SemSeg

Part 2 – Lagrangian Methods

Tutorial: Time-Dependent Flow Visualization

Armin Pobitzer[1], Alexander Kuhn[2]

1) University of Bergen, Norway
2) University of Magdeburg, Germany

Overview

1. **Flow Map**
2. **Lagrangian features**
3. **Finite Time Lyapunov Exponent (FTLE)**
4. **Ridge Extraction**
5. **Efficient FTLE computations**

Lagrangian Methods

Natural Flow Phenomena

- transport of particles
- analysis of spatio-temporal features
- define important structural features

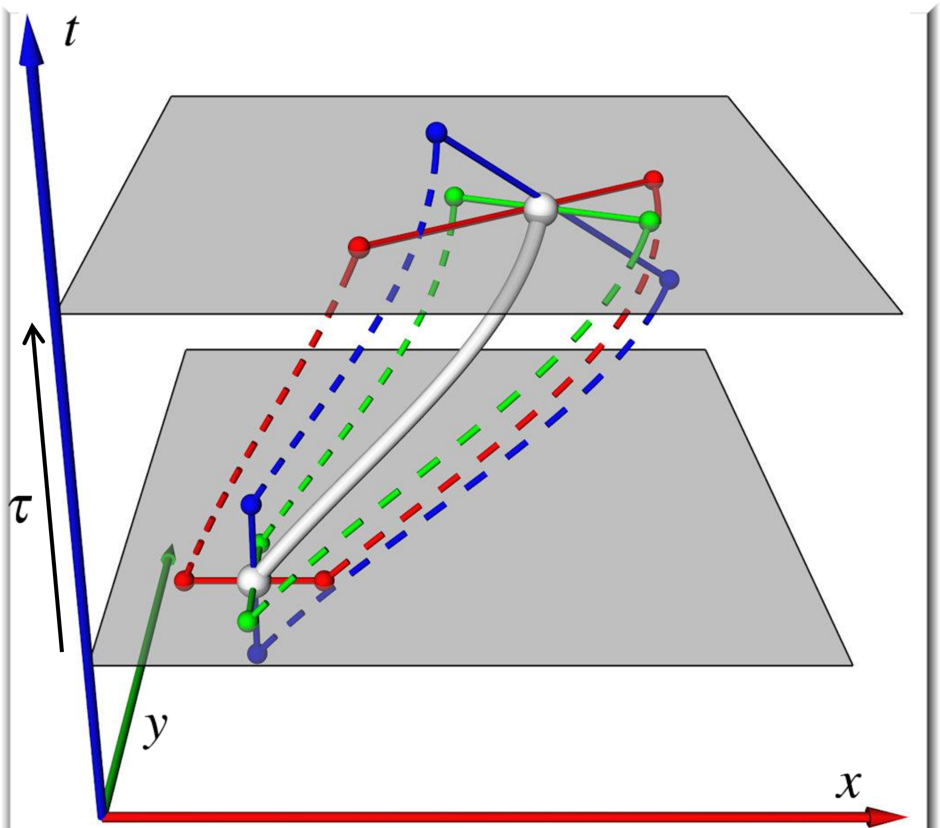
→ **Lagrangian Features**

Wake vortex, smoke injection [NASA LaRC, Wallops Island]

Glacier structures Groenland [Picture Alliance, YPS]

Lagrangian Methods

Flow Map



[Tino Weinkauf, MPI]

- flow map

$$\phi : D \to D \qquad \phi_t^\tau(\mathbf{x}) = \phi(\mathbf{x}, t, \tau)$$

- gradient of flow map

$$\nabla \phi_t^\tau(\mathbf{x}) = \frac{\partial \phi}{\partial \mathbf{x}}$$

- flow map & temporal component

$$\bar{\phi} : D \times T \to D \times T$$

$$\bar{\phi}(\mathbf{x}, t, \tau) = \bar{\phi}_t^\tau(\mathbf{x}) = \begin{pmatrix} \phi_t^\tau(\mathbf{x}) \\ t + \tau \end{pmatrix}$$

- temporal gradient

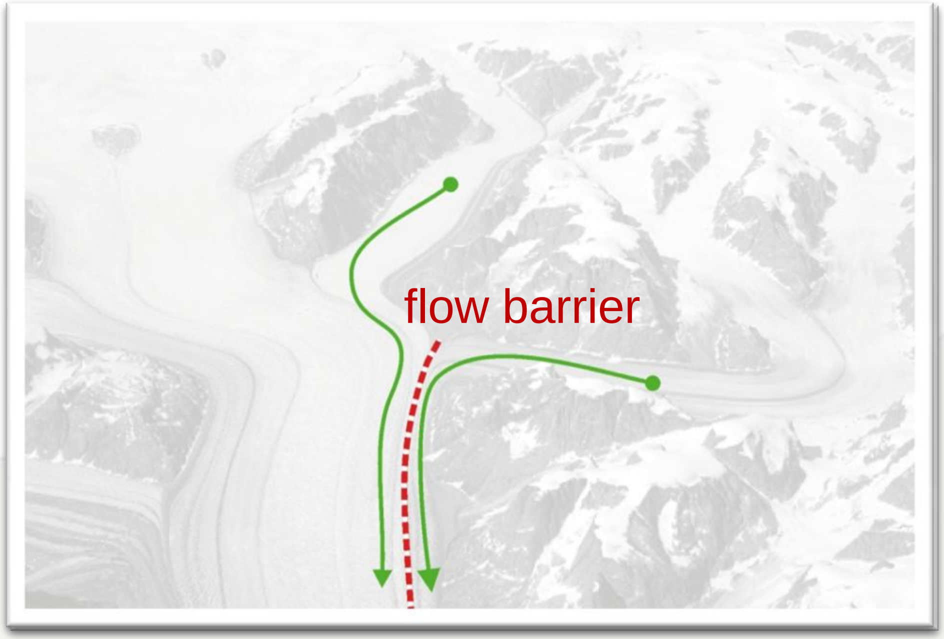
$$\nabla \bar{\phi}(\mathbf{x}, t, \tau) = \begin{pmatrix} \nabla \phi & \frac{\partial \phi}{\partial t} \\ 0 \,..\, 0 & 1 \end{pmatrix}$$

Lagrangian Methods

Lagrangian Coherent Structures (LCS)

- **Properties:**
  - take particle perspective
  - observe properties over pathlines
  - characterize flow transport
    - flow barriers
      → material structures
    - coherent flow behavior
    - invariant regions

→ Analyze Flow Map

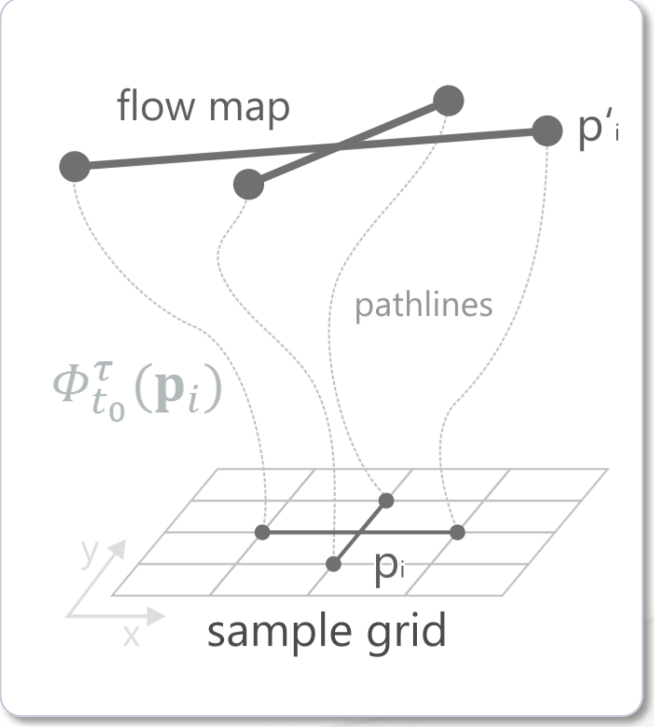


Glacier structures Groenland [Picture Alliance, YPS]

Lagrangian Methods

Finite Time Lyapunov Exponent (FTLE) [Haller2001]

- **Properties:**
  - flow behavior over finite-time intervall
  - rate of separation
  - ridges relate to LCS [Haller2010]



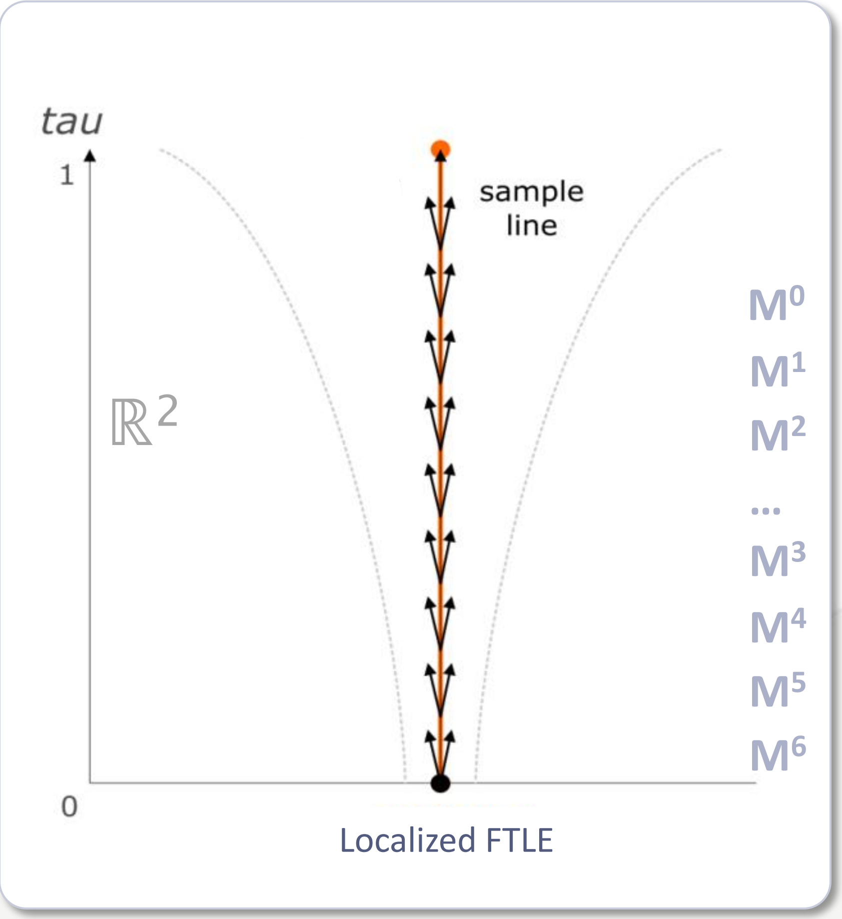
- **Formal definition :**
  - Flow Map gradient: $\nabla\Phi_{t_0}^{\tau}(\mathbf{p}_0) = \frac{\partial\Phi(\mathbf{p}_0, t_0, \tau)}{\partial\mathbf{p}_0}$
  - FTLE: $FTLE(\mathbf{p}_0, t_0, \tau) = \frac{1}{\tau} \cdot ln\sqrt{\lambda_{max}(\nabla^{T}\nabla)}$

(Cauchy-Green Tensor)

Lagrangian Methods

Finite Time Lyapunov Exponent (FTLE)

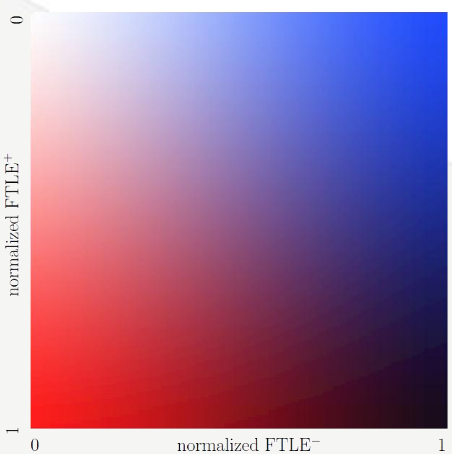
- **Classic FTLE** [Haller2001]
  - four sample points in distance *h*
  - discrete flow map approximation
- **FTLE with Reseeding**
  - five samples
  - different renormalization strategies
- **Localized FTLE** [Kasten2009]
  - one sample + derivatives
  - local deformation by Jacobians
  - accumulation of local derivative tensors

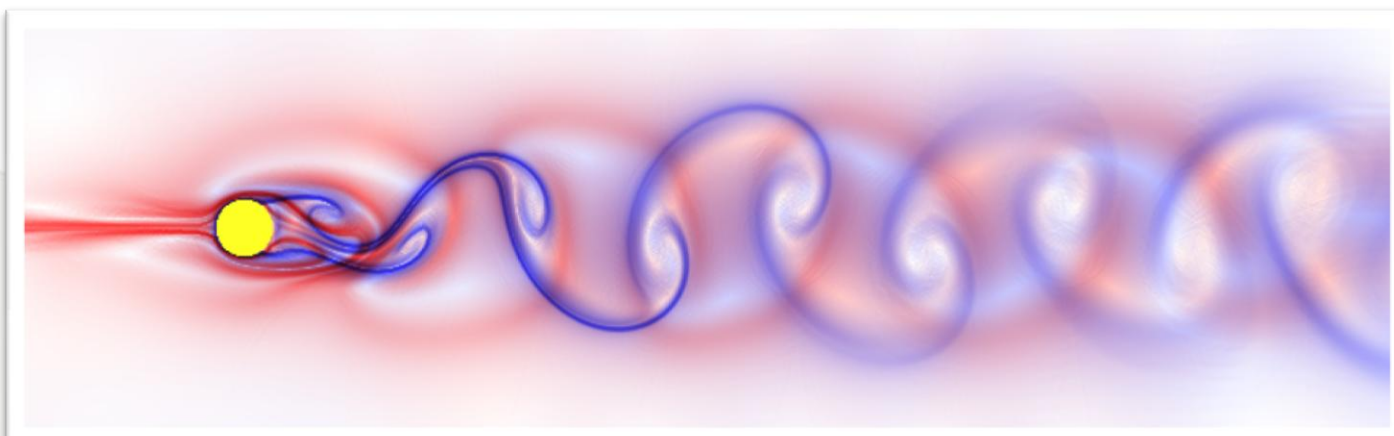


Lagrangian Methods

Finite Time Lyapunov Exponent (FTLE) [Haller2001]

- **Properties:**
  - single scalar field to describe time interval
  - information about transport behavior & barriers
  - low *flux rates* across sharp ridges
  - defined in forward & backward direction (FTLE+ / FTLE-) [Garth2007]

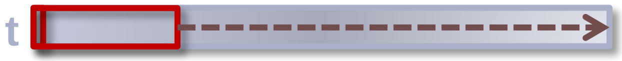


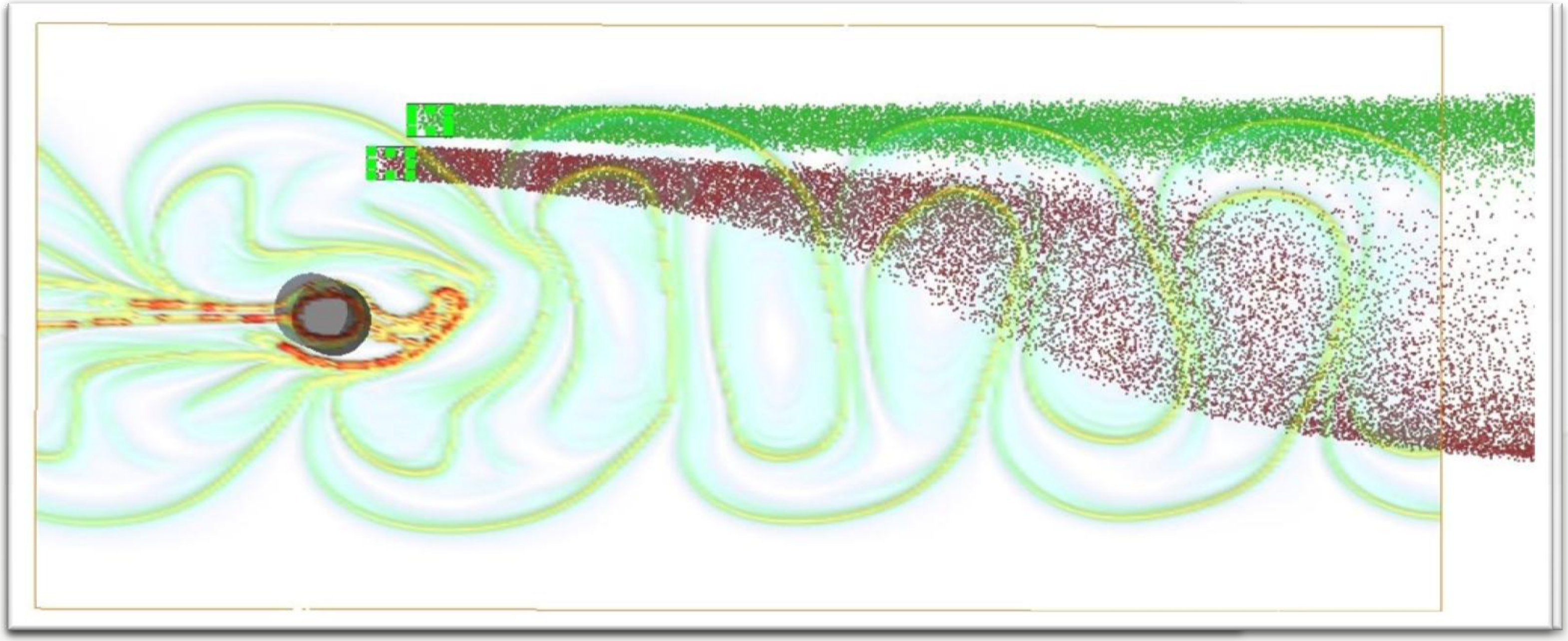


Lagrangian Methods

Finite Time Lyapunov Exponent (FTLE) [Haller2001]

- **Example 2D Cylinder:**

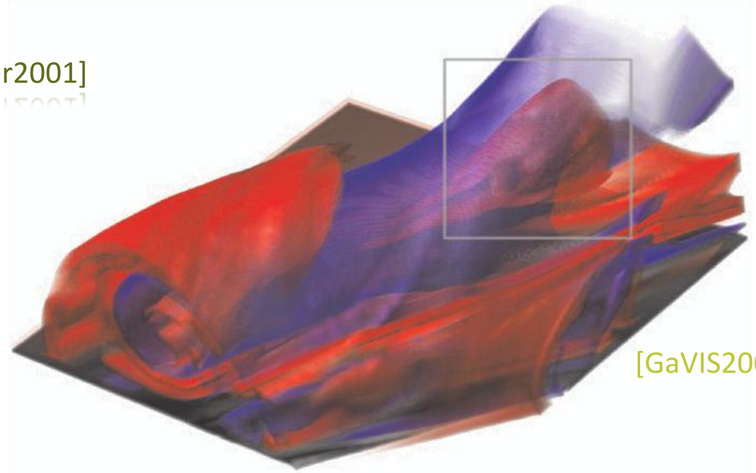




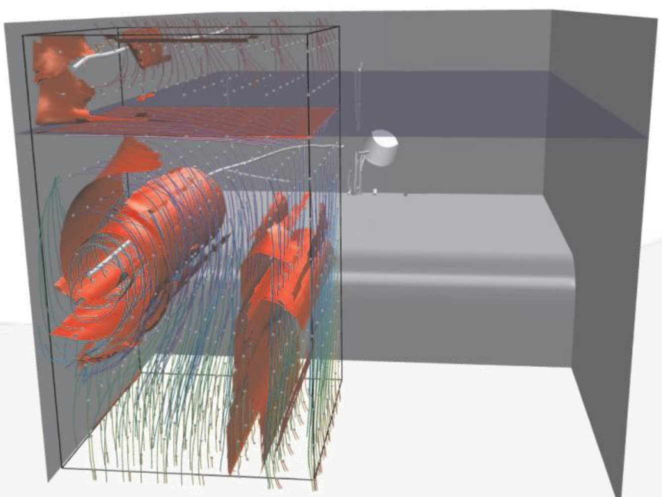
Lagrangian Methods

Finite Time Lyapunov Exponent (FTLE) [Haller2001]

- **Visualization**
  - direct volume rendering [GaVIS2007]
  - slicing / orthogonal planes [GGTH07]
  - ridge surface extraction [Sadlo2007]
    - adaptive refinement
    - filtering



[GaVIS2007]



[Sadlo2007]

Lagrangian Methods

More Lagrangian Definitions

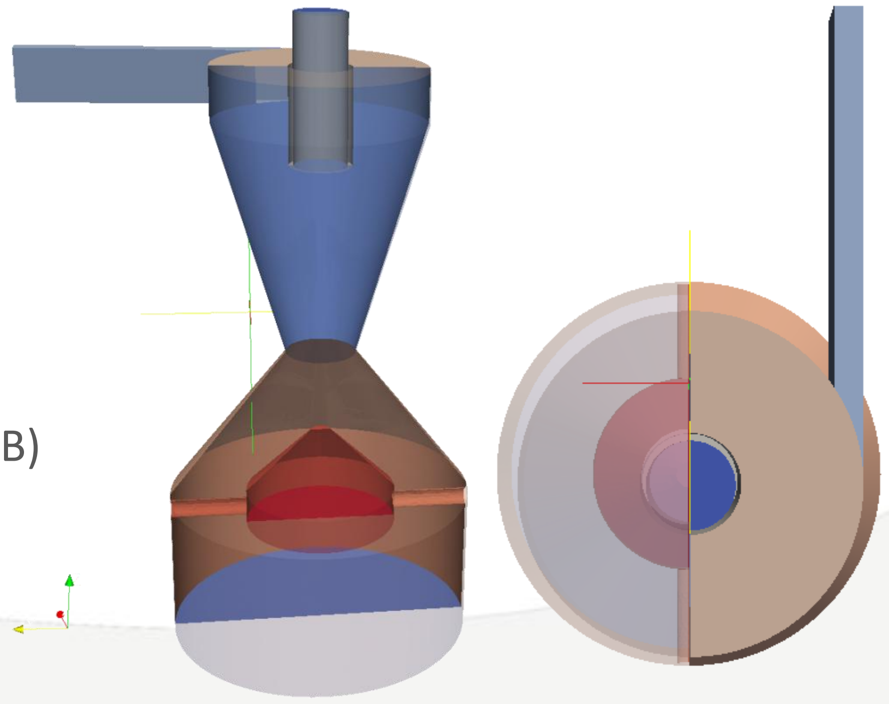
- Minima of spatio-temporal acceleration [KHNH09]
  → filtering based on feature living time
- An objective definition of a vortex [Haller2005]
  → measure time trajectories spend in strain regions
- LCS with guaranteed material separation [Germer2011]
- pathline predicates [Salz2008]
  → boolean flags to cluster similar behavior
- Lagrangian topoloy concept: unsteadiness [Fuchs2010]
  → construct local frame of reference

- **(and many more…)**

Lagrangian Methods

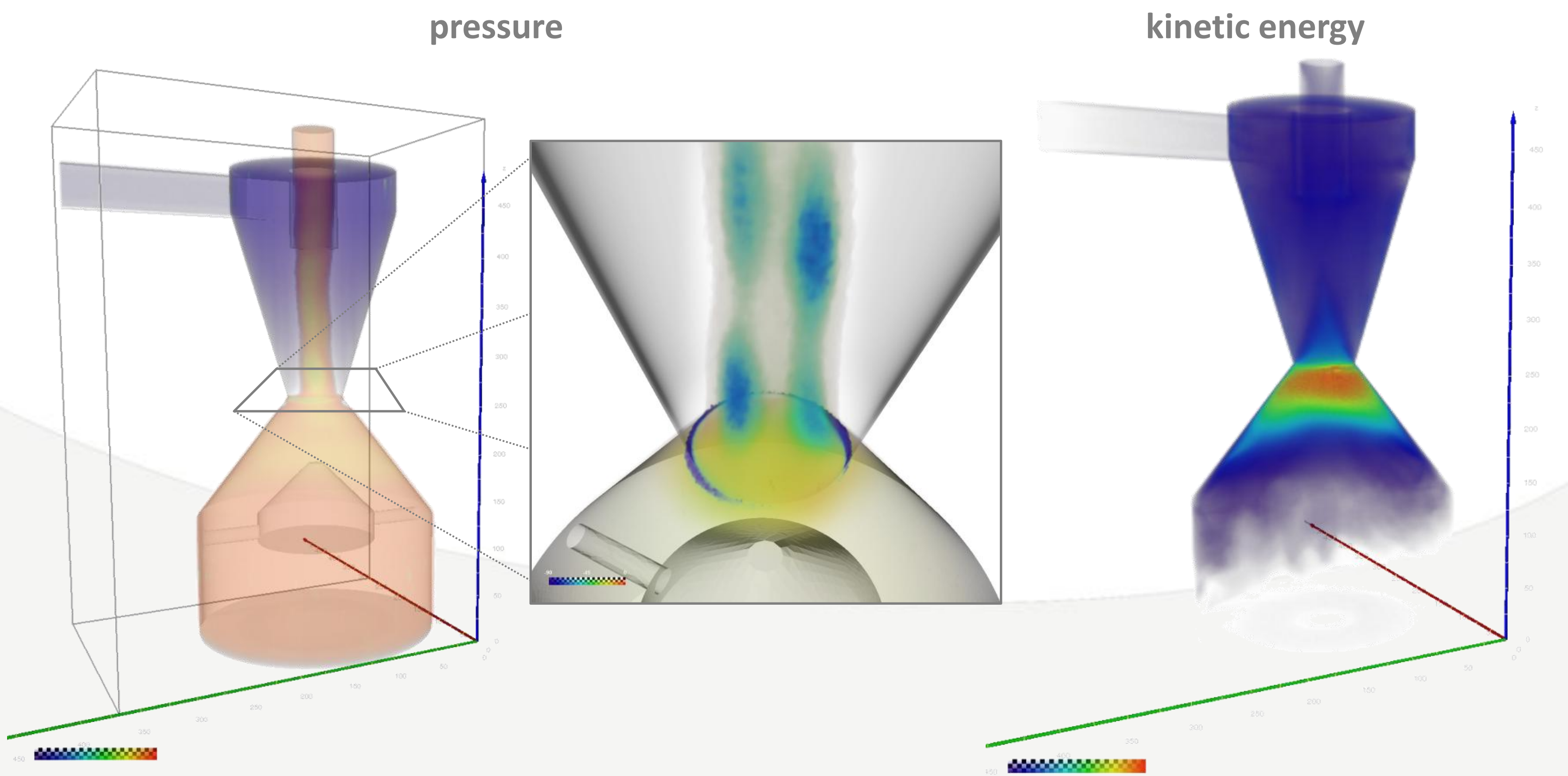
Application Example: Hydrocyclone

- **Physical Properties:**
  - separate particles in suspension
  - centripedal force & fluid resistance
  - separation on varying density
- **Simulation:** (Markus Rütten, DLR)
  - geometry (~2GB) + scalar fields (~2.6GB)
  - steady flow field
  - scalar fields:
    - velocity
    - pressure
    - kinetic energy



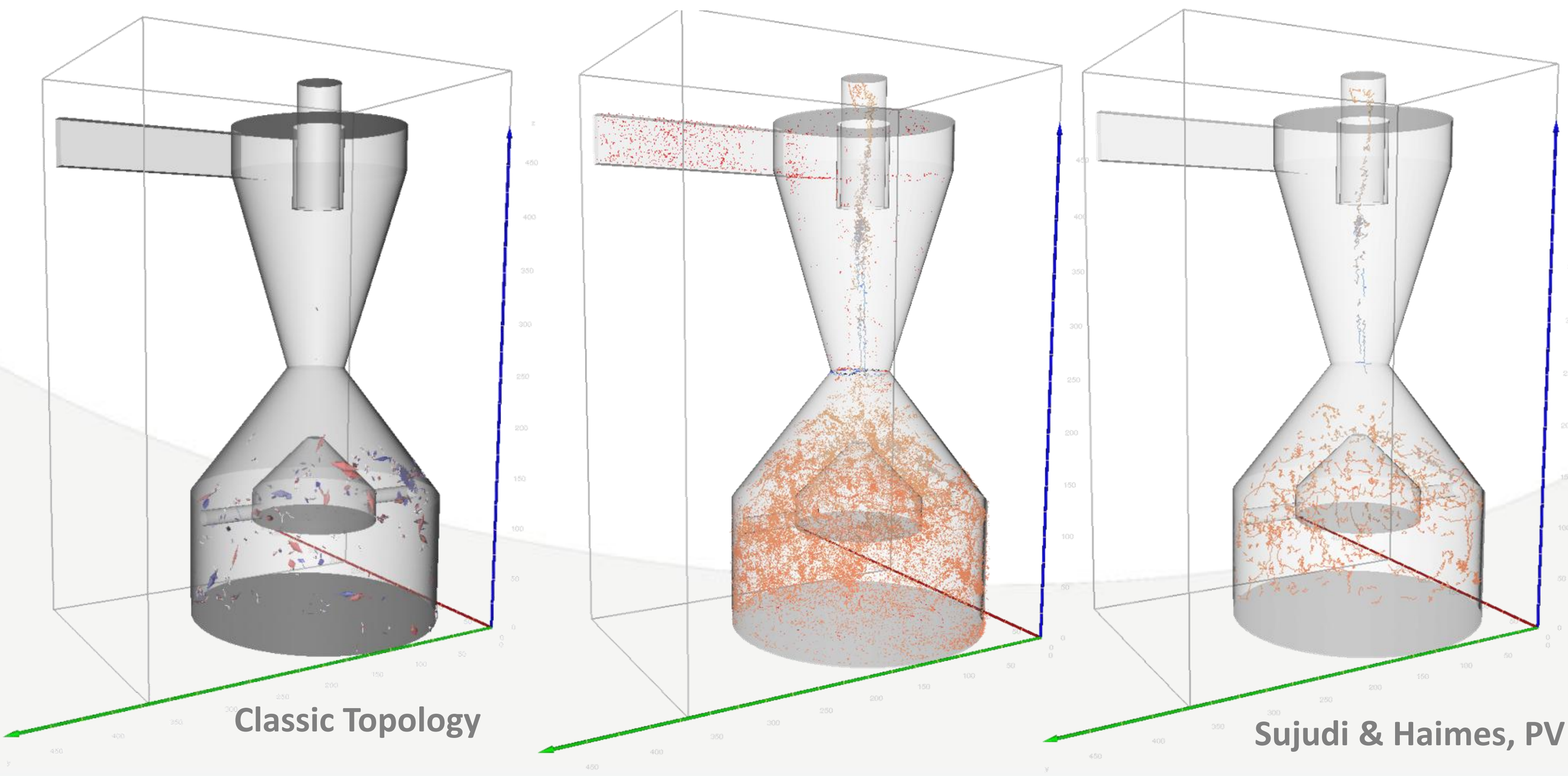
Lagrangian Methods

Application Example: Hydrocyclone



Lagrangian Methods

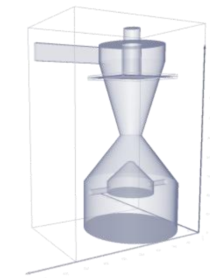
Application Example: Hydrocyclone



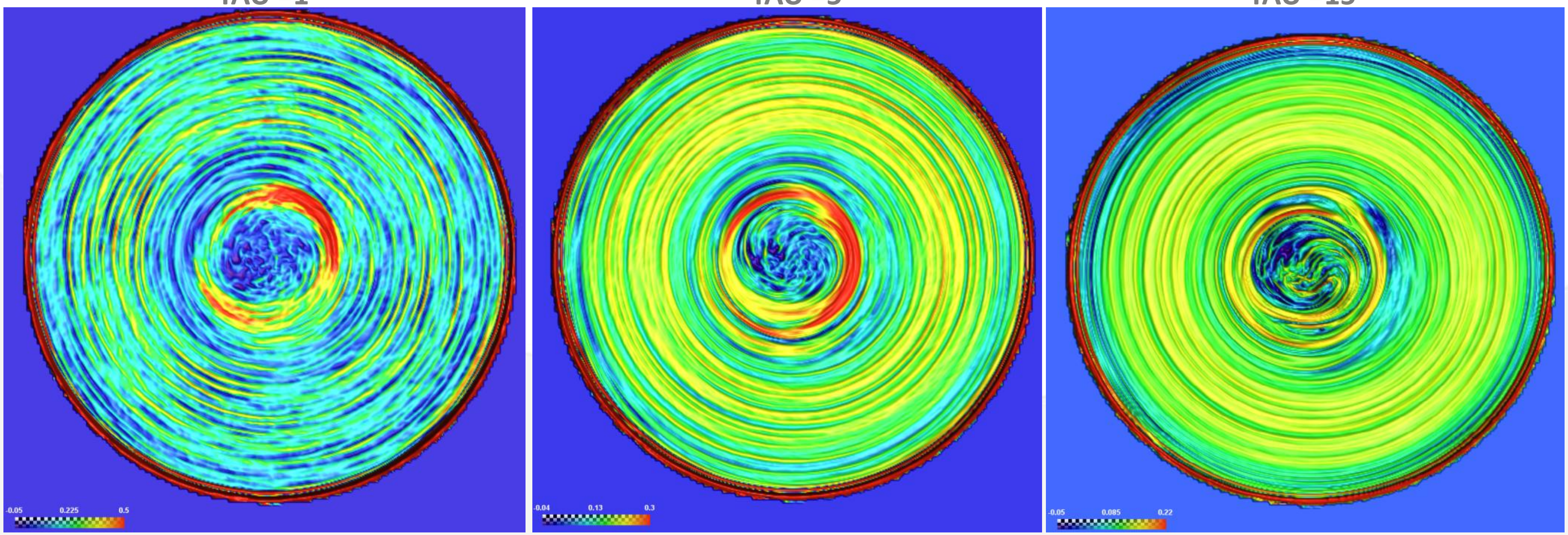
Lagrangian Methods

Application Example: Hydrocyclone

- Classic FTLE: Top Slice



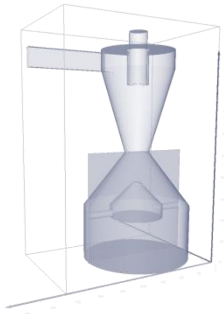
TAU =1 TAU =5 TAU =15



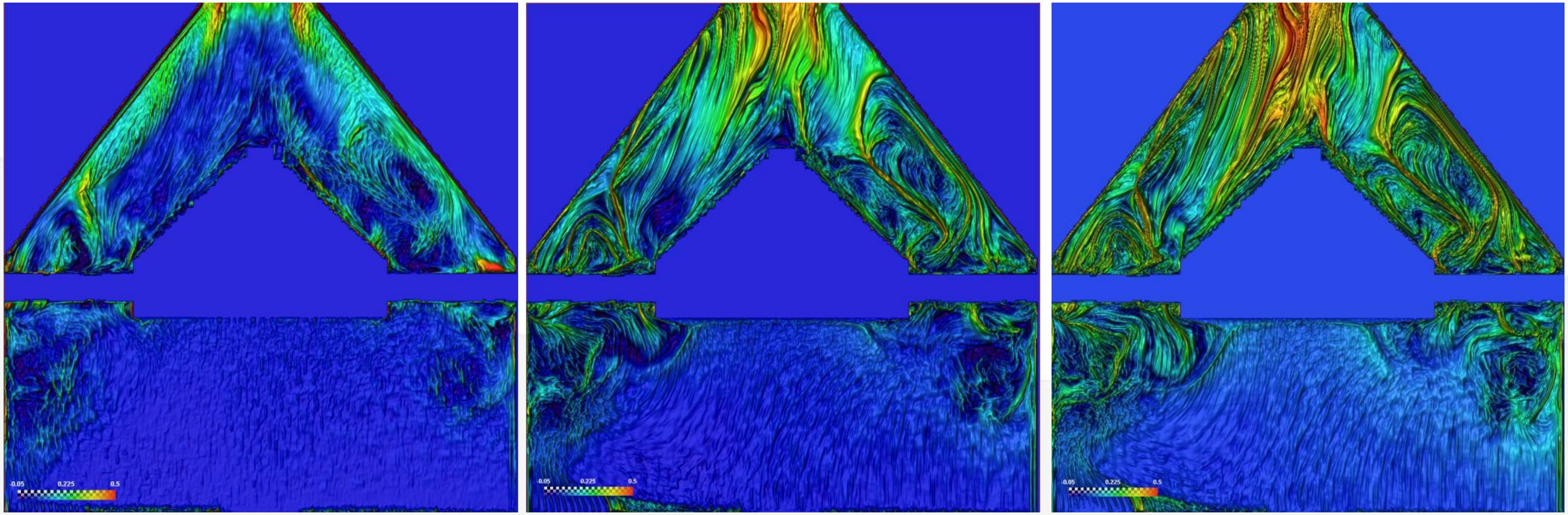
Lagrangian Methods

Application Example: Hydrocyclone

- Classic FTLE: Lower Slice

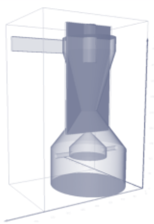


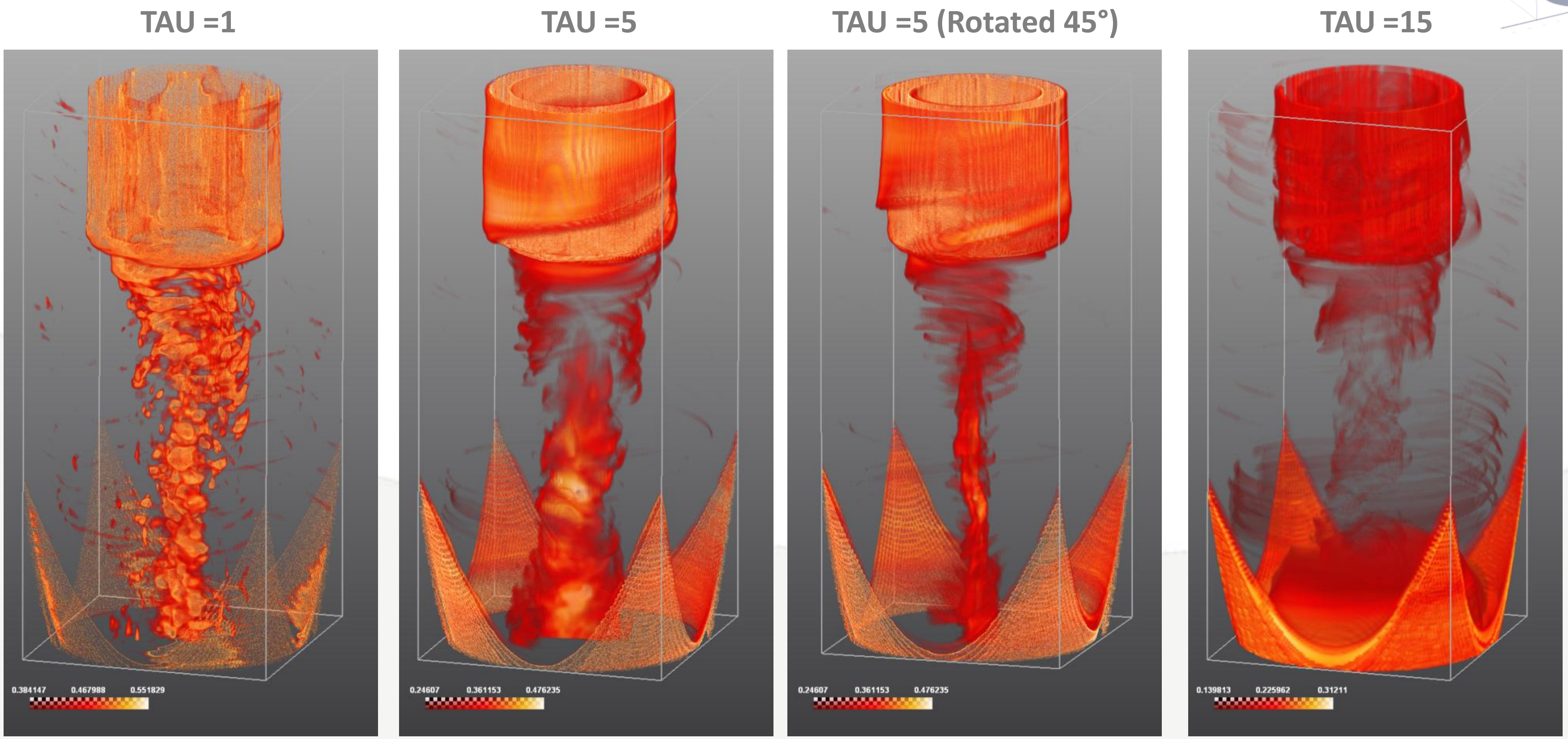
TAU =15 TAU =30 TAU =50



Lagrangian Methods

Application Example: Hydrocyclone

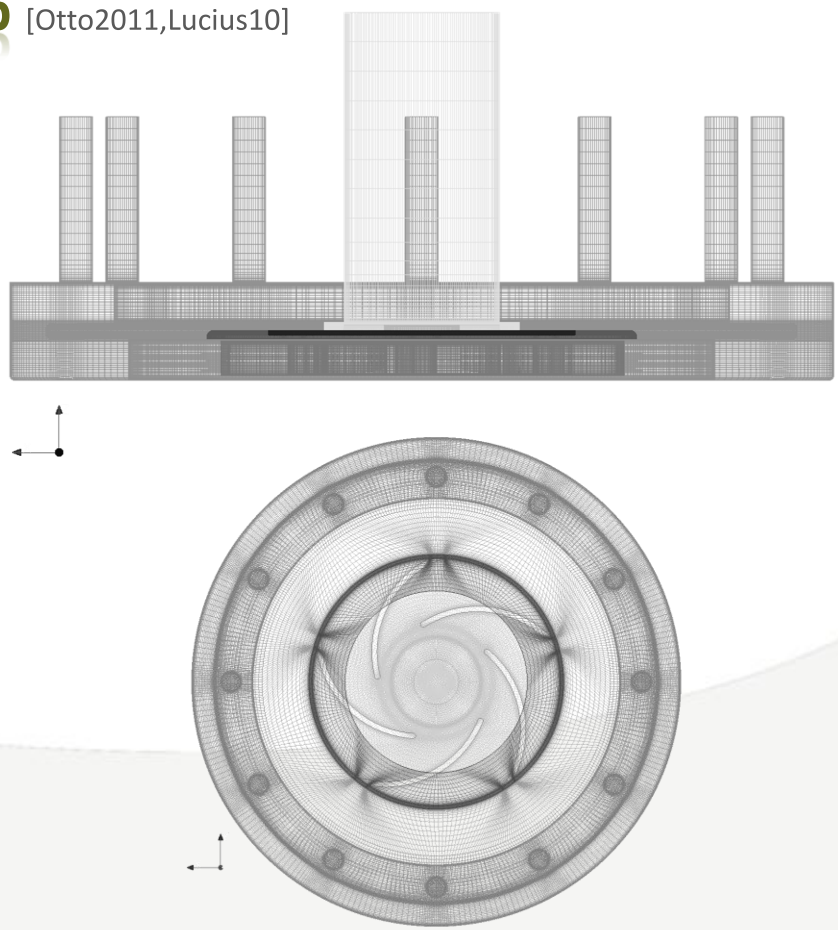




Lagrangian Methods

Application Example: Centrifugal Pump [Otto2011,Lucius10]

- 3 simulation models (SST, DES, SAS)
- Ensight data format with rotating parts
- 80 time steps
- 6.7 million nodes
- 6.5 million hexahedral cells
- 142 GB per model



Lagrangian Methods

t

Application Example: Centrifugal Pump

Q Criterion [Haller05]

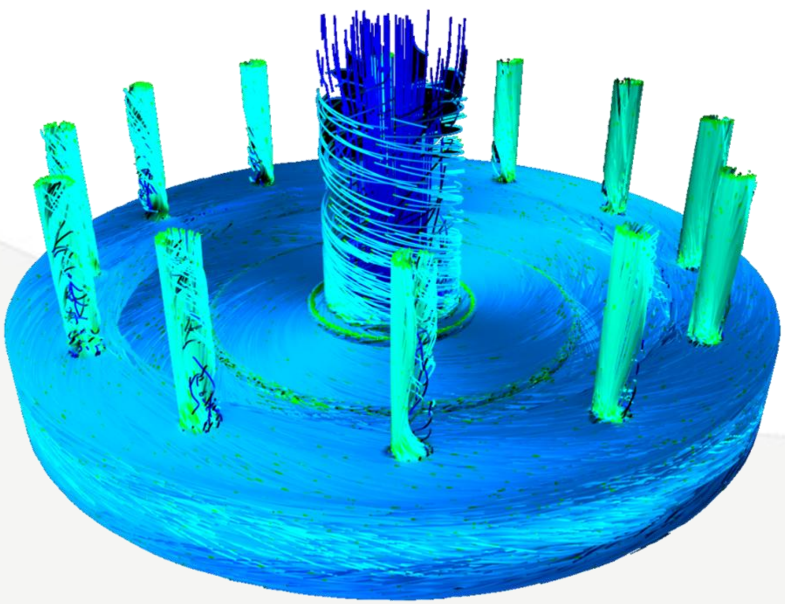
λ_2 [Leong95]

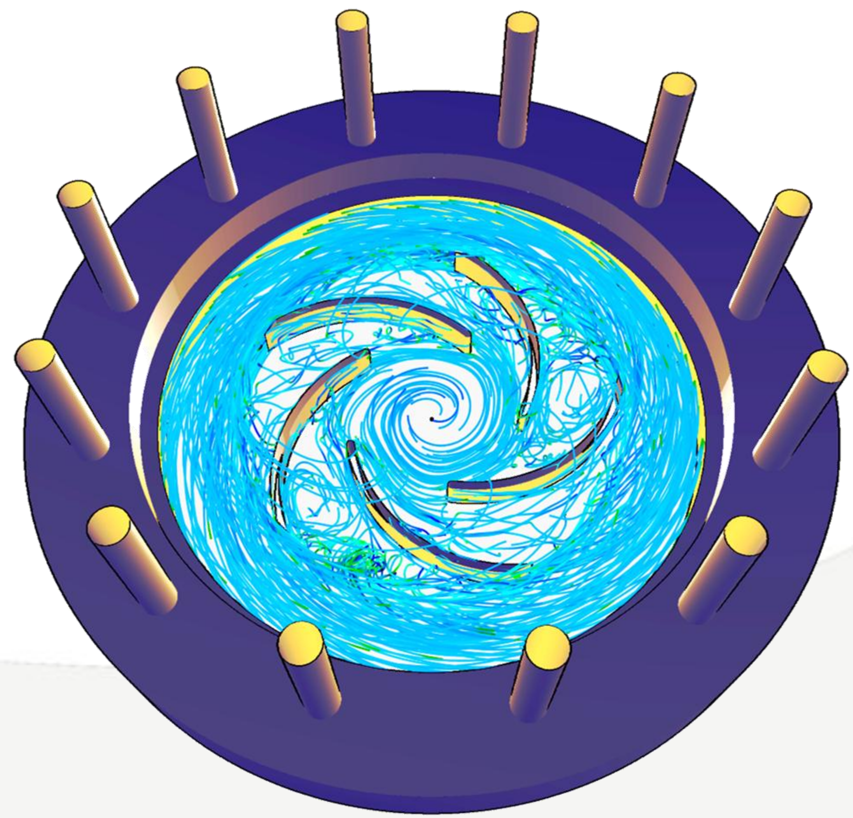
Lagrangian Methods

t

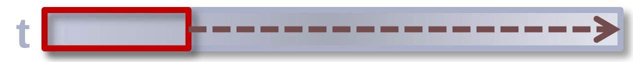
Application Example: Centrifugal Pump

- **Pathlines**: colored z-direction
  - **upwards**
  - **downwards**



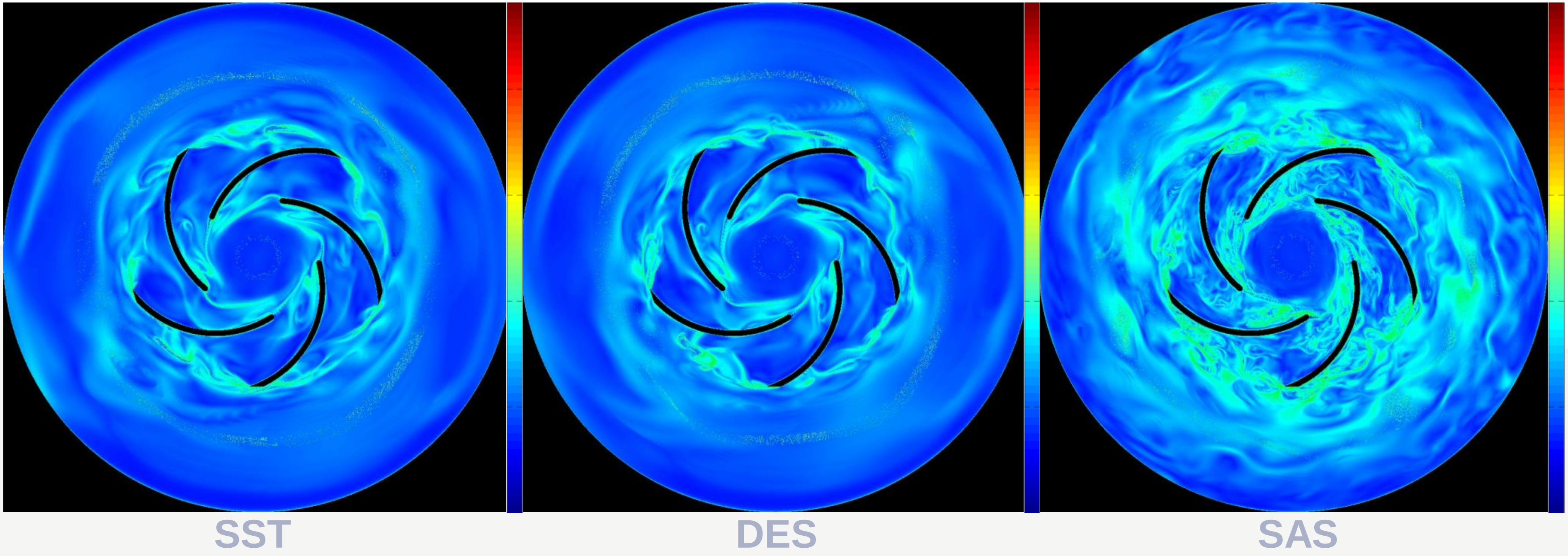


Lagrangian Methods



Application Example: Centrifugal Pump

- **Simulation model comparison:** FTLE

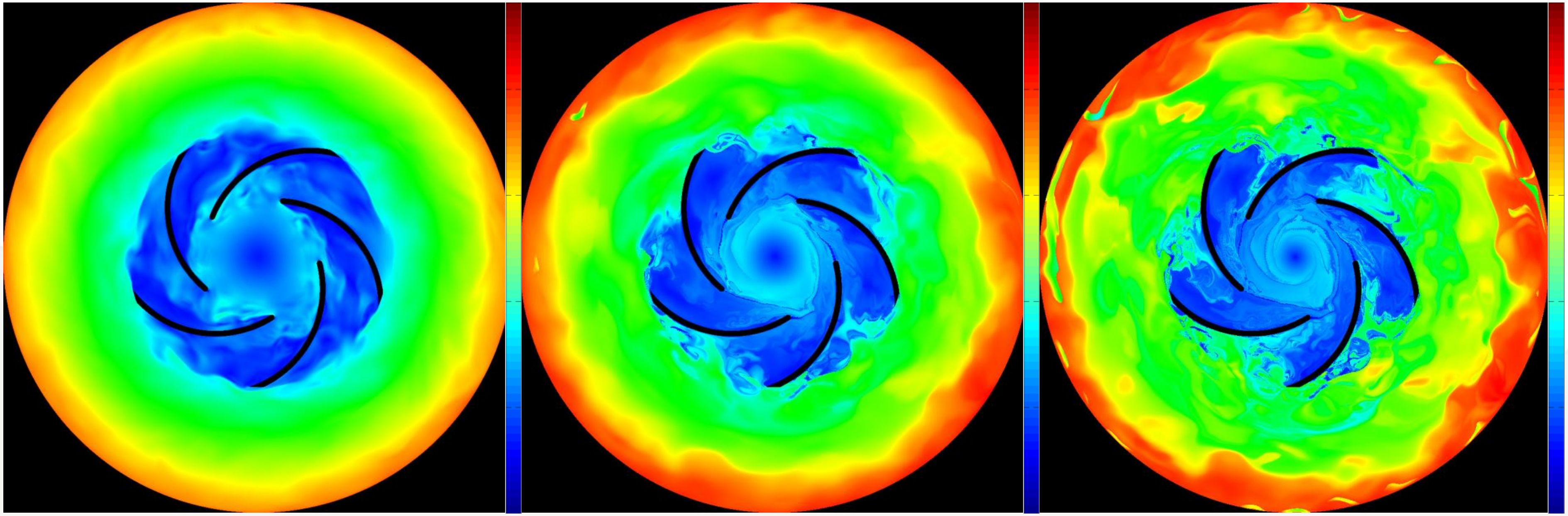


Lagrangian Methods

t

Application Example: Centrifugal Pump

- **Simulation model SAS:** pathline arc length



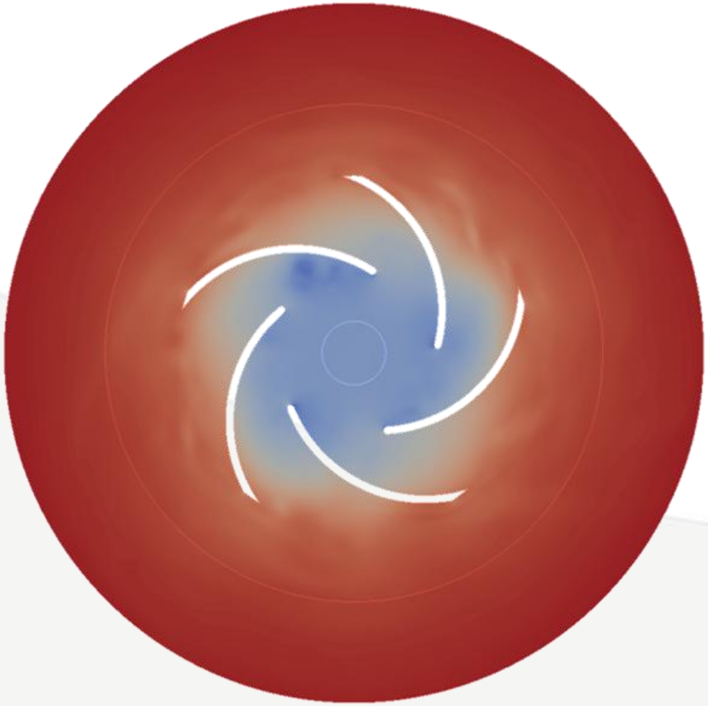
SAS *TAU=0* *TAU=39* *TAU=79*

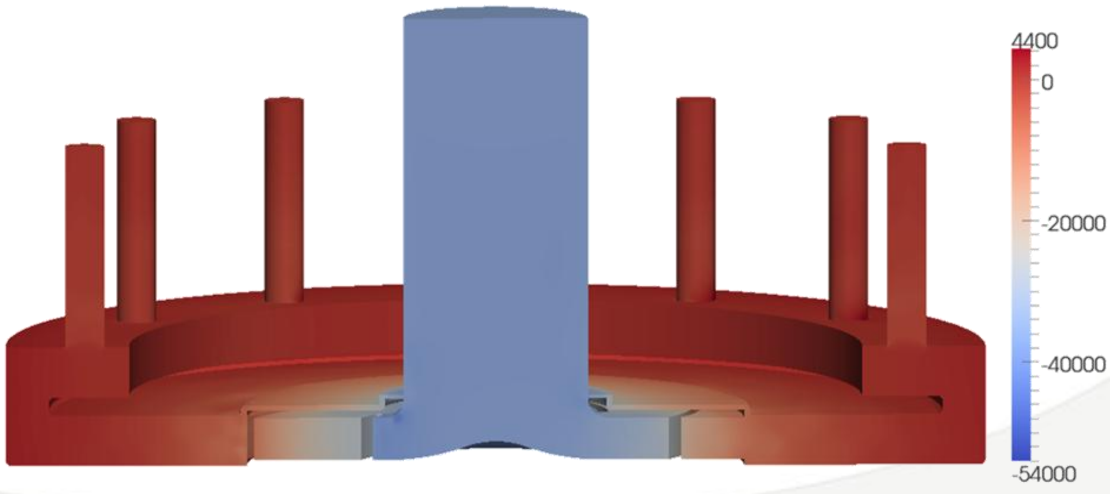
Lagrangian Methods

t

Application Example: Centrifugal Pump

- **Pressure field** (t=0)



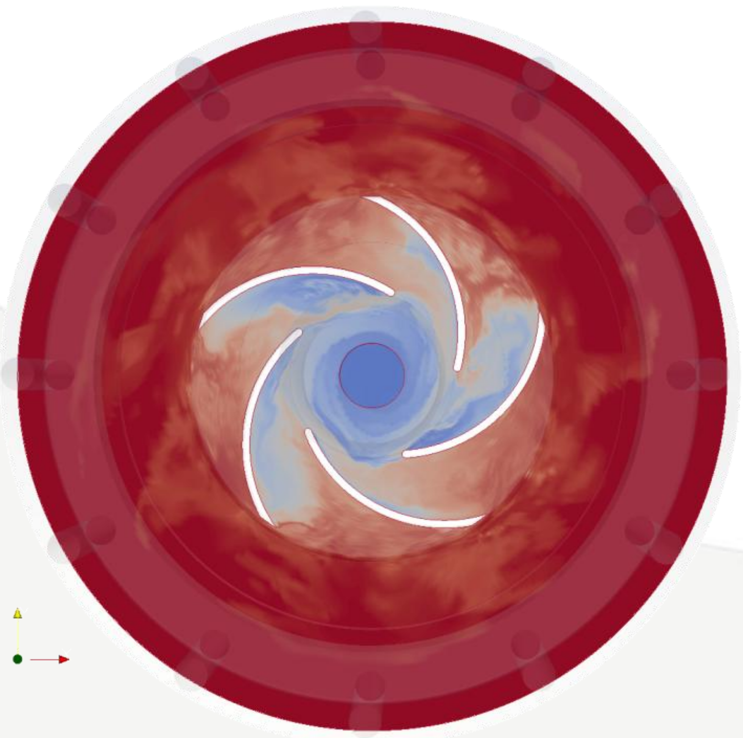


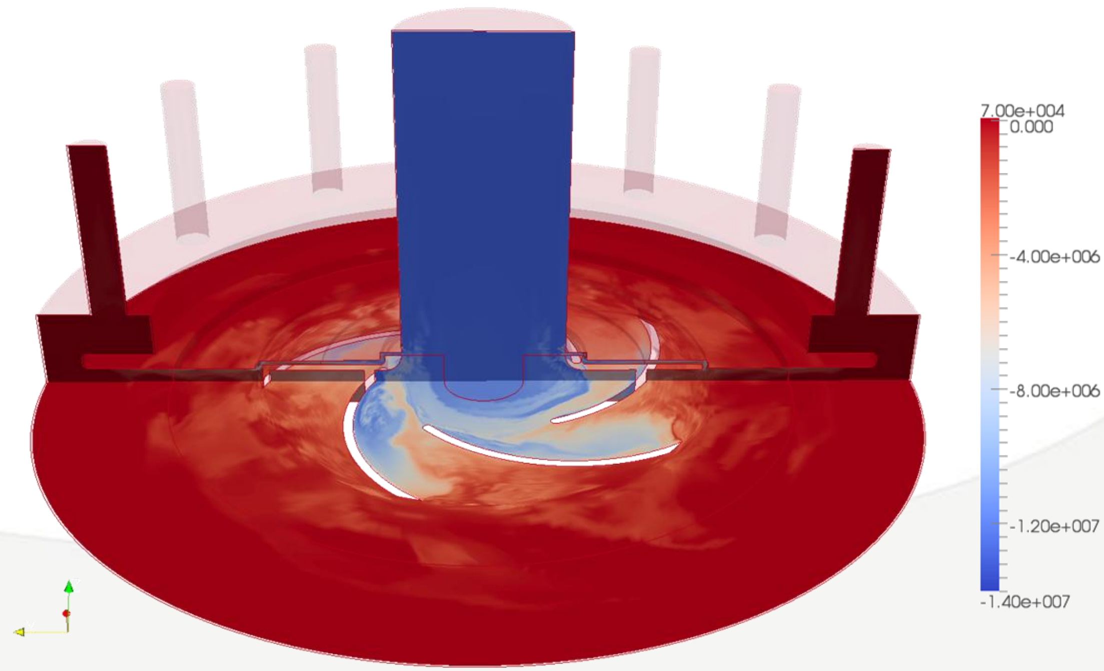
Lagrangian Methods

t

Application Example: Centrifugal Pump

- **Simulation model comparison:** integral pressure



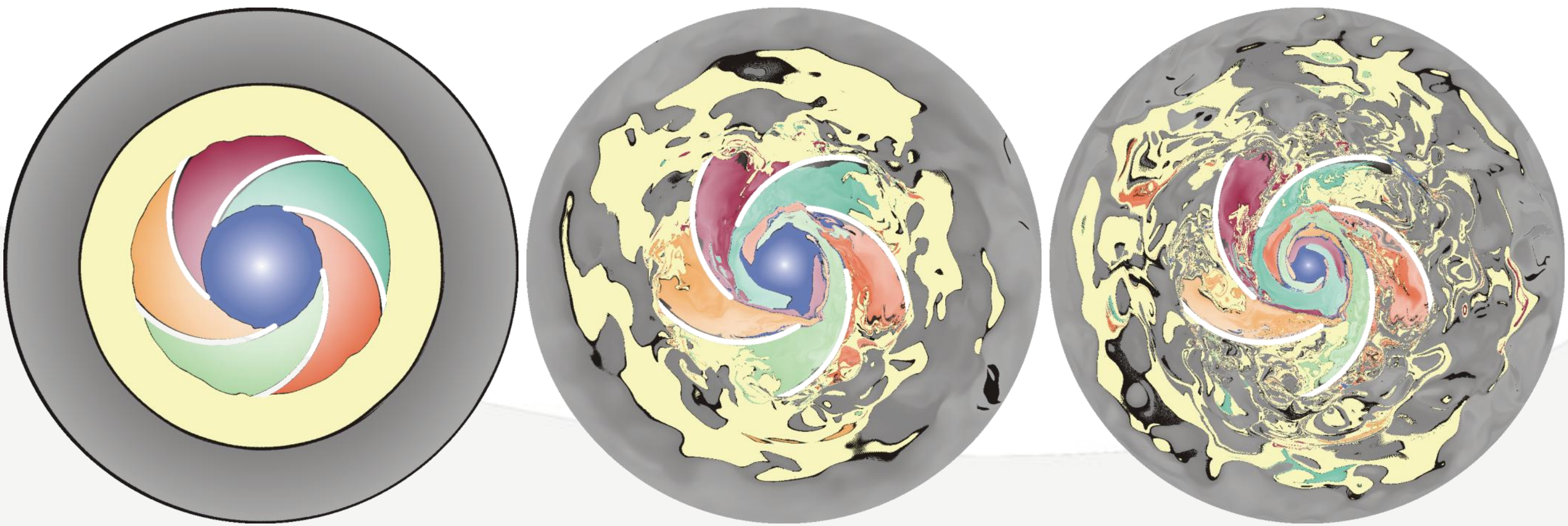


Lagrangian Methods

t

Application Example: Centrifugal Pump

- **Simulation model SAS:** texture advection



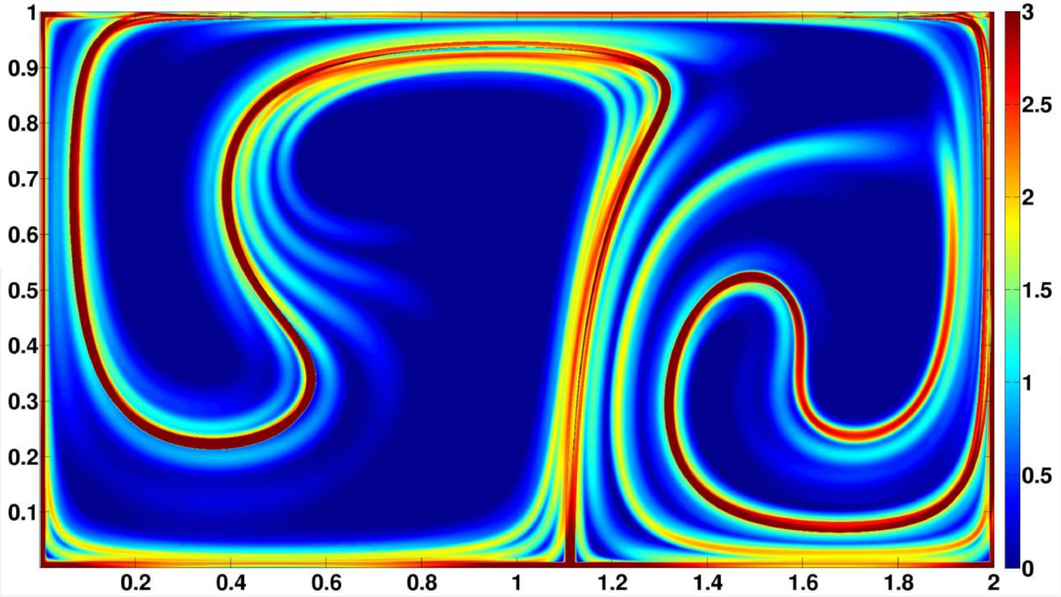
SAS *TAU=0* *TAU=39* *TAU=79*

Overview

1. Flow Map
2. Lagrangian features
3. Finite Time Lyapunov Exponent (FTLE)
4. **Ridge Extraction**
5. Extensions to FTLE

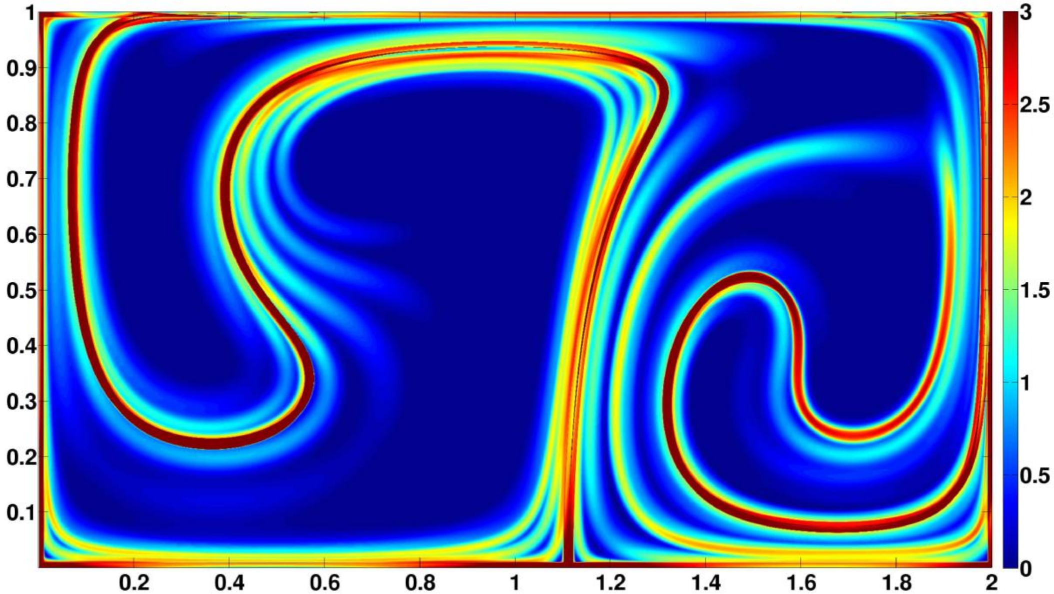
Rigdes – From FTLE to LCS

- **FTLE gives a separation rate, but looking for large values is not enough**
  - **What is a “large” separation can vary within a data set**
  - **Material property is proven under additional assumptions [Shadden, 2005]**
- **Visualizing LCS by thresholding, volume rendering, … of FTLE is not possible from theoretical point of view**



Rigdes – From FTLE to LCS

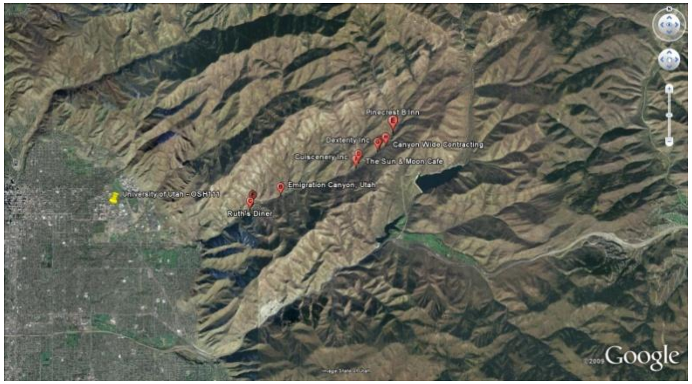
- **LCS ≈ ridges of FTLE field** **[Haller, 2001; Shadden 2005,…]**

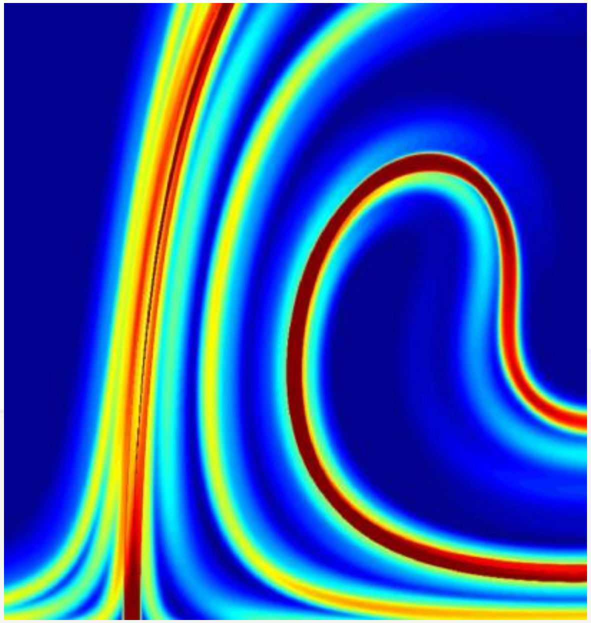


… but ridges are tricky

Rigdes – From FTLE to LCS

- Ridges are easily detectable by human eye
- Mathematical definition less clear
- Intuition: line (2D) or surface (3D) that is maximal with respect to its transversal direction
- Definition of “transvers” open (usually: height ridges)



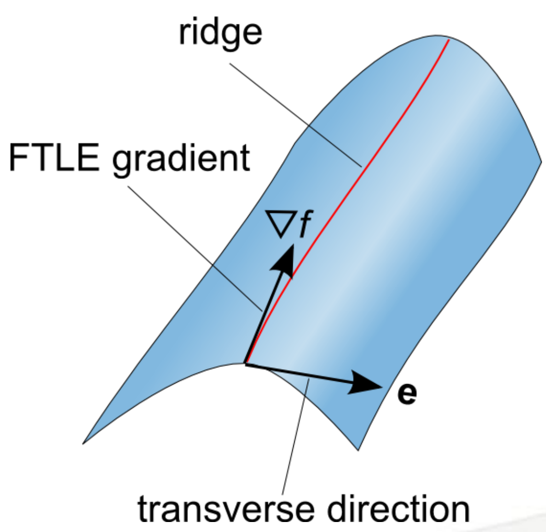


Rigdes – From FTLE to LCS

- **Height ridges: transverse direction given by eigenvector ass. with the smallest eigenvalue of the Hessian**
- **Ridge = points that fulfill**

$$\langle \nabla f, \mathbf{e} \rangle = 0$$

$$\mathbf{e}^{\mathrm{T}} \mathcal{H} \mathbf{e} < 0$$

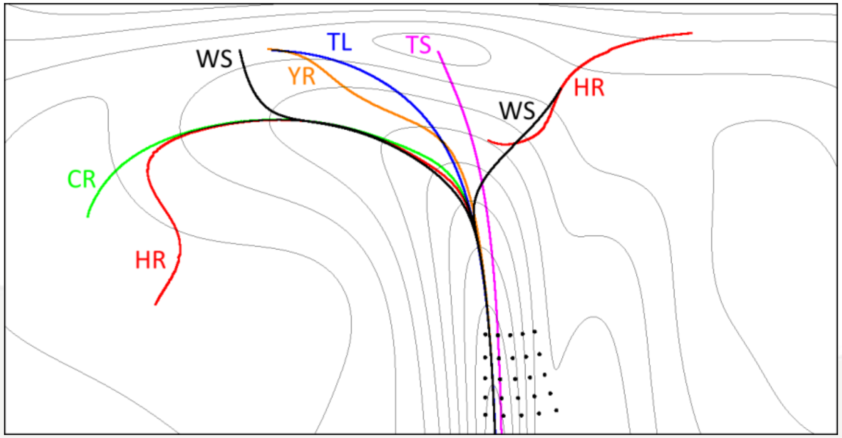


Rigdes – From FTLE to LCS

Other definitions of definitions of e are possible!

- Watersheds
- “C”-ridges
- …

Further comparison in paper by Schindler et al.



[Schindler et al., 2012]

Efficient FTLE computation

- **High quality FTLE ridges…**
  - Require dense seeding of particles
  - accurate integration scheme
- **… are computationally expensive!**
  - Number of path lines + integration are main bottle neck in FTLE computation
  - Has to be done in precomputation step
  - Current state of the art: interactive computation not possible

	with setup time			without setup time		
steps	direct	hier.	ratio	direct	hier.	ratio
2 × 4	46.85	46.36	1.01	0.51	0.42	1.21
2 × 8	48.94	46.65	1.05	2.41	0.46	5.24
2 × 16	53.08	47.19	1.12	6.67	0.56	11.91
2 × 32	60.78	48.11	1.26	13.51	1.11	12.17
2 × 64	76.27	48.41	1.58	30.08	1.94	15.51
2 × 128	107.20	48.91	2.19	60.69	2.18	27.84
2 × 256	168.79	48.99	3.45	121.61	2.75	44.22
2 × 512	291.89	50.31	5.80	245.05	3.61	67.88
2 × 1024	538.29	50.10	10.74	491.64	3.99	123.22

Timing for 2D FTLE on a regular grid (512^2)
[Hlawatsch et al., 2011]

Timings in perspective…

Some examples for simulated flow scenarios [Wasberg et al., 2009]

- 112 x 113 x 112 [Wasberg et al., 2009] Re=180
- 128 x 129 x 128 [Moser et al., 1999] Re=180
- 1536 x 257 x 1536 [del Alamo and Jimenez, 2003] Re=550
- 3072 x 385 x 2304 [del Alamo et al., 2004] Re=950

Typical Reynolds numbers: Blood flow in aorta ca 1000, large ships ca 5 x 10^9 [Wikipedia]

For realistic scenarios efficient computation essential to be able to apply FTLE-based methods!

Timing for 2D FTLE on a regular grid (512^2)
[Hlawatsch et al., 2011]

	with setup time			without setup time		
steps	direct	hier.	ratio	direct	hier.	ratio
2 × 4	46.85	46.36	1.01	0.51	0.42	1.21
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2 × 256	168.79	48.99	3.45	121.61	2.75	44.22
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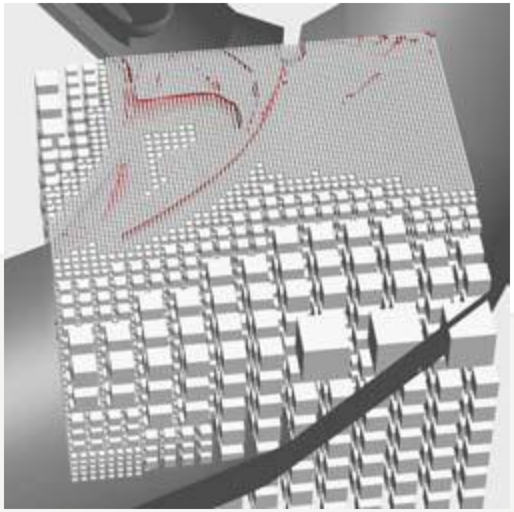
2 × 1024	538.29	50.10	10.74	491.64	3.99	123.22

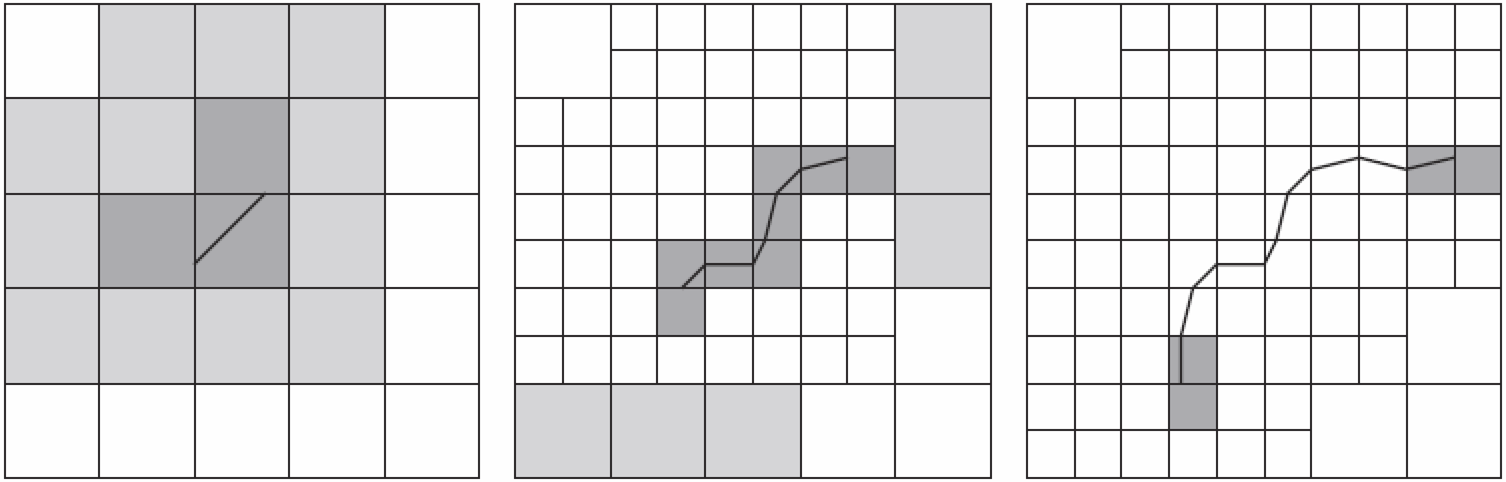
Two principle time saving strategies…

- **Less integrations**
  Adaptive mesh refinement [Sadlo and Peikert, 2007]
- **Cheaper integrations**
  Hierarchical integration [Brunton and Rowley, 2010; Hlawatsch et al., 2011]
- **Combination of both**
  Ridge tracking algorithm [Lipinski and Mohseni, 2010]



[Sadlo and Peikert, 2007]

Adaptive mesh refinement [Sadlo and Peikert, 2007]

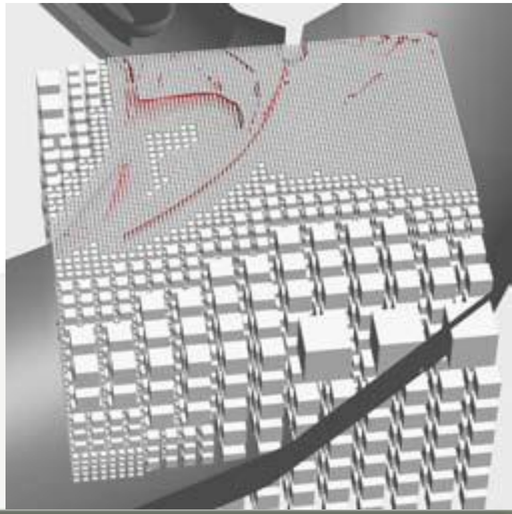


Main loop:

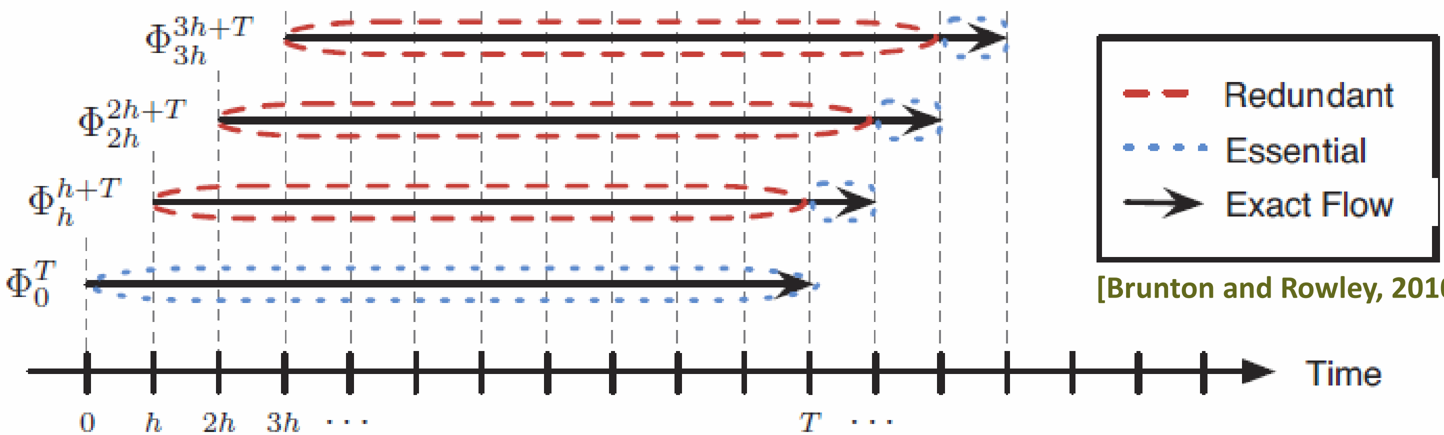
1. Coarse seeding, pointwise verification of ridge detection
2. Subdivision of detected ridge cells and neighbors
3. New pointwise ridge detection

Pro and Cons:

+ Exact ridges
- Relatively low speed-up (factor 4)



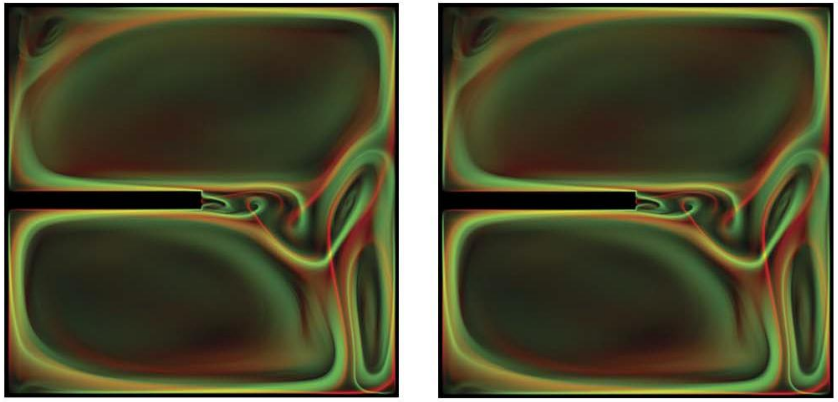
Hierarchical Integration [Brunton and Rowley, 2010; Hlawatsch et al., 2011]



[Brunton and Rowley, 2010]

Main loop:
1. Integrate from each time step to next
2. Concatenate integration by interpolation

Pro and Cons:
+ Large speed-up (factor 10)
+ Animations easily possible
- Interpolation error



[Hlawatsch et al., 2011]

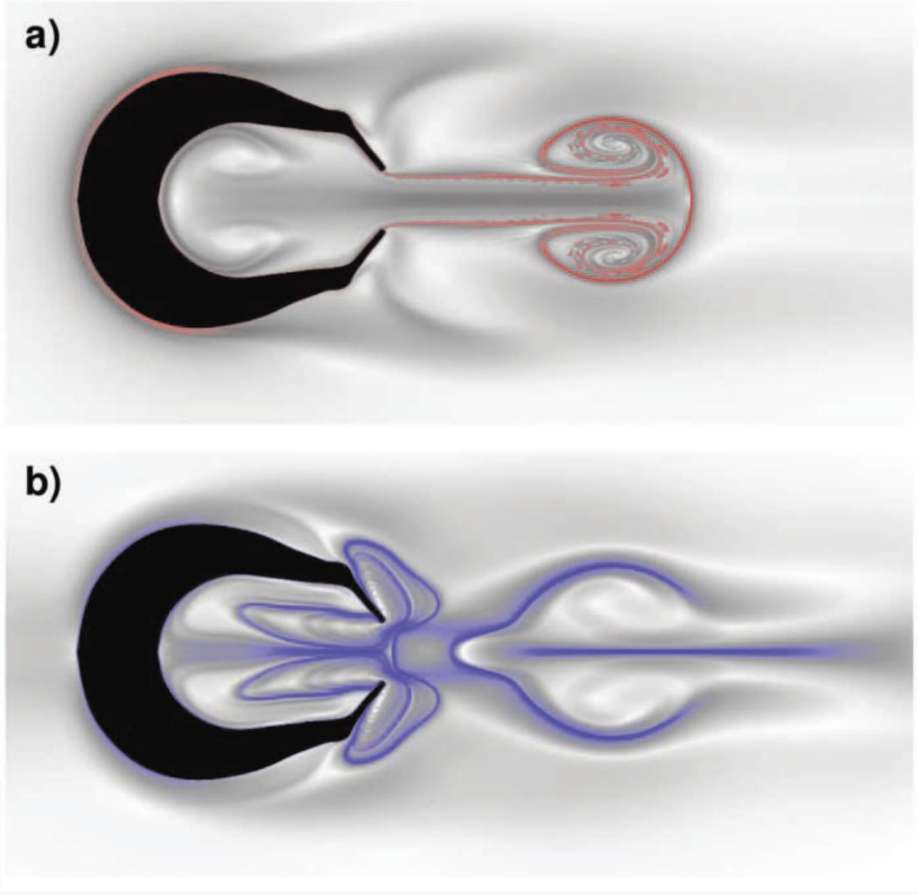
Ridge tracking algorithm [Lipinski and Mohseni, 2010]

Main loop:

1. Compute FTLE ridge
2. Advect ridge
3. Recompute ridge anew if advection error too large

Pro and Cons:

+ Large speed-up (factor 35)
+ Possibly combinable with ridge refinement
- At current 2D only



Thank you for your attention!

Tutorial: Time-Dependent Flow Visualization

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1) University of Bergen, Norway
2) University of Magdeburg, Germany

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Acknowledgements

- **Based on the following references:**
  - A. Pobitzer, R. Peikert, R. Fuchs, B. Schindler, A. Kuhn, H. Theisel, K. Matkovic and H. Hauser
    The State of the Art in Topology-Based Visualization of Unsteady Flow
    Computer Graphics Forum, 2011
  - **Scientific Visualization** — **Tino Weinkauf,** MPI Saarbrücken, 2012
  - **Flow and Tensor Visualization** — **Holger Theisel,** University of Magdeburg, 2011
  - **Flow Visualization** — **Helwig Hauser,** University of Bergen, 2011

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Literature

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