Direct Volume Rendering

R. Daniel Bergeron
Department of Computer Science
University of New Hampshire
Durham, NH 03824

Based on:

Brodlie and Wood, *Recent Advances in Visualization of Volumetric Data*, Eurographics 2000 **State of the Art Report.**

Drebin et al., *Volume Rendering*, **Siggraph 88**.

Sabella, *A Rendering Algorithm for Visualizing 3D Scalar Fields,* **Siggraph 88.**

Levoy, Westover, et al., *Introduction to Volume Rendering*, Siggraph 90 Course Notes

Overview

- Model data as a translucent gas or gel
  - need to assign material properties to data values
- ***Classification*** – assign *color* / *opacity* to data val
  - *Opacity transfer function* – maps data value and other parameters (such as gradient) to opacity value
  - *Color transfer function* – same for color
- ***Segmentation*** – applic-dependent “labeling” of data values, typically *a priori.*
  - gradient often used as *ad hoc* effort to segment

Volume Rendering Integral

- Treat volume as particles with density μ.
- Send ray through each pixel in image plane; for each wavelength λ, the light reaching pixel is

$$I_\lambda = \int_0^L C_\lambda(s)\mu(s)e^{-\int_0^s \mu(t)dt} ds$$

- where L is ray length, $C_\lambda(s)$ is light reflected at s in ray direction.

  $\mu(s)$ is a weight based on density – larger density means more reflected light.
  Integral accumulates intensity, but attenuates it (the exponential) as it passes through material.
  μ defines rate at which light is occluded per unit length due to scattering or extinction

Volume Rendering Integral Approximation

- Using Riemann sum approximation and using n as # samples

  $I_\lambda = \sum_{i=0..n} C_\lambda(i\Delta s)\ \mu(i\Delta s)\ \Delta s \prod_{j=0..i-1} exp(-\mu(j\Delta s)\ \Delta s))$

- Now replace exponential term with 2 terms of Taylor expans.

  $exp(-\mu(j\Delta s)\ \Delta s)) = 1- \mu(j\Delta s)\ \Delta s$

  and define *transparency* $t(j\Delta s)$ as

  $t(j\Delta s) = exp(-\mu(j\Delta s)\ \Delta s))$

  and *opacity,* $\alpha(j\Delta s) = 1 - t(j\Delta s) = \mu(j\Delta s)\ \Delta s$

  and: $I_\lambda = \sum_{i=0..n} C_\lambda(i\Delta s)\ \alpha(i\Delta s) \prod_{j=0..i-1} (1- \alpha(j\Delta s))$

  for Δs=1, we get: $I_\lambda = \sum_{i=0..n} C_\lambda(i)\ \alpha(i) \prod_{j=0..i-1} (1- \alpha(j))$

- Do this for R,G,B: summing intensities of individual samples, each of which is attenuated by the product of transparencies accumulated as light passes from sample to pixel.

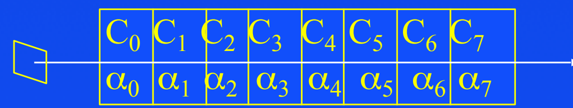
Recursive Approximation

- Dropping λ, and expanding we get

  $C = C_0\alpha_0 + C_1\alpha_1(1-\alpha_0) + C_2\alpha_2(1-\alpha_1)(1-\alpha_0) + \ldots$
- Can compute recursively using

  $C_{out} = C_{in} + (1-\alpha_{in})\,\alpha_i\,C_i$

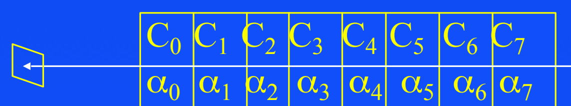
  $\alpha_{out} = \alpha_{in} + (1-\alpha_{in})\,\alpha_i$



  This is *front-to-back* image composition (Duff's ***over*** operator).

  back-to-front ordering only needs to recursively compute color component

  $C_{out} = \alpha_i\,C_i + C_{in}\,(1-\alpha_i)$



  Note: compositing is associative, but
  not commutative: order matters

DVR Approaches

- Image order approach: process from the image plane to the object
  - also called *backward rendering*
  - *ray casting* is classic image order algorithm
- Object order approach: process from the object to the image plane
  - also called *forward rendering*
  - *splatting* is the classic object order algorithm

Image Order Issues

- Volume rendering equation approximation
  - improve accuracy and/or speed
- Interpolation
  - calculating data values between grid points is vital
- Curvilinear and unstructured grids
  - basic approaches map nicely to rectilinear grids, others are more difficult to handle
- Faster ray traversal
- Hardware designed for volume rendering

Volume Rendering Eqn. 2

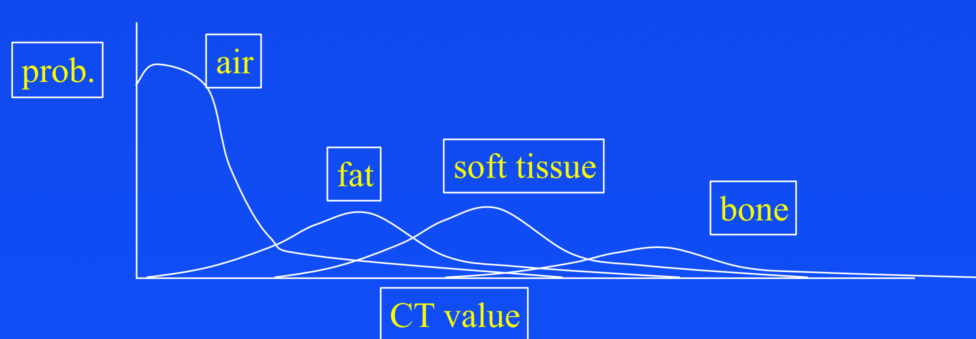
- Has been much work to make integration faster and more accurate
- Alternative is to dramatically simplify the approximation at the cost of accuracy:
  - *Maximum Intensity Projection* (MIP): simply find the maximum data value along the ray and project its “color”.
  - works well for angiography (highlight blood vessels)

Drebin et al., Siggraph ‘88

- CT data
- Basic *segmentation* based on probabilities
  - from segmentation, produced *density*, *color* and *opacity*
- Estimated gradient by simple forward differencing
  - Used gradient to infer surfaces for reflections

Segmentation

- Segmentation is often *ad hoc,* but *shouldn*’t make binary decisions
  - for CT, X-ray absorption of materials is known *a priori* as a probability distribution function (pdf)



Segmentation 2

- Given a voxel has the value I,

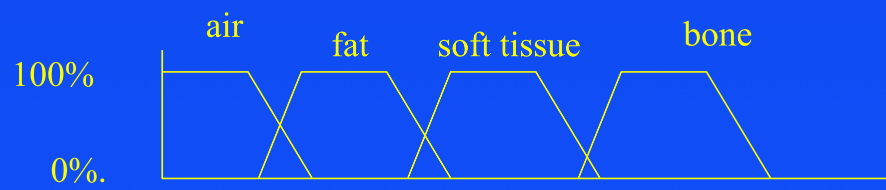
  probability of getting I, $P(I) = \sum_i p_i P_i(I)$

  where p_i is the probability of getting material *i* and

  $P_i(I)$ is probability that material *i* has value I

  Using Bayesian estimation,

  $p_i(I) = P_i(I)/(\sum_j P_j(I))$ which can be implemented as lookup
- Only 2 materials overlap: get simple relationship:



Density, color, opacity

- "density", D, computed as
  $D(I) = \sum_i \rho_i \, p_i(I)$ where ρ_i is density of material i
- color and opacity (rgbα)
  - $C(I) = \sum_i p_i(I) \, \alpha_i \, (R_i, G_i, B_i)$
- For each x,y,z, estimate by forward differences
  - gradient: $N(x,y,z) = (D_{x+1}-D_x, D_{y+1}-D_y, D_{z+1}-D_z)$
  - normalized gradient: $n(x,y,z) = N(x,y,z)/|| N(x,y,z)$
  - strength: $|| N(x,y,z) ||$
- n(x,y,z) is used in lighting model for reflected light from a light source.

Ray Tracing Volume Data

(Notes from Levoy in *Introduction to Volume Rendering*, Siggraph 91 tutorial.)

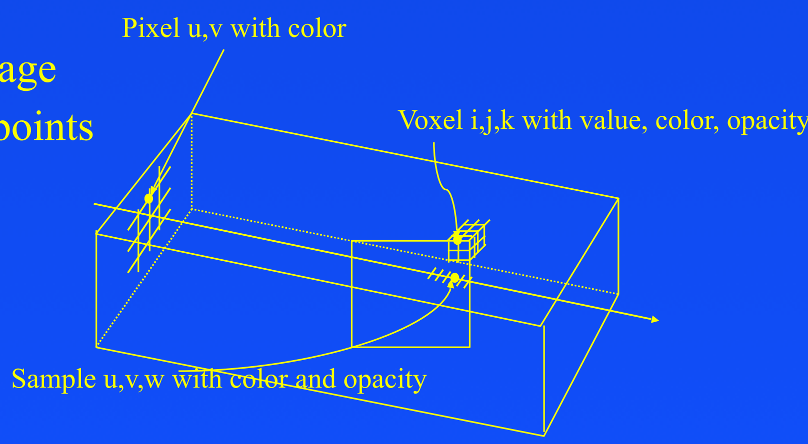
- Data assumed to be samples of a continuous scalar function (voxel as point not volume)
- Sampling lattice is rectilinear and uniformly spaced
- Pixel spacing < voxel spacing
- Other typical simplifications
  - one ray per pixel (no supersampling)
  - parallel projection

View Specification

- Need view specification, image plane, volume location
  - Parallel projection along major axis
    - » integral mapping of voxel address space to pixel address space: 1-1 is easiest; usually have projection of a voxel map to k x k pixels
    - » arbitrary mapping requires interpolation
  - Arbitrary parallel projection
    - » need view direction and size of image space
    - » usually voxel address space as "world coordinates"

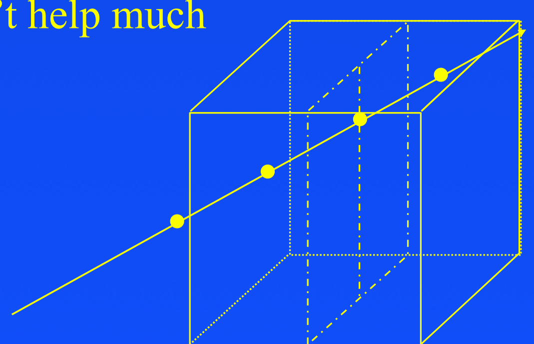
Coordinate Systems

- Object space
  – coordinate axes correspond to volume array indices
  – typically NxNxN
- Image space
  – PxP pixels in image
  – PxPxW sample points



Resampling

- Calculating color/opacity inside a voxel is resampling the functions
- Sample at even spacing along ray
- Sampling rate (for typical CT and MR data)
  – less than voxel spacing introduces artifacts
  – more than twice per voxel doesn't help much
- Use trilinear interpolation



Trilinear Interpolation

From *Graphics Gems V,* p. 521

- Linear interpolation between 2 sample values:

  $v_x=(1-f_x)v_0 + f_x v_1$ where $0 \leq f_x \leq 1$, also written as

  $v_x= v_0 + f_x (v_1- v_0)$
- In 2-dimensions, interpolate from 4 points

  $v_{xy}=(1-f_x) (1-f_y)v_{00} + (1-f_x) f_y v_{01} + f_x(1-f_y) v_{10} + f_x f_y v_{11}$
- But, more efficient (3 mults) to do 2 linear steps:

  $v_{x0}= v_{00} + f_x (v_{10}- v_{00})$

  $v_{x1}= v_{01} + f_x (v_{11}- v_{01})$

  $v_{xy}= v_{x0} + f_y (v_{x1}- v_{x0})$

Trilinear Interpolation – 2

- And in 3D, interpolate from 8 points
  Use 3 linear steps (7 mults)

  $v_{x00}= v_{000} + f_x (v_{100}- v_{000})$

  $v_{x01}= v_{001} + f_x (v_{101}- v_{001})$

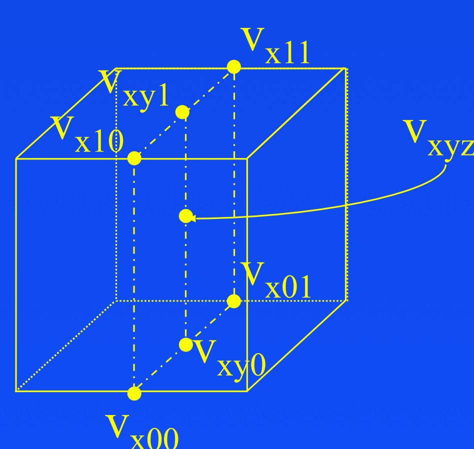
  $v_{x10}= v_{010} + f_x (v_{110}- v_{010})$

  $v_{x11}= v_{011} + f_x (v_{111}- v_{011})$

  $v_{xy0}= v_{x00} + f_y (v_{x10}- v_{x00})$

  $v_{xy1}= v_{x01} + f_y (v_{x11}- v_{x01})$

  $v_{xyz}= v_{xy0} + f_z (v_{xy1}- v_{xy0})$

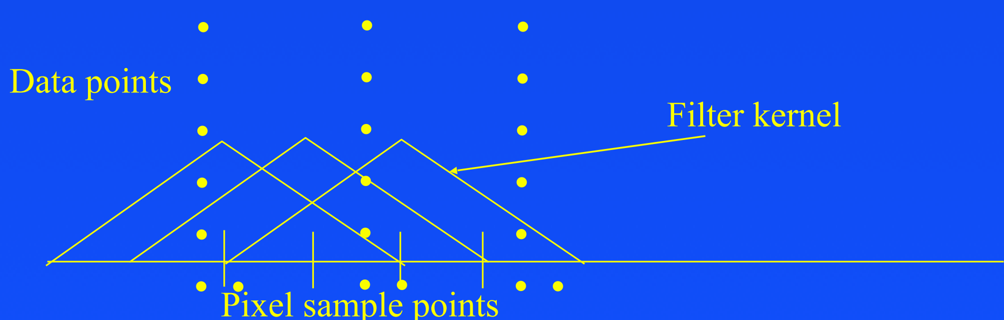


Splatting

- Westover, *VolVis Symposium 89* and *Siggraph 90*
- Each voxel drawn on image plane as a cloud of points (footprint), covering many pixels
- Voxel treated as a single value "thrown at the screen"
- Example of *feed forward convolution* as opposed to a *feed backward convolution*

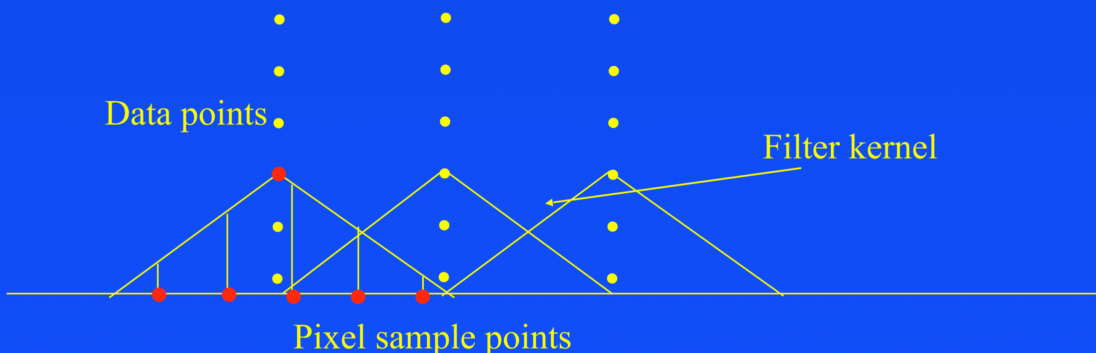
Feed Backward Convolution

- Output (pixel value) is weighted average of input data
- Center a convolution kernel at the output (pixel) location and gather data points that project onto kernel
- Touch each output sample once
- Touch each input data point many times



Feed Forward Convolution

- Input energy spread to many outputs (pixels)
- Center kernel at data point and distribute to output pixels (really a 3D convolution)
- Touch each input data point once
- Touch each output often



Splatting: Ideal

- Feed forward and incremental reconstruction
- Ideal splatting
  - center kernel at D
  - evaluate kernel
  - multiply by input value at D

    $contribution_D(x,y,z) = h(x-x_D, y-y_D, z-z_D)\rho(D)$

    where h evaluates the convolution function
  - of course, this is terribly expensive

Splatting: Dimension Reduction

- Want 2D image from 3D data
  - given pixel at (x,y), want the contribution for each point, D
  - center kernel at D
  - project weighted kernel onto (x,y) plane (assumes parallel projection along the z-axis)

    $\text{contribution}(x,y) = \rho(D)\int h(x-x_D, y-y_D, w)\,dw$
  - Note integral is independent of the density (ρ); it depends only on (x,y) projected location; leads to *footprint function:*

    $\text{footprint}(x,y) = \int h(x, y, w)\,dw$

    where (x,y) is the displacement from projected sample point

Footprint Function Tables

- Can integrate the kernel function into a generic footprint table
- for each voxel

  transform to screen space

  for each pixel in the extent of the footprint

    map back to precomputed table

    composite the weighted contribution