Shear-Warp Volume Rendering

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From:

Lacroute and Levoy, *Fast Volume Rendering Using a Shear-Warp- Factorization of the Viewing Transformation*, Siggraph ’94

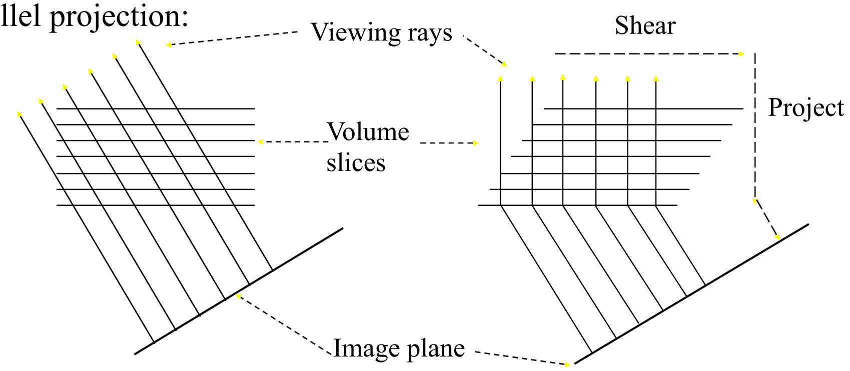
Volume Rendering Overview

- Spatial data structures
  - can lower costs without sacrificing quality
  - e.g., octrees, k-d trees, distance trees
- Image-order algorithms – casting rays through pixels
  - traverse spatial d.s. for every ray; multiple traversals
- Object-order algorithms – splatting
  - process data once, but hard to terminate processing early
- Shear-warp algorithms
  - efficient data traversal with possibility of early exit

Shear-Warp: Parallel Projection

- *Sheared object space*
  - simple transformation of volume allowing efficient projection
  - in this space all viewing rays are parallel to a coordinate axis

Parallel projection:



Viewing rays

Shear

Project

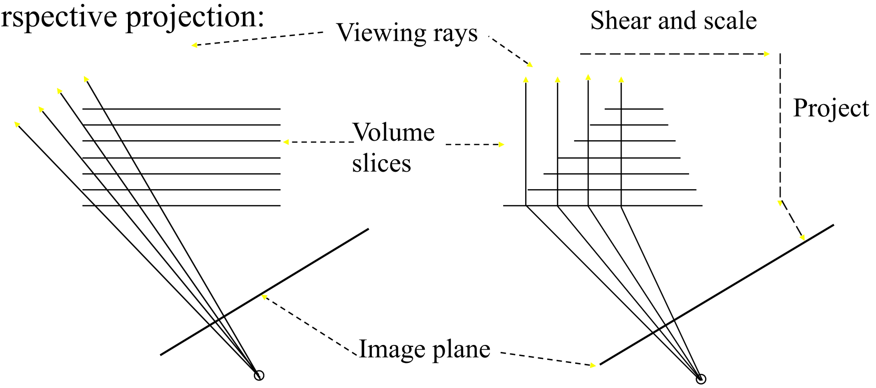
Volume slices

Image plane

Shear-Warp: Perspective Projection

- Perspective projection more complex
  - requires each slice to be scaled based on the view

Perspective projection:



Viewing rays

Shear and scale

Project

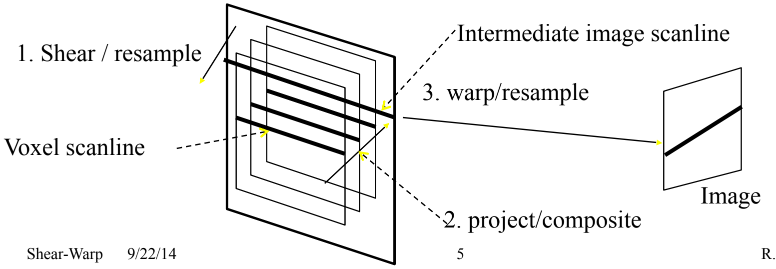
Volume slices

Image plane

Basic Algorithm

Determine which of 3 possible slicing directions to use (P).

1. Transform volume data to sheared object space by *translating* and *resampling* each slice (S).
2. Composite resampled slices in front-to-back order. This produces a 2D intermediate image in sheared object space.
3. Transform intermediate image to image space by warping (M_{warp}). This is a 2d *resampling* step.



Shear-Warp Factorization

- Shear-Warp can be expressed as factorization of the view transform matrix: $M_{view} = M_{warp2d} \cdot M_{shear3d} = M_{warp2d} \cdot S \cdot P$
  - P permutes axes that so shear is parallel to slices that are most perpendicular to viewing direction
  - S is shear whose terms can be extracted from M_{view}

$$S_{par} = \begin{bmatrix} 1 & 0 & s_x & 0 \\ 0 & 1 & s_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$S_{per} = \begin{bmatrix} 1 & 0 & s_x & 0 \\ 0 & 1 & s_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & s_w & 1 \end{bmatrix}$$

  - M_{warp2d} transforms sheared object coords to image coords:

$$M_{warp2d} = M_{view} \cdot P^{-1} \cdot S^{-1}$$

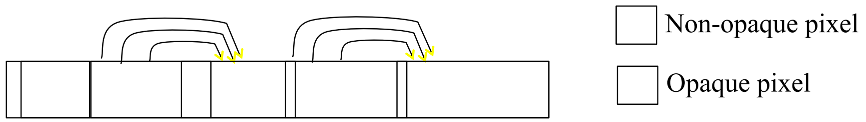
Shear-Warp Properties

- Projection in sheared object space has properties that allow more efficient compositing:
  1. Scanlines in intermediate space are parallel to volume scanlines
  2. All voxels in a given slice are scaled by same factor.
  3. For parallel projections: every slice has same scale factor and that is arbitrary. Usually choose 1, so get 1-1 mapping of voxels to intermediate image pixels.

Lacroute and Levoy describe 3 different rendering algorithms based on Shear-Warp.

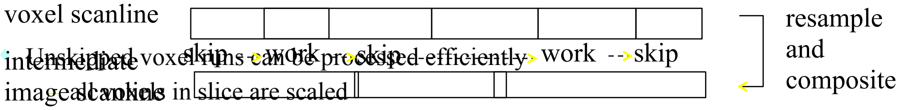
Parallel Projection Rendering 1

- Parallel view allows *run-length encoding* for data.
  - most data has lots of “empty” space
  - sheared, resampled volume stored as run-length encoded *voxel scanlines,* with 2 kinds of runs: transparent and non-transparent, defined by user-specified threshold
  - intermediate image scanline also stores run information: each opaque pixel (based on user threshold) has pointer to next non-opaque pixel in the scanline. Can skip quickly over runs of opaque pixels.

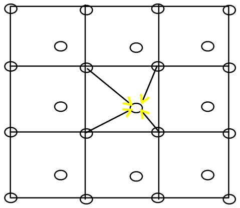


Parallel Projection Rendering 2

- For each slice and for each volume scanline
  - Walk through volume scanline and intermed. image
  - use voxel run-length encoding to skip transparent voxels
  - use image encoding to skip occluded voxels

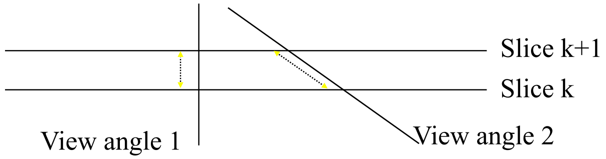


- Unskipped voxel runs can be processed efficiently
  - all voxels in slice are scaled by same factors, so resampling to get values at image pixel centers uses same weights:



Parallel Projection Rendering 3

- Use bilinear interp. & backward projection convolution
  - 2 voxel scanlines are traversed simultaneously to produce one intermediate image scanline (intermediate image scanline lies between two voxel scanlines)
- Use lookup table for shading
- Use lookup table to correct voxel opacity for view angle
  - apparent slice thickness depends on angle



Parallel Projection Rendering 4

- After compositing, need to warp 2D intermediate image to final image
  - use general purpose affine image warper with bilinear filter
  - image is small compared to volume, so this is minor part
- Run length encoded data structure
  - created on the fly, but it is (nearly) view-independent
  - create 3 encodings, one for each principal view direction
  - because transparent voxels are not stored, size is usually tractable
  - value of P matrix used to select which version to use

Perspective Projection Rendering 1

- Perspective rays diverge, so uniform sampling is hard
  - ray tracing solutions:
    - as distance along ray increases, split ray into multiple rays, or
    - use each sample point to sample larger portion of volume using a mip-map
  - splatting: resampling filter footprint must be recomputed for each voxel
  - shear-warp: adaptive area sampling is part of the algorithm
    - each slice is scaled differently, so farther slices are smaller and each ray is, in effect, sampling a larger portion of volume as it gets farther away

Perspective Projection Rendering 2

- Algorithm nearly same as parallel rendering, except
  - each voxel scaled as well as translated during resampling, so
    - more than 2 voxel scanlines may need to be traversed simultaneously to contribute to the intermediate image scanline, and
    - voxel scanlines may not be traversed at the same rate as image scanlines
  - choose factors so closest slice has unit scaling (all the rest will have < 1, so no slice will be enlarged)
  - use a box reconstruction filter and a box low-pass filter

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Fast Classification Algorithm

- 2 algorithms presented don’t allow experimentation with transfer function (it’s done in run-length encoding)
- 3rd variation keeps the full volume and evaluates opacity transfer while rendering; need to avoid unnecessary computations
- Key data structures
  - min-max octree: each node stores min/max of all children; built at data loading time; it is not dependent on transfer fcn
  - summed area table: built after transfer fcn defined
  - 3D voxel array

Summed Area Table

- Summed area table developed by Crow (84) for texture mapping
  - entry i,j in summed area table is sum of image entries from 0,0 to i,j
  - can get sum of any rectangle (i_1, j_1) to (i_2, j_2) in the image with
    sum = $sat(i_2, j_2) – sat(i_2, j_1 -1) – sat(i_1 -1, j_2) + sat(i_1 -1, j_1 -1)$

image

13	14	15	16
9	10	11	12
5	6	7	8
1	2	3	4

summed area table

28	60	96	136
15	33	54	78
6	14	24	36
1	3	6	10

Transfer Function Evaluation

- Opacity transfer function can be of form:

  α = f(p, q, …) where p might be data value, q the length of the gradient, or whatever.
  - given a threshold, f partitions the multidim space (defined by p,q, …) into transparent/non-transparent regions
  - for region of volume that just contains current scanline
    1. find extrema of parameters: min and max of p,q,…
    2. determine if opacity is transparent throughout the region
       - if so, discard scanline since it is transparent everywhere
       - else if scanline is small enough, render it
       - else subdivide scanline (and region) and recurse

Region Transparency Test

- Min-max octree contains extrema of opacity function parameter values in each node (subcube of volume)
- For step 2 above, need to integrate f over region of parameter space defined by parameter extrema
  - Build summed area table for opacity function where indexes are discretized values of parameters
  - use pmin, pmax, qmin, qmax to find sum of all possible values of function in the region; if sum is 0, region must be transparent everywhere.
  - if parameters can take on large ranges, need to quantize some or all of the parameters to keep table to manageable size
  - if there are 3 parameters, need 3d summed area table

Fast Classification Rendering Algorithm

- Build min-max octree as preprocessing step; octree is independent of both view and transfer function
- Just before rendering, build summed area table based on current opacity transfer function
- Use either parallel or perspective algorithm accessing 3d array of voxels in scanline order
  - for each scanline, use octree and SA table to skip transparent regions
  - for non-transparent regions, classify each voxel via a lookup table and proceed as before.
  - opaque regions of the image still cause voxel processing to be skipped.
  - note that voxel classification never done in transparent volume regions or opaque image regions; that saves computation

Fast Classification Limitations

- Octree traversal and SA table computations add overhead
  - can be reduced by avoiding re-computation: e.g., transparency test for an octree node is computed once on demand, then saved in the tree
- Opacity transfer function has restrictions
  - parameters must be available and function pre-computable for each voxel in order to build octree
  - domain of parameter space must be manageable
  - context-sensitive segmentation does not satisfy these restrictions
- If major view axis changes, access to scanlines in the 3d array won't follow storage order. For large volumes get thrashing.
  - can reorder the array, but that causes delay
  - best to use this algorithm only for small range of views; once desired opacity function is defined, switch to one of other algorithms.

Performance Results

- Lacroute/Levoy tested on a modest machine: SGI Indigo R4000 with 64Mbytes and no graphics accelerator
- 256x256x225 head MRI data set using gray scale

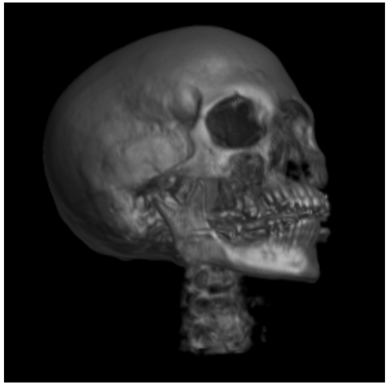
	Parallel	Perspective	Fast classification/Parallel
Avg time (sec)	1.2	3.3	2.8
Memory (Mb)	13	13	61

- Color rendering takes about twice as long
- Ray casting versions were 5 times longer for 128^3 data sets and 10 times longer for 256^3 data sets

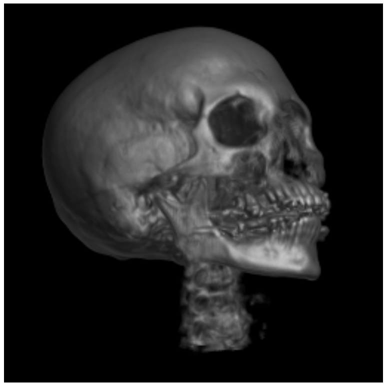
Image Quality

- Many images are virtually identical to ray casting. The 2 resampling steps might lead to blurring, but they don't see it.

Shear-Warp



Ray Casting



Other Images

256x256x159: Parallel 2.2 sec

256x256x110: Perspective 3.8 sec

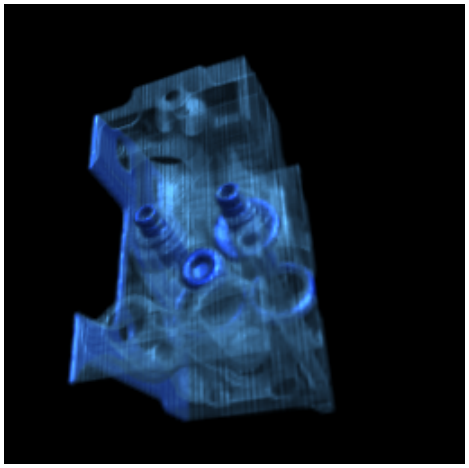


Image Quality Problems

- Shear-warp uses 2d rather than 3d filter to resample volume data. It is 1st order in plane of slice but 0-order between slices.
  - could be a problem with high frequencies perpendicular to slices; example below classifies with extremely sharp ramps to get high freq. and uses worst possible viewing angle (close to 45 degrees

Shear-warp Ray caster Shear-warp w/smoother classification