1 handout: slides
First-order Logic
A logic is a formal system:

- syntax: defines sentences
- semantics: relation to world
- inference rules: reaching new conclusions

three layers: proof, models, reality

flexible, general, and principled form of KR
First-Order Logic

1. Things:
   - constants: John, Chair23
   - functions (thing → thing): MotherOf(John), SumOf(1,2)

2. Relations:
   - predicates (objects → T/F): IsWet(John), IsSittingOn(MotherOf(John), chair23)

3. Complex sentences:
   - connectives: IsWet(John) ∨ IsSittingOn(MotherOf(John), Chair23)
   - quantifiers and variables: ∀person..., ∃person...
\[ \forall \text{person} \ \forall \text{time} \ (\text{ItIsRaining}(\text{time}) \land \neg \exists \text{umbrella} \ \text{Holding}(\text{person}, \text{umbrella}, \text{time})) \rightarrow \text{IsWet}(\text{person}, \text{time}) \]

John loves Mary.

All crows are black.

Dolphin are mammals that live in the water.

Everyone loves someone.

Mary likes the color of one of John’s ties.

I can’t hold more than one thing at a time.
1. Indirect knowledge: $Tall(MotherOf(John))$
2. Counterfactuals: $\neg Tall(John)$
3. Partial knowledge (disjunction): $IsSisterOf(b, a) \lor IsSisterOf(c, a)$
4. Partial knowledge (indefiniteness): $\exists x IsSisterOf(x, a)$
Reasoning in First-order Logic
1. Eliminate → using ¬ and ∨
2. Push ¬ inward using de Morgan’s laws
3. Standardize variables apart
4. Eliminate ∃ using Skolem functions
5. Move ∀ to front
6. Move all ∧ outside any ∨ (CNF)
7. Can finally remove ∀ and ∧
1. Cats like fish.
2. Cats eat everything they like.
3. Joe is a cat.

Prove: Joe eats fish.
First-order Logic

Inference in FOL
- Clausal Form
- Example
- Break
- Unification
- Example
- Semantics
- Terminology
- Refutation
- Completeness
- EOLQs

- asst 6, 7
Unifying Two Terms

1. if one is a constant and the other is
2. a constant: if the same, done; else, fail
3. a function: fail
4. a variable: substitute constant for var
5. if one is a function and the other is
6. a different function: fail
7. the same function: unify the two arguments lists
8. a variable: if var occurs in function, fail
9. otherwise, substitute function for var
10. otherwise, substitute one variable for the other

Carry out substitutions on all expressions you are unifying!
Build up substitutions as you go, carrying them out before checking expressions?
See handout on website.
1. Anyone who can read is literate.
2. Dolphins are not literate.
3. Some dolphins are intelligent.

Skolem, standardizing apart
A possible world is:

**Propositional:** a truth assignment for symbols. Exponential number of worlds.

**First-order:** a set of objects and an interpretation for constants, functions, and predicates (fixing referent of every term). Unbounded number of worlds.

No unique names assumption: constants not distinct.
No closed world assumption: unknown facts not false.

\[ \alpha \text{ valid iff true in every world} \]
\[ \alpha \models \beta \text{ iff } \beta \text{ true in every model of } \alpha \]
Formally,

**Interpretation:** maps constant symbols to objects in the world, each function symbol to a particular function on objects, and each predicate symbol to a particular relation.

**Model of** $P$: an interpretation in which $P$ is true. Eg, $\text{Famous}(\text{BarbaraBush})$ is true under the intended interpretation but not when the symbol $\text{BarbaraBush}$ maps to Joe Shmoe.

**Satisfiable:** $\exists$ a model for $P$. Eg, $P \land \neg P$ is not satisfiable.

**Entailment:** if $Q$ is true in every model of $P$, then $P \models Q$.

Eg, $P \land Q \models P$.

**Valid:** true in any interpretation. Eg, $P \lor \neg P$. 


Recall $\alpha \models \beta$ iff $\beta$ true in every model of $\alpha$.

1. Assume $\text{KB} \models \alpha$.
2. So if a model $i$ satisfies $\text{KB}$, then $i$ satisfies $\alpha$.
3. If $i$ satisfies $\alpha$, then doesn’t satisfy $\neg \alpha$.
4. So no model satisfies $\text{KB}$ and $\neg \alpha$.
5. So $\text{KB} \land \neg \alpha$ is unsatisfiable.

The other way:

1. Suppose no model that satisfies $\text{KB}$ also satisfies $\neg \alpha$. In other words, $\text{KB} \land \neg \alpha$ is unsatisfiable (= inconsistent = contradictory).
2. In every model of $\text{KB}$, $\alpha$ must be true or false.
3. Since in any model of $\text{KB}$, $\neg \alpha$ is false, $\alpha$ must be true in all models of $\text{KB}$.
Gödel’s Completeness Theorem (1930) says a complete set of inference rules exists for FOL.

Herbrand base: substitute all constants and combinations of constants and functions in place of variables. Potentially infinite!

Herbrand’s Theorem (1930): If a set of clauses $S$ is unsatisfiable, then there exists a finite subset of its Herbrand base that is also unsatisfiable.

Ground Resolution Thm: If a set of ground clauses is unsatisfiable, then the resolution closure of those clauses contains $\bot$.

Robinson (1965): If there is a proof on ground clauses, there is a corresponding proof in the original clauses.

FOL is semi-decidable: if entailed, will eventually know
Please write down the most pressing question you have about the course material covered so far and put it in the box on your way out.

Thanks!