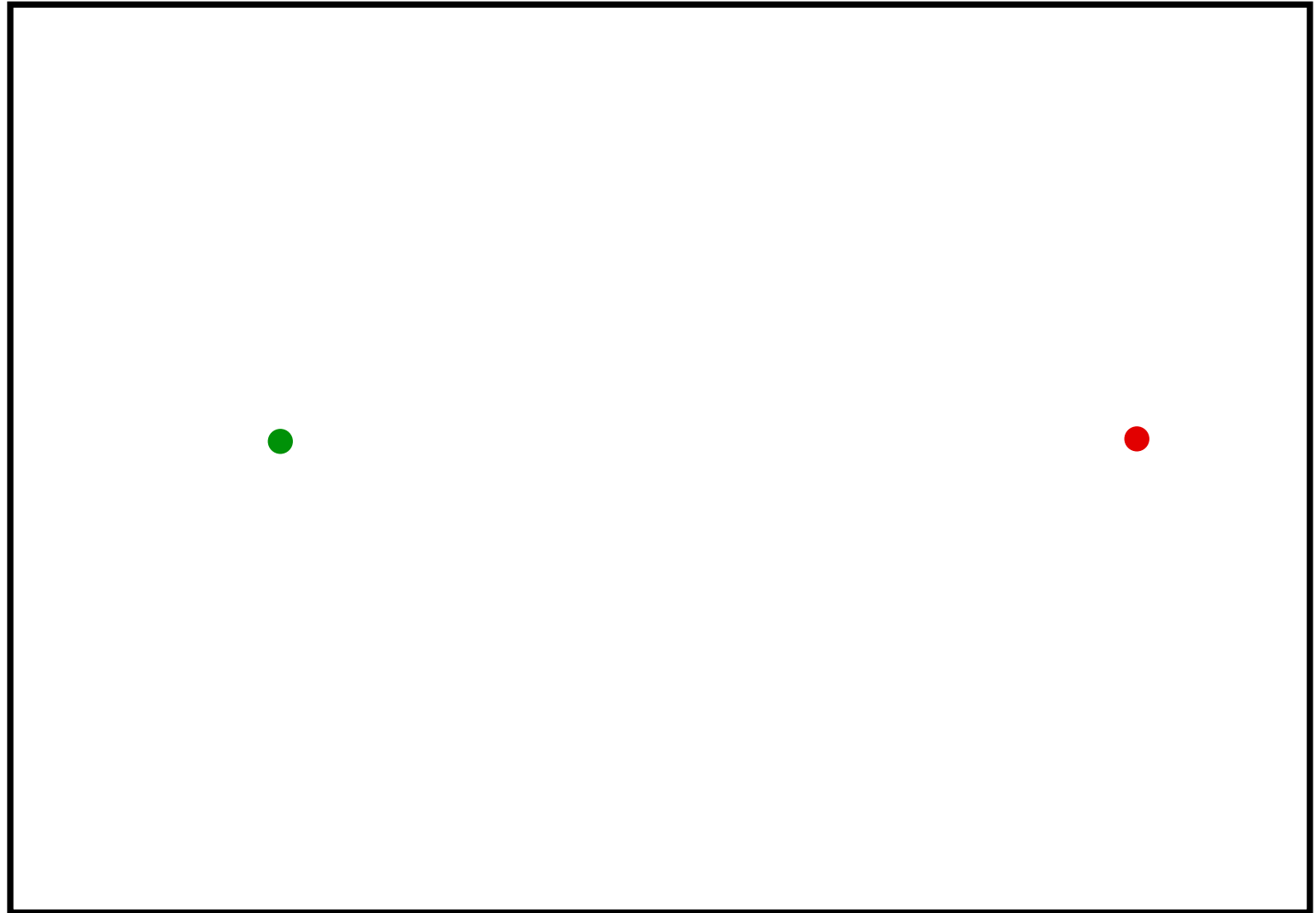
Dijkstra Behavior

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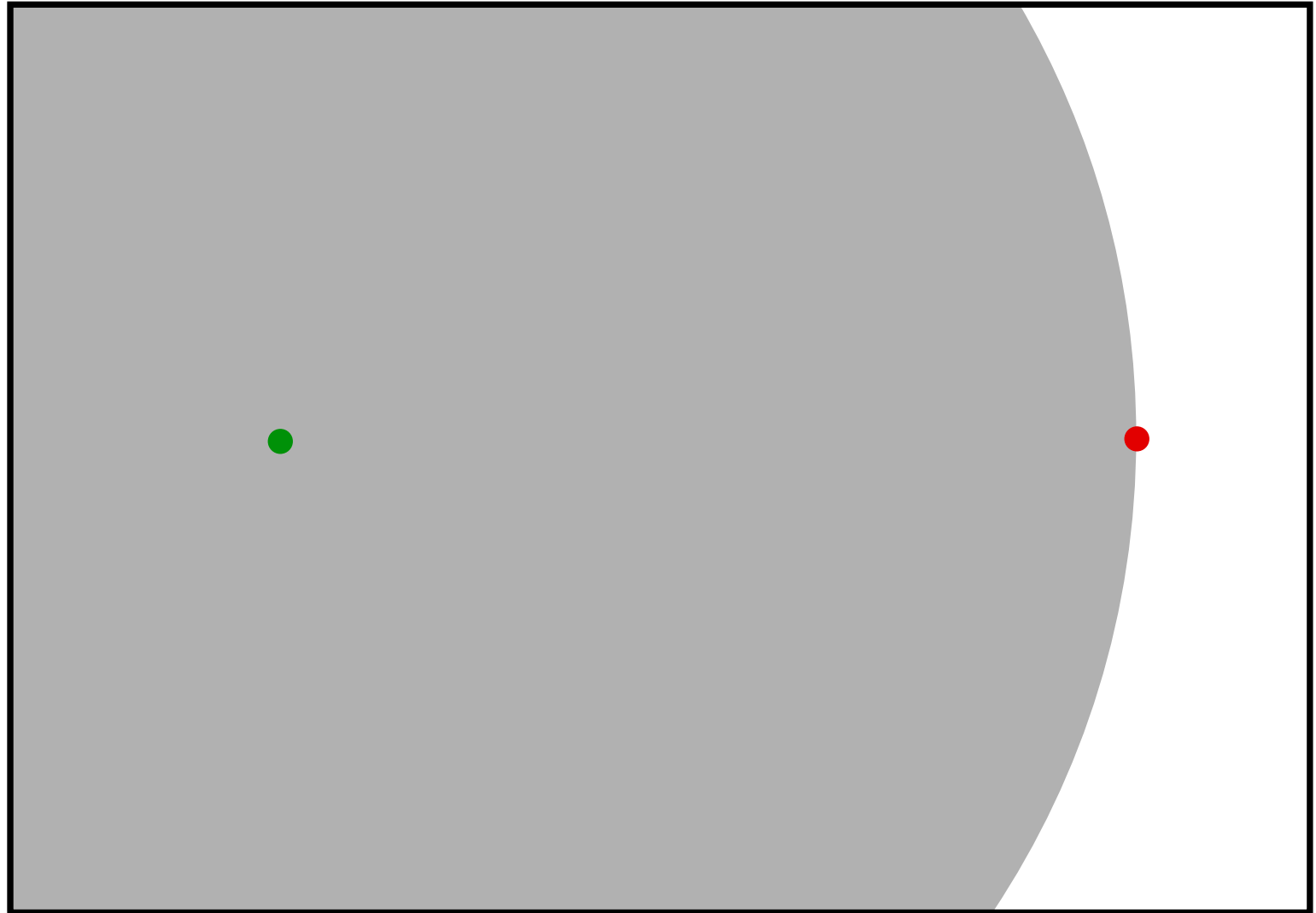
Dijkstra Behavior

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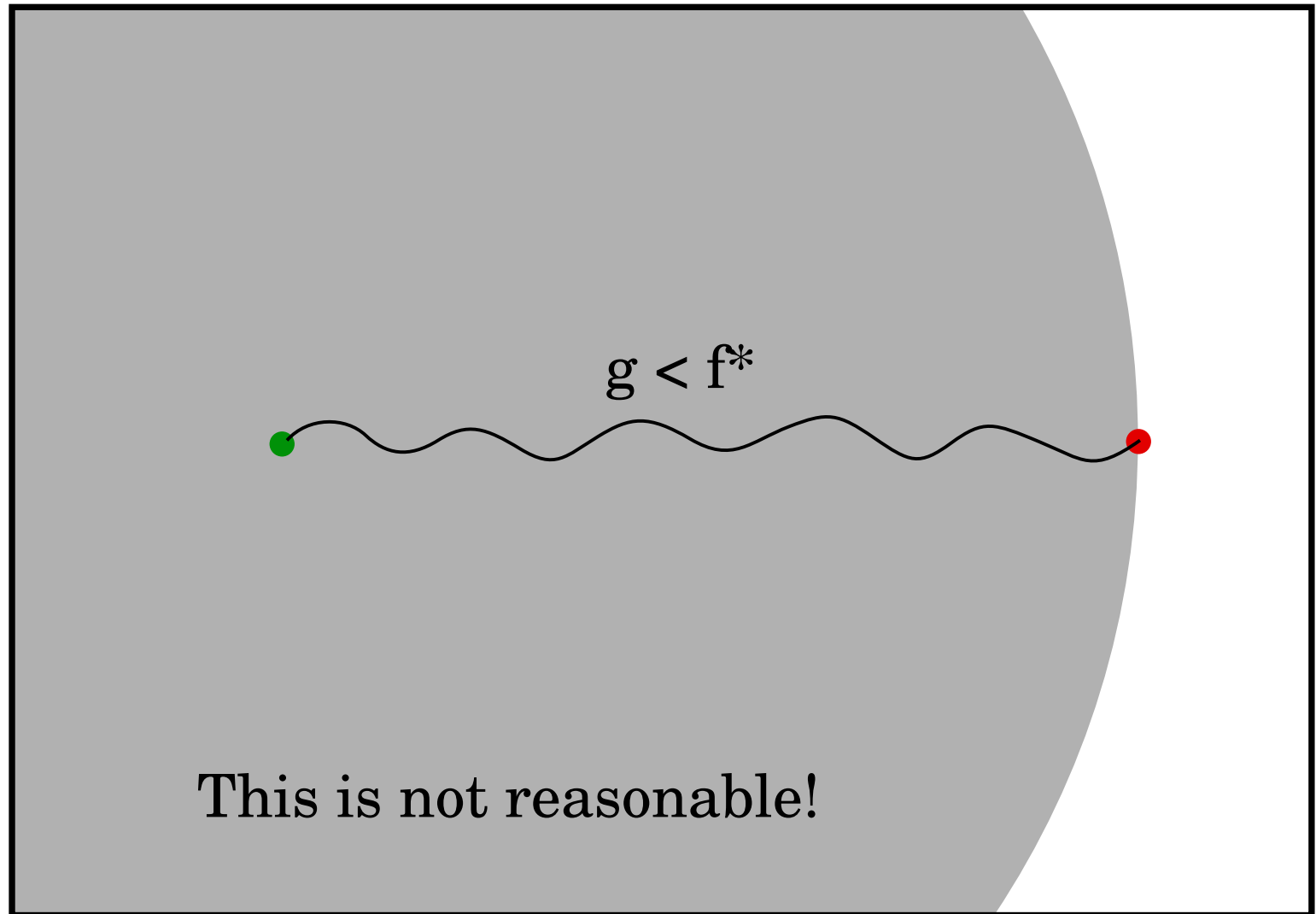
Dijkstra Behavior

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Heuristic Evaluation

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Heureka!

— *Archimedes*

Heuristic knowledge is useful, but not necessarily correct.

Heuristic algorithms use heuristic knowledge to solve a problem.

A *heuristic function* takes a state and returns an estimate of the distance to a goal.

(Newell and Ernst, 1965; Lin, 1965)

Greedy Search

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$Q \leftarrow$ an ordered list containing just the initial state.

Loop

If Q is empty,
then return failure.

$Node \leftarrow \text{Pop}(Q)$.

If $Node$ is a goal,
then return $Node$ (or path to it)

else

$Children \leftarrow \text{Expand}(Node)$.

Merge $Children$ into Q , keeping **sorted by heuristic**. \leftarrow

Greedy on the 8-puzzle

$h(n)$ = number of tiles out of place. (The blank is not a tile.)

	2	8	3		1	2	3
Start state:	1	6	4	Goal state:	8	□	4
	7	□	5		7	6	5

Please draw the tree resulting from the first two node expansions.

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Evaluating Greedy

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Assume branching factor b and solution at depth d .

Completeness:

Time:

Space:

Admissibility:

Break

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- slides: handout, website
- recitation
- blog entries due on Wednesdays, 8am
- asst 1
 - ◆ duplicates
 - ◆ make script, submission
 - ◆ transcript but no hardcopy for milestone
 - ◆ reference solution
 - ◆ standard libraries
- office hours
- projects (Apr 2), UROP (Mar 1), UNH AI group (see wiki)

A* Search

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Consider estimated final path cost! $f(n) = g(n) + h(n)$

$Q \leftarrow$ an ordered list containing just the initial state.

Loop

If Q is empty,
then return failure.

$Node \leftarrow \text{Pop}(Q)$.

If $Node$ is a goal,
then return $Node$ (or path to it)

else

$Children \leftarrow \text{Expand}(Node)$.

Merge $Children$ into Q , keeping **sorted by** $f(n)$. \leftarrow

A* on the 8-puzzle

A* doesn't get sidetracked like best-first.

Recap

Heuristic Search

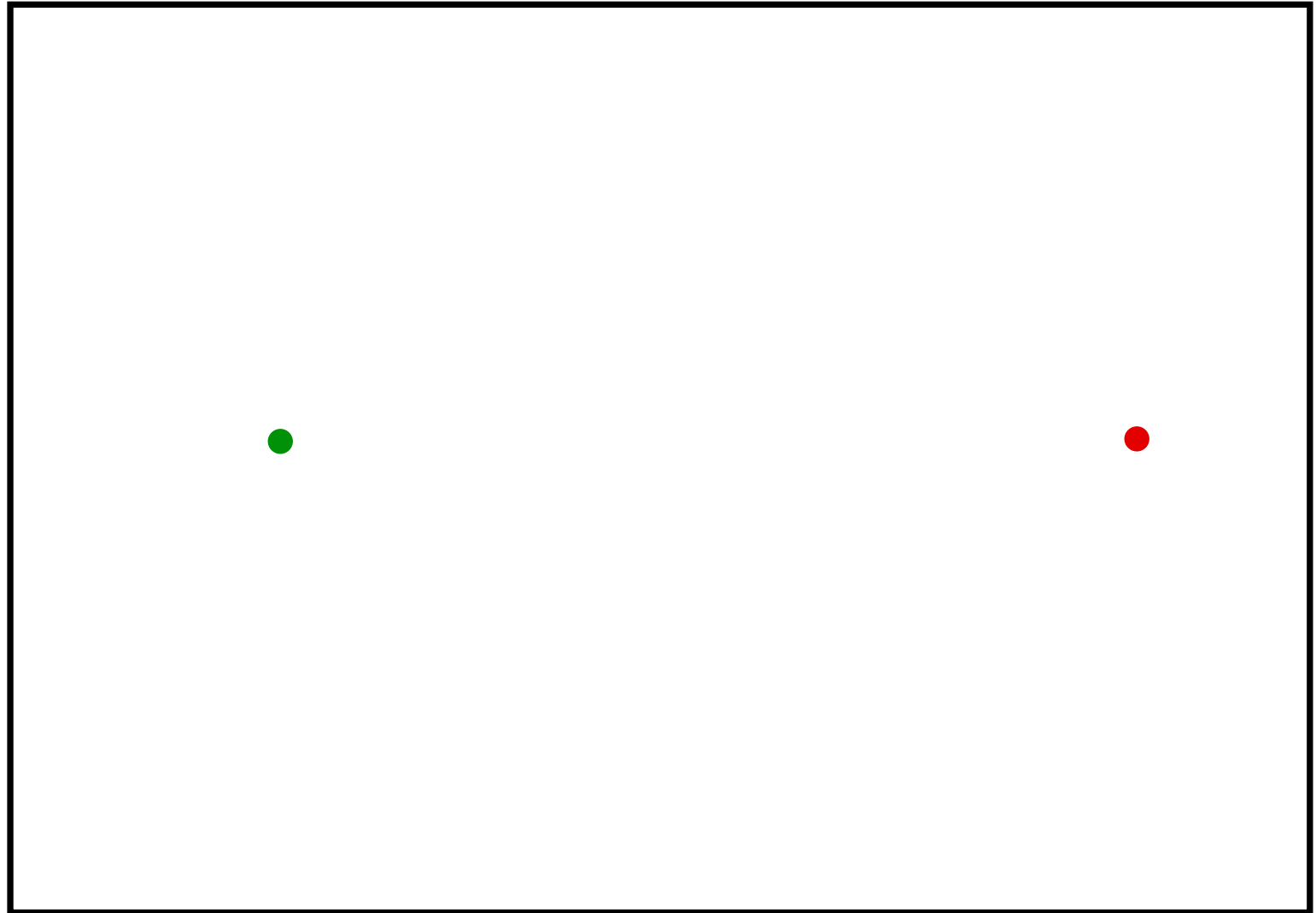
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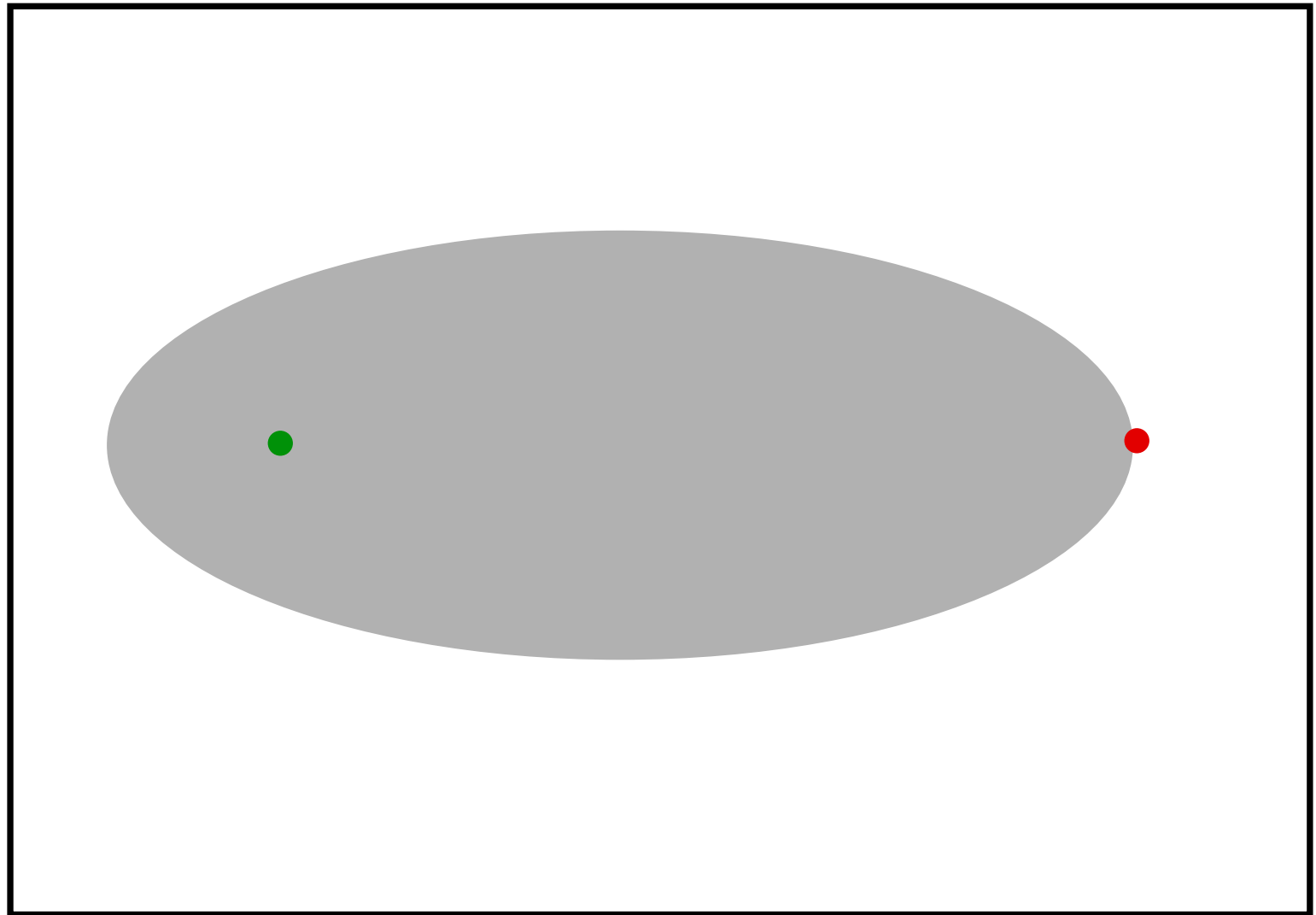


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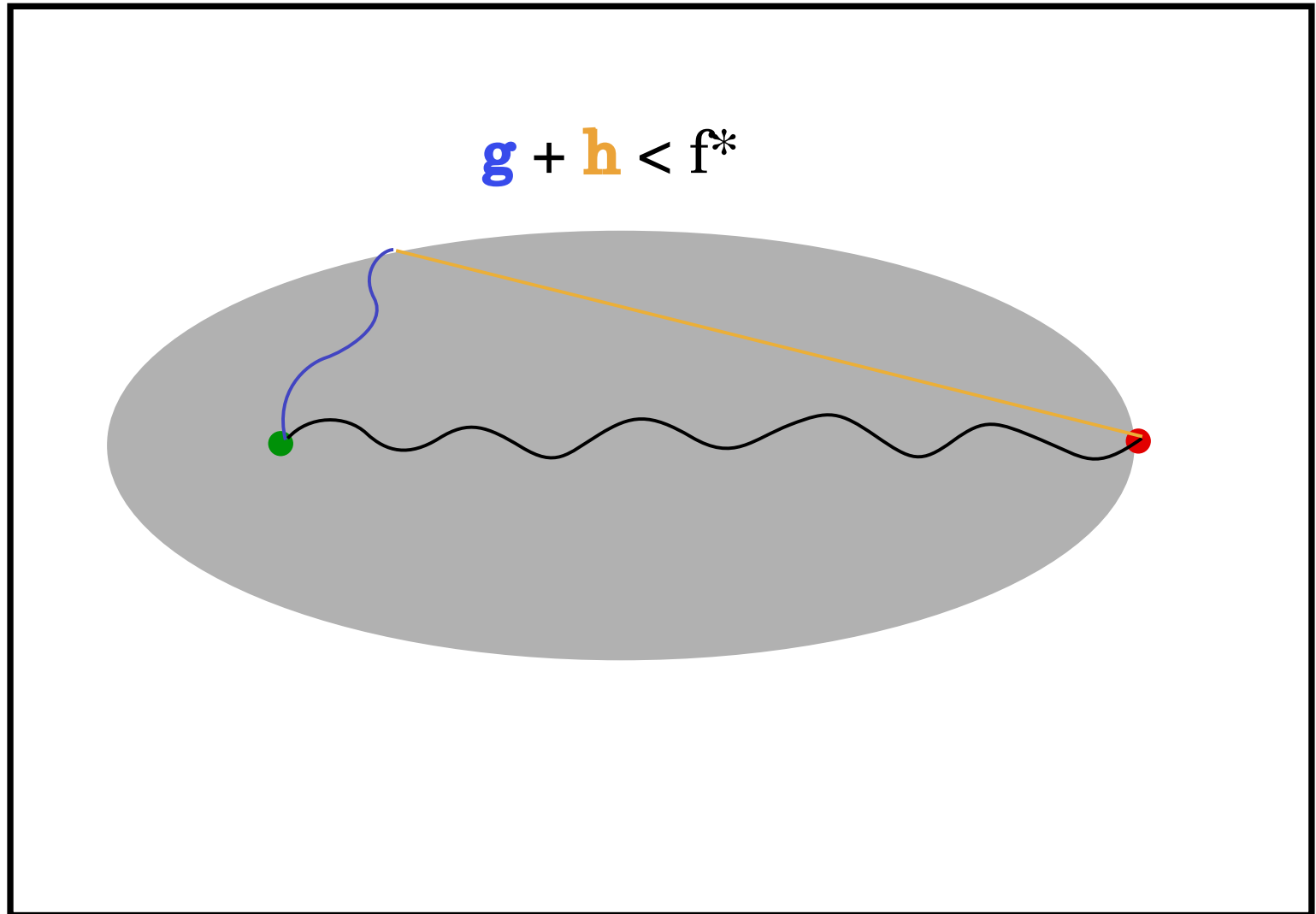


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Uniform-cost Search Behavior

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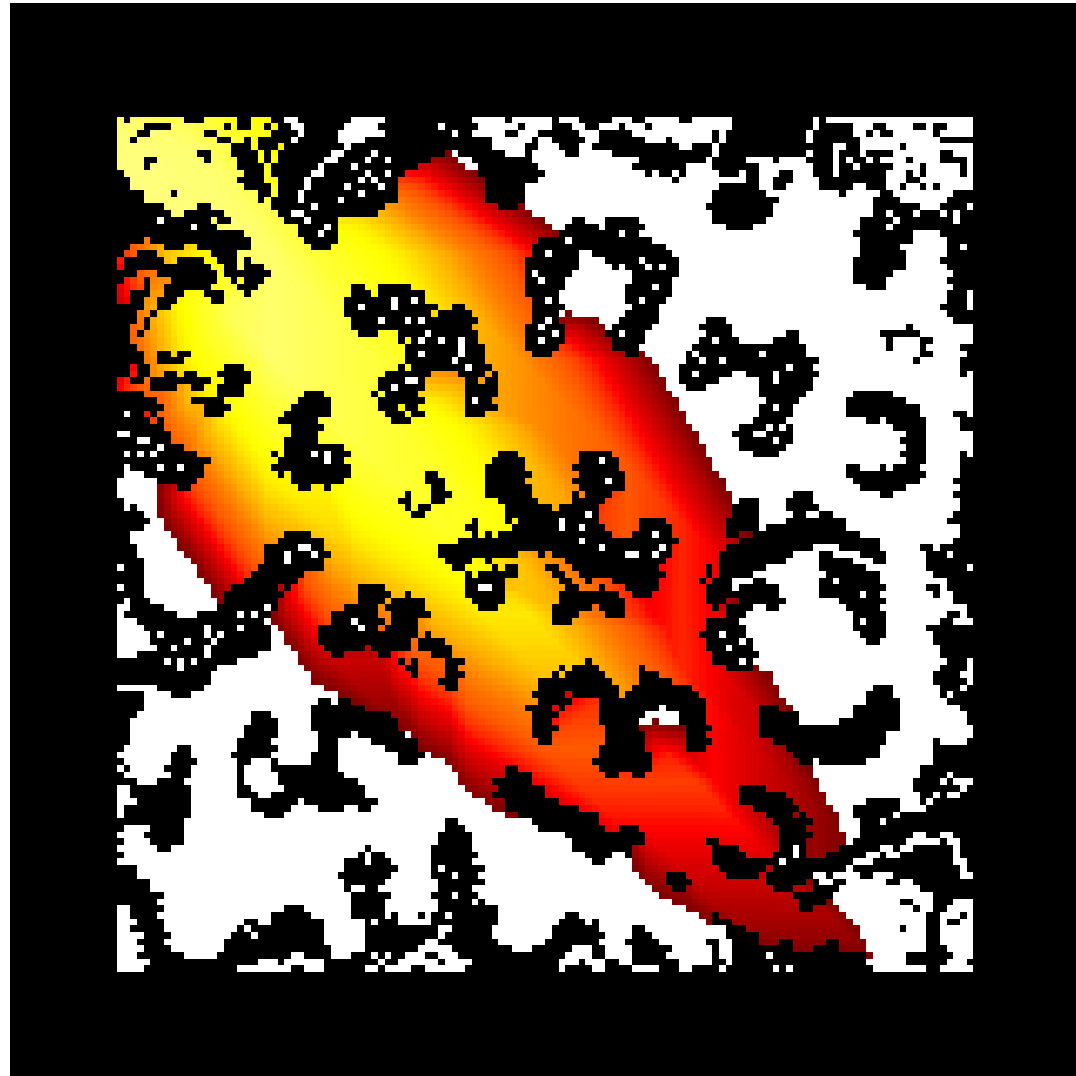


A* Behavior

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Evaluating A*

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Assume branching factor b and solution at depth d .

Completeness:

Time:

Space:

Admissibility:

Admissibility of A*

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Optimality of A*

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1. For admissible h , f can be made non-decreasing.
2. A* expands nodes in order of non-decreasing f .
3. Must examine all nodes with $f < f^*$.

Heuristic Functions

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1. Find by relaxing problem
2. Want cheap computation
3. Want highest value
 - (a) If $h_1(n) \leq h_2(n)$ for all n , h_2 dominates h_1

Beam Search

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Truncate queue to hold k nodes.

k is the *beam width*.

Can be used with any queue ordering (such as $g(n)$ or $f(n)$).

Other Shortest-path Algorithms

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- IDA*
- SMA*, IE
- RBFS
- RTA*, LRTA*
- A_{ϵ}^*

Course projects!

Search Algorithms

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- Uninformed methods
 - ◆ Depth-first
 - ◆ Breadth-first
 - ◆ Uniform-cost
- Informed methods ('best-first')
 - ◆ Greedy
 - ◆ A*
- Bounding memory
 - ◆ Iterative deepening
 - ◆ Beam search

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Please write down the most pressing question you have about the course material covered so far and put it in the box on your way out.

Thanks!