"Spock had a big, big effect on me. I am so much more Spock-like today than when I first played the part in 1965 that you wouldn't recognize me. I'm not talking about appearance, but thought processes. Doing that character, I learned so much about rational logical thought that it reshaped my life."

– Leonard Nimoy (1931–2015)

First-Order Inference

Clausal Form

1. Eliminate $\rightarrow$ using $\neg$ and $\vee$
2. Push $\neg$ inward using de Morgan's laws
3. Standardize variables apart
4. Eliminate $\exists$ using Skolem functions
5. Move $\forall$ to front
6. Move all $\wedge$ outside any $\vee$ (CNF)
7. Can finally remove $\forall$ and $\wedge$

Another Example

1. Anyone whom Mary loves is a football star.
2. Any student who does not pass does not play.
3. John is a student.
4. Any student who does not study does not pass.
5. Anyone who does not play is not a football star.
6. Prove: If John does not study, then Mary does not love John.

Break

- asst 6, 7
- project idea sharing next class
- proposals due Mar 11
  - now is the time to talk!
  - wait to start project until I comment on your proposal

Semantics

Semantics

A *possible world* is:

Propositional: a truth assignment for symbols. Exponential number of worlds.
First-order: a set of objects and an interpretation for constants, functions, and predicates (fixing referent of every term). Unbounded number of worlds.

No unique names assumption: constants not distinct.
No closed world assumption: unknown facts not false.

α valid iff true in every world
$\alpha \models \beta$ iff β true in every model of α

Terminology

Formally,

Interpretation: maps constant symbols to objects in the world, each function symbol to a particular function on objects, and each predicate symbol to a particular relation.

Model of P: an interpretation in which P is true. Eg, *Famous(LadyGaga)* is true under the intended interpretation but not when the symbol *LadyGaga* maps to Joe Shmoe.

Satisfiable: $\exists$ a model for P. Eg, $P \wedge \neg P$ is not satisfiable.

Entailment: if Q is true in every model of P, then $P \models Q$. Eg, $P \wedge Q \models P$.

Valid: true in any interpretation. Eg, $P \vee \neg P$.

The Basis for Refutation

Recall $\alpha \models \beta$ iff β true in every model of α.

1. Assume KB $\models \alpha$.
2. So if a model i satisfies KB, then i satisfies α.
3. If i satisfies α, then doesn't satisfy $\neg\alpha$.
4. So no model satisfies KB and $\neg\alpha$.
5. So KB $\wedge\neg\alpha$ is unsatisfiable.

Another way:

1. Suppose no model that satisfies KB also satisfies $\neg\alpha$. In other words, KB $\wedge\neg\alpha$ is unsatisfiable ($=$ inconsistent $=$ contradictory).
2. In every model of KB, α must be true or false.
3. Since in any model of KB, $\neg\alpha$ is false, α must be true in all models of KB.

Resolution is not complete: cannot derive $P \wedge \neg P$

FOL Odds and Ends

Refutation Completeness

Gödel's Completeness Theorem (1930) says a complete set of inference rules exists for FOL.

Herbrand base: substitute all constants and combinations of constants and functions in place of variables. Potentially infinite!

Herbrand's Theorem (1930): If a set of clauses is unsatisfiable, then there exists a finite subset of the Herbrand base that is also unsatisfiable.

Ground Resolution Theorem: If a set of ground clauses is unsatisfiable, then the resolution closure of those clauses contains $\perp$.

Robinson's Lifting Lemma (1965): If there is a proof on ground clauses, there is a corresponding proof in the original clauses.

FOL is semi-decidable: if entailed, will eventually know

Equality

Equality: $\forall xy\,(Holding(x) \wedge \neg(x = y) \rightarrow \neg Holding(y))$

Unique: $\exists! x P(x) \equiv \exists x\,(P(x) \wedge \forall y(\neg(x = y) \rightarrow \neg P(y)))$

Specific Answers

Use the "answer literal":

1. FatherOf(Alice, Bob)
2. FatherOf(Caroline, Bob)
3. FatherOf(x, y) $\rightarrow$ ParentOf(x, y)

Query: Who is Caroline's parent?

Resolution Strategies

Breadth-first: all first-level resolvents, then second-level...

- Complete, slow

Set of Support: at least one parent comes from SoS

- Complete if non-SoS are satisfiable, nice

Input Resolution: at least one parent from the input set

- Complete for Horn KBs

Simplifications: remove tautologies, subsumbed clauses, and pure literals.

EOLQs

Please write down the most pressing question you have about the course material covered so far and put it in the box on your way out.

Thanks!