Part 1 – General Methods

Tutorial: Time-Dependent Flow Visualization

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Based on the following references:

  The State of the Art in Topology-Based Visualization of Unsteady Flow
  *Computer Graphics Forum, 2011*

- **Scientific Visualization**
  Tino Weinkauf, MPI Saarbrücken, 2012

- **Flow and Tensor Visualization**
  Holger Theisel, University of Magdeburg, 2011

- **Flow Visualization**
  Helwig Hauser, University of Bergen, 2011

The project *SemSeg* acknowledges the financial support of the Future and Emerging Technologies (FET) programme within the Seventh Framework Programme for Research of the European Commission, under FET-Open grant number 226042.
Overview

1. Line integral convolution (LIC)
2. Vortex detectors
3. The parallel vectors operator
4. Stream- vs pathlines
5. Limits of Vector Field Topology
Line Integral Convolution – A FlowVis Classic

- Introduced by Cabral and Leedom in 1993
- Space-filling visualization of instantaneous flow field
- Local processing, global information
- Automatic critical point detection and classification
Basic idea: convolute (white noise) texture with stream line

(Weighted) averaging of noise along part of stream line

- High correlation along stream line
- Low correlation orthogonal to stream line

Formal computation of intensity for every space point

\[ I(x_0) = \int_{s_0-1}^{s_0+1} K(s_0 - s)n(x(s))ds \]

- \( I \) ... intensity
- \( K \) ... convolution kernel, bandwidth 2
- \( n \) ... noise texture
- \( x(s) \) ... streamline (arc length param.)
1. **Contains no information on velocity magnitude**
   1. Add colour

2. **Stream lines = steady velocity field**
   1. Animation (2D)
   2. Convolute with path lines [Shen and Kao, 1998]

3. **Extension to 3D**
   1. Algorithm: not limited to specific dimension
   2. Rendering: ???
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**Line Integral Convolution – A FlowVis Classic**
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Vortices are...
- One of the most prominent flow features
- One of the least understood flow features

No (or several) mathematical definitions available
- Common intuition: “something swirling”

Vortex detectors
- Vortex as area/volume
- Vortex as core
Vortex Regions – Similar, but Different

- By thresholding
  - Vorticity \( \omega = \nabla \times \mathbf{v} \)
  - Lambda 2 \( \lambda_2 \) [Jeong and Hussain, 1995]
  - Delta criterion \( \Delta \) [Chong et al., 1990]
  - Hunt’s Q criterion [Hunt et al., 1988]

- Some properties /relations between the criteria
  - Strong shear: no vorticity thresholding!
  - Hunt’s Q more restrictive than Delta
  - Near wall: Lambda 2 better than Hunt’s Q
  - For compressible flow: problems with Lambda 2
  - Delta corresponds with VFT

- In to deep discussion of different definitions [Chakraborty et al. 2005]
Advantages

- Mostly applicable to steady AND unsteady flow, without modifications
- Easy visualization by direct volume rendering/isosurfacing

Drawbacks

- A good threshold?
- Vortices are “fuzzy”

[Helgeland et al., 2007]
Common agreement on vortex as swirling motion around common axis

- Clearer visualization
- (Ideally) parameter-free
- Again, no mathematical definition

Popular vortex core criteria

- Helicity method [Levy et al., 1990]
- Sujudi and Haimes [Sujudi and Haimes, 1995]
- “Unsteady Sujudi and Haimes” [Fuchs et al., 2008]
- Cores of swirling motion [Weinkauf et al., 2007]
- Acceleration minima [Fuchs et al, 2010; Kasten et al., 2011]
Vortex Core Lines – The Central Part of Vortical Motion

- **Advantages**
  - Less visual clutter
  - Crisp feature

- **Drawbacks**
  - Region of influence of vortex unclear
  - More sensitive to stable/unstable
  - Assume tube-like vortices

[Weinkauf et al., 2007]

[Fuchs et al., 2008]
The parallel vectors operator [Peikert and Roth, 1999]:
Unified formulation for line and surface extraction in vector fields.
Detection of
- Zero curvature lines of velocity field
- Extremal lines in magnitude for general vector fields
- Local parallelism of two arbitrary vector fields

One application: vortex core line

- All previous presented approaches can be formulated as
  "find space points where vector field \( a \) is parallel to vector field \( b \)"
- Choice of \( a \) and \( b \)
  - In [Peikert and Roth, 1999]: normalised helicity, Sujudi and Haimes,
    acceleration minimum
  - In [Fuchs et al., 2007]: “unsteady Sujudi and Haimes”
  - In [Weinkauf et al., 2007]: cores of swirling motion
The original algorithm, as described by Sujudi and Haimes:

1. Linearly interpolate the velocity within the cell.
2. Compute the rate-of-deformation tensor $A$. Since a linear interpolation of the velocity within the cell can be written as

$$ u_i = C_i + \frac{\partial u_i}{\partial x} \Delta x + \frac{\partial u_i}{\partial y} \Delta y + \frac{\partial u_i}{\partial z} \Delta z $$

then $A$ can be constructed from the coefficients of the linear interpolation function of the velocity vector.
3. Find the eigenvalues of $A$. Processing continues only if $A$ has one real ($\lambda_R$) and a pair of complex-conjugate eigenvalues ($\lambda_C$).
4. At each node of the tetrahedron, subtract the velocity component in the direction of the eigenvector corresponding to $\lambda_R$. This is equivalent to projecting the velocity onto the plane normal to the eigenvector belonging to $\lambda_R$, and can be expressed as

$$ \tilde{w} = \tilde{u} - (\tilde{u} \cdot \tilde{n})\tilde{n} $$

where $\tilde{n}$ is the normalized eigenvector corresponding to $\lambda_R$, and $\tilde{w}$ is the reduced velocity.
5. Linearly interpolate each component of the reduced velocity to obtain

$$ w_i = a_i + b_i x + c_i y + d_i z \quad (3) $$

$$ i = 1, 2, 3 $$

6. To find the center, we set $w_i$ in equation (3) to zero. Since the reduced velocity lies in a plane, it has only 2 degrees of freedom. Thus, only 2 of the 3 equations in equation (3) are independent. Any 2 can be chosen as long as their coefficients are not all zero. Now we have

$$ 0 = a_i + b_i x + c_i y + d_i z \quad (4) $$

$$ i = 1, 2 $$

which are the equations of 2 planes, whose solution (the intersection of 2 planes) is a line.
7. If this line intersects the cell at more than 1 point, then the cell contains a center of a local swirling flow. The center is defined by the line segment formed by the 2 intersection points.

[Sujudi and Haimes., 1995]
... in the parallel vector formulation, as described by Peikert and Roth:
... in the parallel vector formulation, as described by Peikert and Roth:

\[
\mathbf{v} \parallel (\nabla \mathbf{v}) \mathbf{v} = \left\{ \mathbf{x} \left| (\mathbf{v}(\mathbf{x})) \times (\nabla \mathbf{v}(\mathbf{x}) \mathbf{v}(\mathbf{x})) = 0 \right. \right\}
\]

Find set by intersection of isosurfaces (e.g., marching lines), Newton iteration on cell faces, analytic solution (for triangular faces),...
... in the parallel vector formulation, as described by Peikert and Roth:

\[
\mathbf{v} \parallel (\nabla \mathbf{v}) \mathbf{v} = \left\{ \mathbf{x} \mid (\mathbf{v} \mathbf{v}) \times (\nabla \mathbf{v} \mathbf{v}) = 0 \right\}
\]

Find set by intersection of isosurfaces (e.g., marching lines), Newton iteration on cell faces, analytic solution (for triangular faces),...
Vortices – A Few General Remarks

- Steady vs. unsteady flow
  - Correction by consideration of spatial AND temporal velocity fluctuations (Material derivative)
  - (Usually) small effects, vortices are mostly instantaneous features
  - In unsteady flows vortices can be created, merge, split, braid, be dissipated,…

- Region vs. core line
  - Region: fails to detect rotation axis
  - Core line: no information on region of influence
Overview

1. **Line integral convolution (LIC)**
2. **Vortex detectors**
3. **The parallel vectors operator**
4. **Streamlines vs. Pathlines**
5. **Limits of Vector Field Topology**
Streamlines vs. Pathlines

**Characteristic Curves**

- **tangent curves:**
  
  \[ \mathbf{s}(t) \quad \rightarrow \quad \dot{\mathbf{s}}(t) = \mathbf{v}(\mathbf{s}(t)) \]

  - solve initial value problem
  - describes path of a mass less particle

[Tino Weinkauf, MPI]
Streamlines vs. Pathlines

Characteristic Curves

- **tangent curves:**
  - tangent curves do not intersect
  - unique for any point in \( \mathbf{v} \)
    - exception: *critical points*

- description of tangent curves:
  - parametric description (only linear fields)
  - numerical integration

[Tino Weinkauf, MPI]
Streamlines vs. Pathlines

Space-time domain approach

- concept to handle time-dependent data
  - lift problem to higher dimension
  - time as additional spatial dimension
  - unsteady case → steady case
  - space and time can be handled in one set
  - extendable to arbitrary dimensions
Streamlines vs. Pathlines

Space-time domain approach

- **vector field:**
  \[ \mathbf{v}(x, y, t) = \begin{pmatrix} u(x, y, t) \\ v(x, y, t) \end{pmatrix} \]

- **steady case:**
  \[ \mathbf{s}(x, y, t) = \begin{pmatrix} u(x, y, t) \\ v(x, y, t) \\ 0 \end{pmatrix} \Rightarrow \text{streamlines} \]

- **unsteady case:**
  \[ \mathbf{p}(x, y, t) = \begin{pmatrix} u(x, y, t) \\ v(x, y, t) \\ 1 \end{pmatrix} \Rightarrow \text{pathlines} \]
Streamlines vs. Pathlines

Simple Example:

\[ \mathbf{v}(x,t) = (1-t) + t \]
**Steady Case 2D**

\[ \mathbf{v}(x, y) \]

\[ \frac{d}{d\tau} \mathbf{x}(\tau) = \mathbf{v}(\mathbf{x}(\tau)) \]

with \( \mathbf{x}(0) = \mathbf{x}_0 \)

*streamlines*

---

**Unsteady Case 2D**

\[ \mathbf{v}(x, y, t) \]

\[ \frac{d}{d\tau} \mathbf{x}(\tau) = \mathbf{v}(\mathbf{x}(\tau), t_0) \]

with \( \mathbf{x}(0) = \mathbf{x}_0 \)

*streamlines*

\[ \frac{d}{dt} \mathbf{x}(t) = \mathbf{v}(\mathbf{x}(t), t) \]

with \( \mathbf{x}(t_0) = \mathbf{x}_0 \)

*pathlines*

[Tino Weinkauf, MPI]
**Steady Case 3D**

\[ \mathbf{v}(x, y, z) = \begin{pmatrix} u(x, y, z) \\ v(x, y, z) \\ w(x, y, z) \end{pmatrix} \]

*streamlines*

**Unsteady Case 3D**

\[ \mathbf{s}(x, y, z, t) = \begin{pmatrix} u(x, y, z, t) \\ v(x, y, z, t) \\ w(x, y, z, t) \end{pmatrix} \]

\[ \mathbf{p}(x, y, z, t) = \begin{pmatrix} u(x, y, z, t) \\ v(x, y, z, t) \\ w(x, y, z, t) \end{pmatrix} \]

*streamlines*  
*pathlines*

[Tino Weinkauf, MPI]
Streamlines vs. Pathlines

Example Pathlines:

Streamlines vs. Pathlines

Characteristic Curves Overview:

- Curve types:
  - **streamlines:** parallel to $v(p,t)$ in each point $p$ for a fixed time
  - **pathlines:** motion of a particles over the time in an *unsteady* field $v(p,t)$
  - **streaklines:** location of all particles set out at a *fixed point* over time
  - **timelines:** evolution of a curve set out at a time $t_0$
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Example 1: The Beads Problem [Wiebel et al., TopoInVis 2009]

- Vector field:
  - modeling aggregating cell behavior
  - simple rotating attractor

\[ \mathbf{v}(x, y, t) = \left( -\left( y - \frac{1}{3} \sin(t) \right) - \left( x - \frac{1}{3} \cos(t) \right) \right) \]

- applied methods:
  - critical points \((\text{green})\)
  - swirling pathline cores \((\text{blue})\)
Example 2: Double Gyre [Shadden06]

- **static case:**
  - isolated critical features
  - two sliding orbits
  - separating structure

- **unsteady case:**
  - asymmetric material structure
  - complex ridge evolvement
  - scalar field description
Limits of Vector Field Topology

Example 2: Double Gyre [Shadden06]

- comparison:
  a) pathlines
  b) vector length
  c) Lagrangian feature
  d) combined
Thank you for your attention!

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Toward a Lagrangian Vector Field Topology,
Literature


