Part 1 – General Methods

Tutorial: Time-Dependent Flow Visualization

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Acknowledgements

- **Based on the following references:**
  - A. Pobitzer, R. Peikert, R. Fuchs, B. Schindler, A. Kuhn, H. Theisel, K. Matkovic and H. Hauser
    The State of the Art in Topology-Based Visualization of Unsteady Flow
    Computer Graphics Forum, 2011
  - **Scientific Visualization** — **Tino Weinkauf,** MPI Saarbrücken, 2012
  - **Flow and Tensor Visualization** — **Holger Theisel,** University of Magdeburg, 2011
  - **Flow Visualization** — **Helwig Hauser,** University of Bergen, 2011

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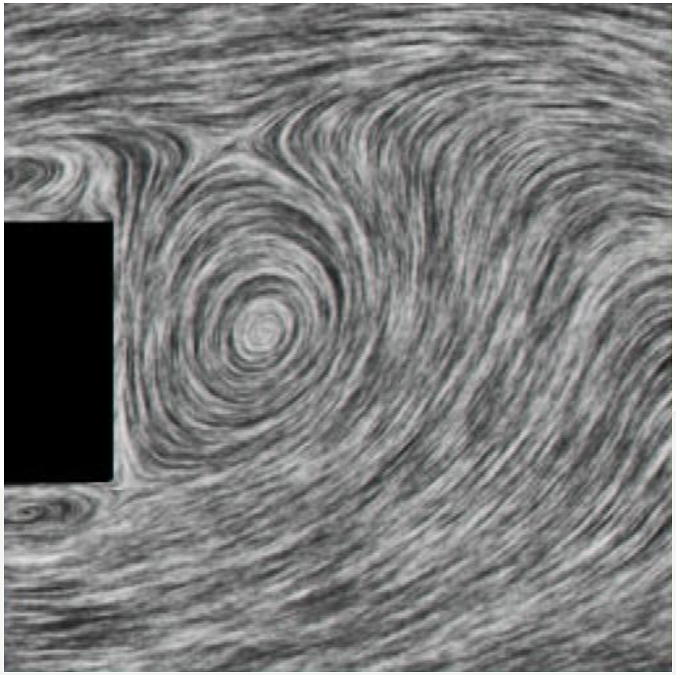
Overview

1. Line integral convolution (LIC)
2. Vortex detectors
3. The parallel vectors operator
4. Stream- vs pathlines
5. Limits of Vector Field Topology

Line Integral Convolution – A FlowVis Classic

- **Introduced by Cabral and Leedom in 1993**
- **Space-filling visualization of instantaneous flow field**
- **Local processing, global information**
- **Automatic critical point detection and classification**



Line Integral Convolution – A FlowVis Classic

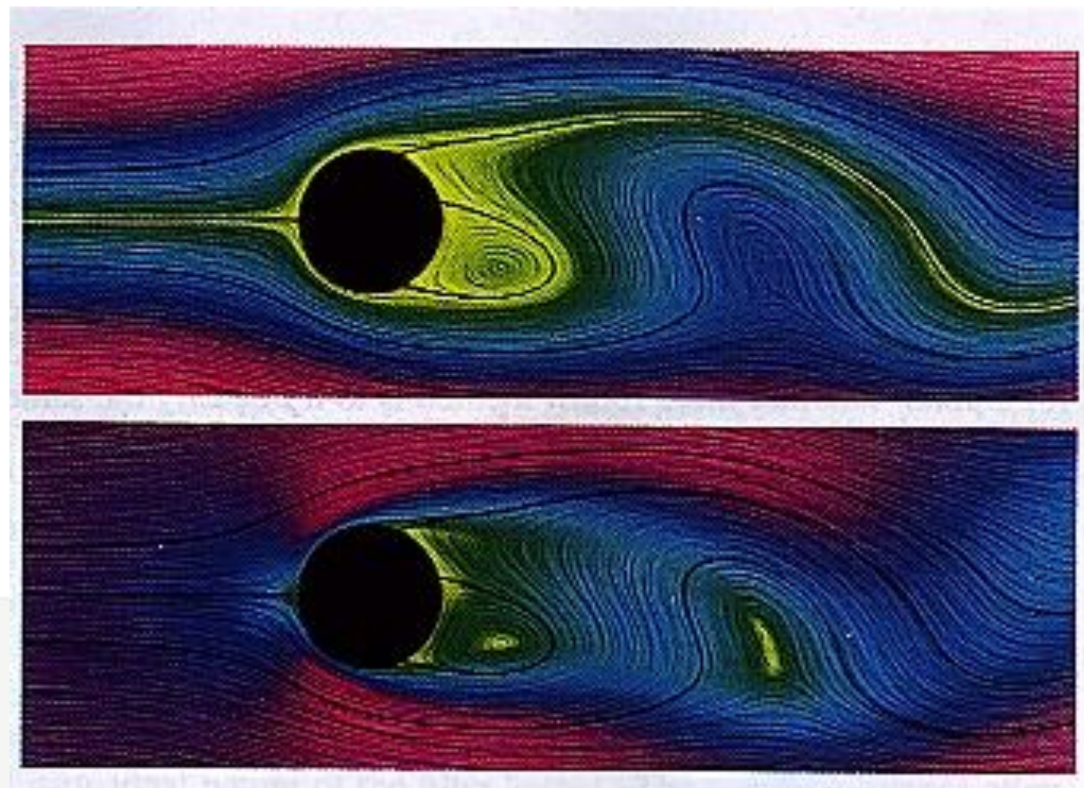
- **Basic idea: convolute (white noise) texture with stream line**
- **(Weighted) averaging of noise along part of stream line**
  - **High correlation along stream line**
  - **Low correlation orthogonal to stream line**
- **Formal computation of intensity for every space point**

$$I(\mathbf{x}_0) = \int_{s_0-l}^{s_0+l} K(s_0 - s)\, n(\mathbf{x}(s))\, ds$$

I… intensity
K… convolution kernel, bandwith 2l
n… noise texture
$\mathbf{x}(s)$… streamline (arc length param.)

Line Integral Convolution – A FlowVis Classic

1. **Contains no information on velocity magnitude**
   1. **Add colour**
2. Stream lines = steady velocity field
   1. Animation (2D)
   2. Convolute with path lines [Shen and Kao, 1998]
3. Extension to 3D
   1. Algorithm: not limited to specific dimension
   2. Rendering: ???



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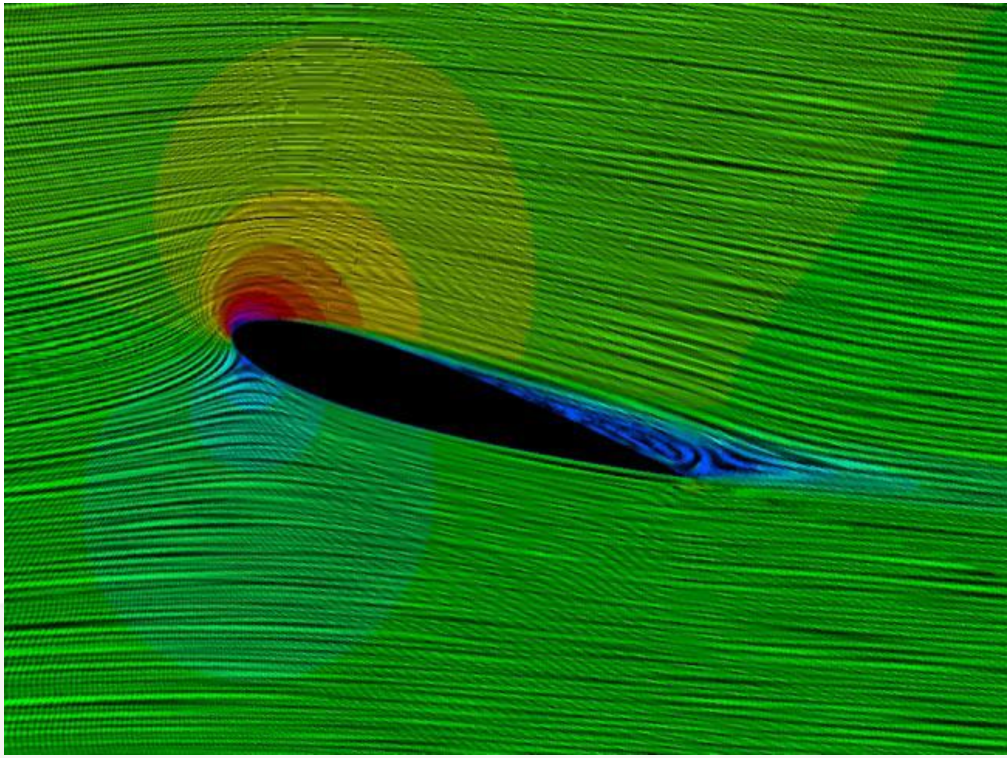

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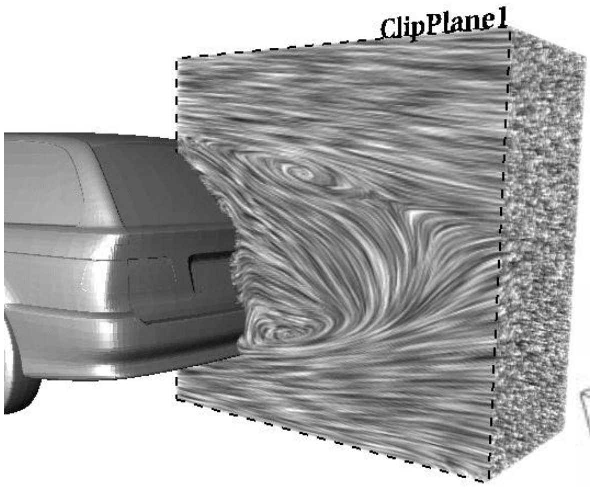
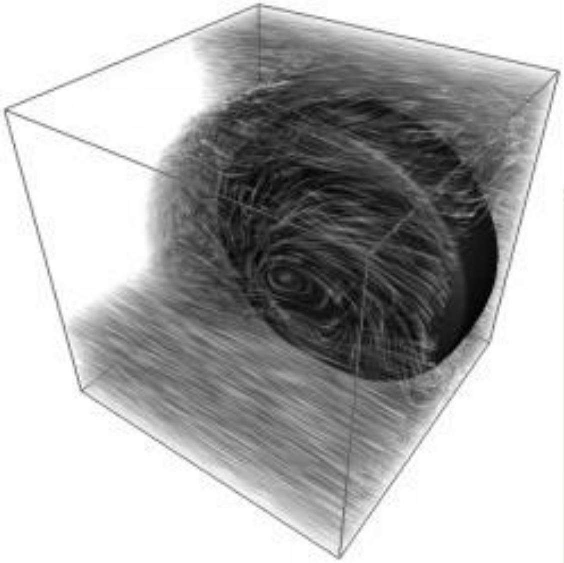


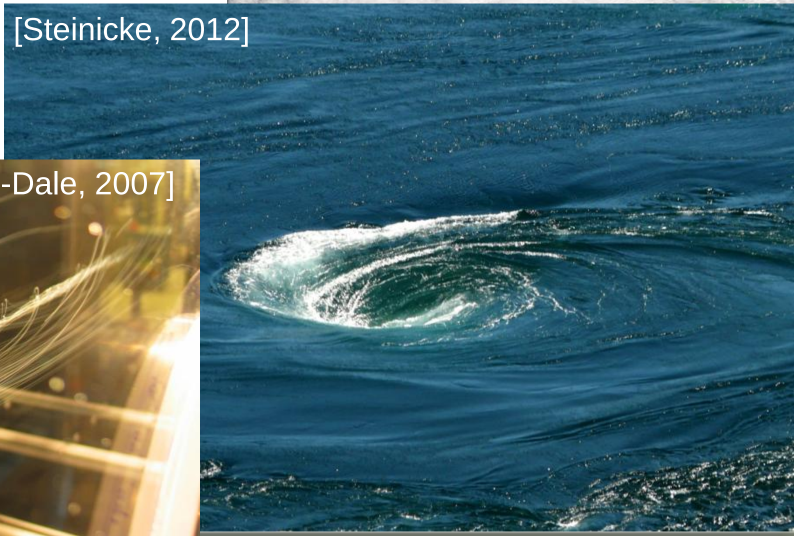
[Rezk-Salama et al., 1999]

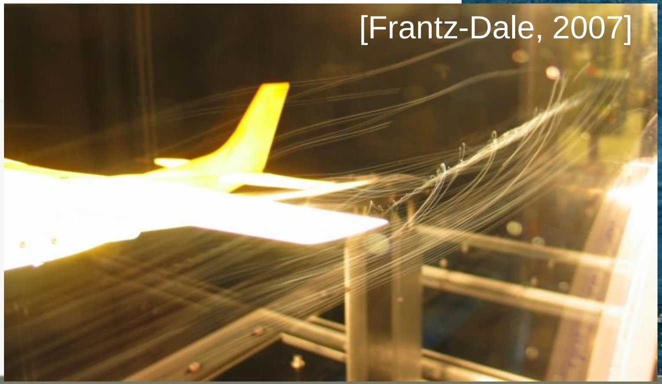


Vortices – Easy to Imagine, Hard to Define

- Vortices are…
  - One of the most prominent flow features
  - One of the least understood flow features
- No (or several) mathematical definitions available
  - Common intuition: "something swirling"
- Vortex detectors
  - Vortex as area/volume
  - Vortex as core


[Descloitres, 2005]


[Steinicke, 2012]


[Frantz-Dale, 2007]

Vortex Regions – Similar, but Different

- **By thresholding**
  - **Vorticity** $\boldsymbol{\omega} = \nabla \times \mathbf{v}$
  - **Lambda 2** λ_2 [Jeong and Hussain, 1995]
  - **Delta criterion** Δ [Chong et al., 1990]
  - **Hunt's Q criterion** [Hunt et al., 1988]

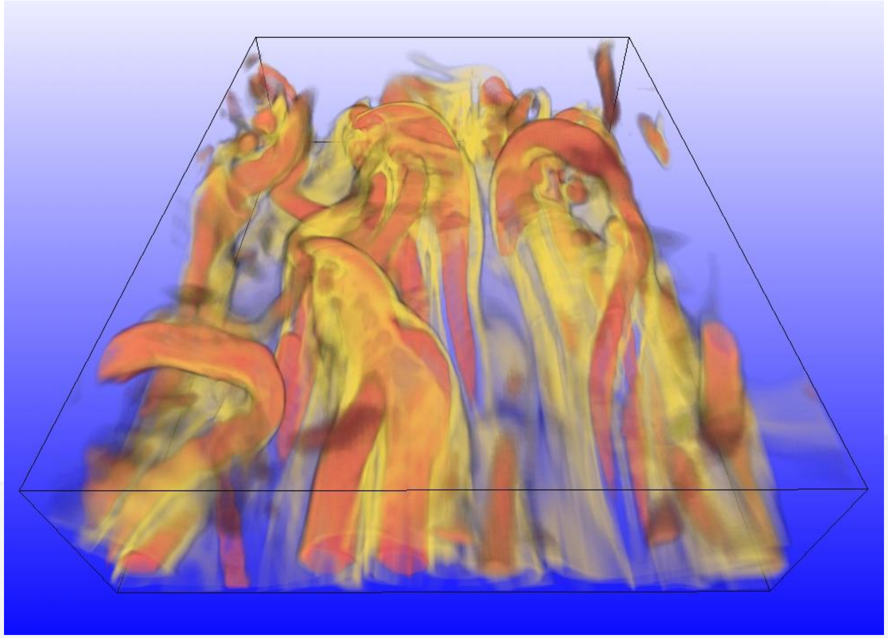
In common for **all** criteria: use of

$$\nabla \mathbf{v}$$

- **Some properties /relations between the criteria**
  - **Strong shear: no vorticity thresholding!**
  - **Hunt's Q more restrictive than Delta**
  - **Near wall: Lambda 2 better then Hunt's Q**
  - **For compressible flow: problems with Lambda 2**
  - **Delta corresponds with VFT**
- **In to deep discussion of different definitions** [Chakraborty et al. 2005]

Vortex regions – Similar, but different

- **Advantages**
  - **Mostly applicable to steady AND unsteady flow, without modifications**
  - **Easy visualization by direct volume rendering/isosurfacing**
- **Drawbacks**
  - **A good threshold?**
  - **Vortices are "fuzzy"**



[Helgeland et al., 2007]

Vortex Core Lines – The Central Part of Vortical Motion

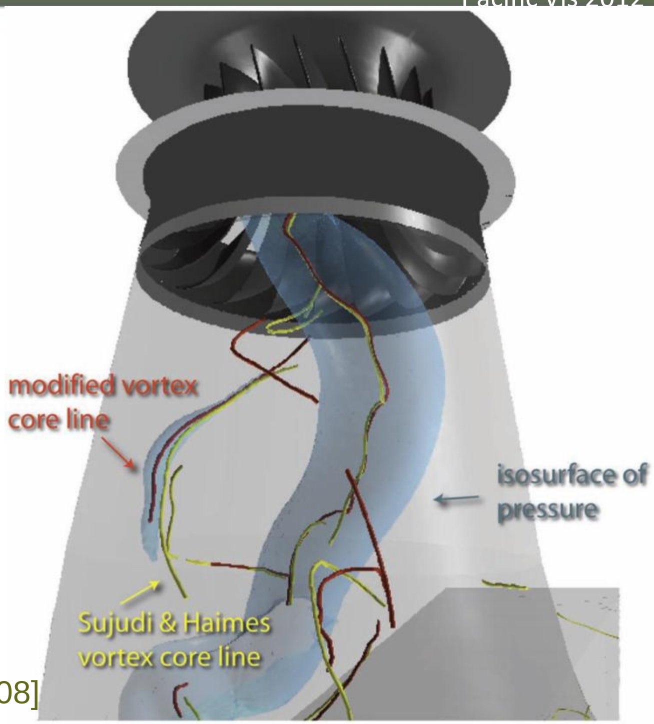
- **Common agreement on vortex as swirling motion around common axis**
  - Clearer visualization
  - (Ideally) parameter-free
  - Again, no mathematical definition

- **Popular vortex core criteria**
  - Helicity method [Levy et al., 1990]
  - Sujudi and Haimes [Sujudi and Haimes, 1995]
  - "Unsteady Sujudi and Haimes" [Fuchs et al., 2008]
  - Cores of swirling motion [Weinkauf et al., 2007]
  - Acceleration minima [Fuchs et al, 2010; Kasten et al., 2011]

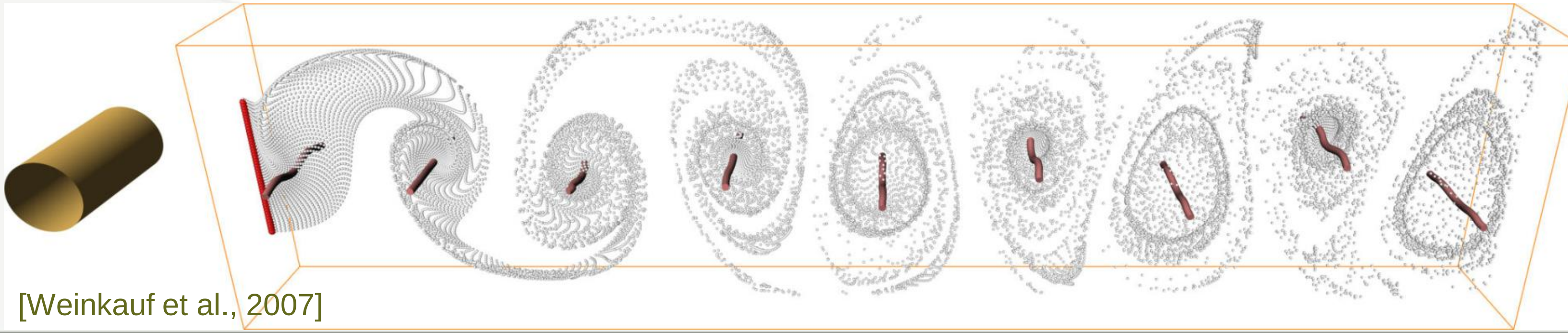
Vortex Core Lines – The Central Part of Vortical Motion

- **Advantages**
  - **Less visual clutter**
  - **Crisp feature**

- **Drawbacks**
  - **Region of influence of vortex unclear**
  - **More sensitive to stable/unstable**
  - **Assume tube-like vortices**



[Fuchs et al., 2008]



[Weinkauf et al., 2007]

Parallel vectors – Multiple Core Lines for the Price of One

- **The parallel vectors operator** [Peikert and Roth, 1999]:
  Unified formulation for line and surface extraction in vector fields.
  Detection of
  - Zero curvature lines of velocity field
  - Extremal lines in magnitude for general vector fields
  - Local parallelism of two arbitrary vector fields

- **One application: vortex core line**
  - All previous presented approaches can be formulated as
    "find space points where vector field a is parallel to vector field b"
  - Choice of a and b
    - In [Peikert and Roth, 1999]: normalised helicity, Sujudi and Haimes, acceleration minumum
    - In [Fuchs et al., 2007]: "unsteady Sujudi and Haimes"
    - In [Weinkauf et al., 2007]: cores of swirling motion

Parallel vectors – Example: Sujudi and Haimes

The original algorithm, as described by Sujudi and Haimes:

The algorithm proceeds one tetrahedral cell at a time, and can be summarized as follows (it is assumed that a velocity vector is available at each node):

1. Linearly interpolate the velocity within the cell.
2. Compute the rate-of-deformation tensor A. Since a linear interpolation of the velocity within the cell can be written as

$$u_i = C_i + \frac{\partial u_i}{\partial x}\Delta x + \frac{\partial u_i}{\partial y}\Delta y + \frac{\partial u_i}{\partial z}\Delta z \qquad (1)$$

then A can be constructed from the coefficients of the linear interpolation function of the velocity vector.

3. Find the eigenvalues of A. Processing continues only if A has one real (λ_R) and a pair of complex-conjugate eigenvalues (λ_C).
4. At each node of the tetrahedron, subtract the velocity component in the direction of the eigenvector corresponding to λ_R. This is equivalent to projecting the velocity onto the plane normal to the eigenvector belonging to λ_R, and can be expressed as

$$\vec{w} = \vec{u} - (\vec{u} \cdot \vec{n})\vec{n} \qquad (2)$$

where $\vec{n}$ is the normalized eigenvector corresponding to λ_R, and w is the reduced velocity.

5. Linearly interpolate each component of the reduced velocity to obtain

$$w_i = a_i + b_i x + c_i y + d_i z \qquad (3)$$
$$i = 1, 2, 3$$

6. To find the center, we set w_i in equation (3) to zero. Since the reduced velocity lies in a plane, it has only 2 degrees of freedom. Thus, only 2 of the 3 equations in equation (3) are independent. Any 2 can be chosen as long as their coefficients are not all zero. Now we have

$$0 = a_i + b_i x + c_i y + d_i z \qquad (4)$$
$$i = 1, 2$$

which are the equations of 2 planes, whose solution (the intersection of 2 planes) is a line.

7. If this line intersects the cell at more than 1 point, then the cell contains a center of a local swirling flow. The center is defined by the line segment formed by the 2 intersection points.

[Sujudi and Haimes., 1995]

… in the parallel vector formulation, as described by Peikert and Roth:

Parallel vectors – Example: Sujudi and Haimes

… in the parallel vector formulation, as described by Peikert and Roth:

$$\mathbf{v} \parallel (\nabla \mathbf{v})\mathbf{v} = \{\mathbf{x} \mid (\mathbf{v}(\mathbf{x})) \times (\nabla \mathbf{v}(\mathbf{x})\mathbf{v}(\mathbf{x})) = 0\}$$

Find set by intersection of isosurfaces (e.g., marching lines), Newton iteration on cell faces, analytic solution (for triangular faces),…

Parallel vectors – Example: Sujudi and Haimes

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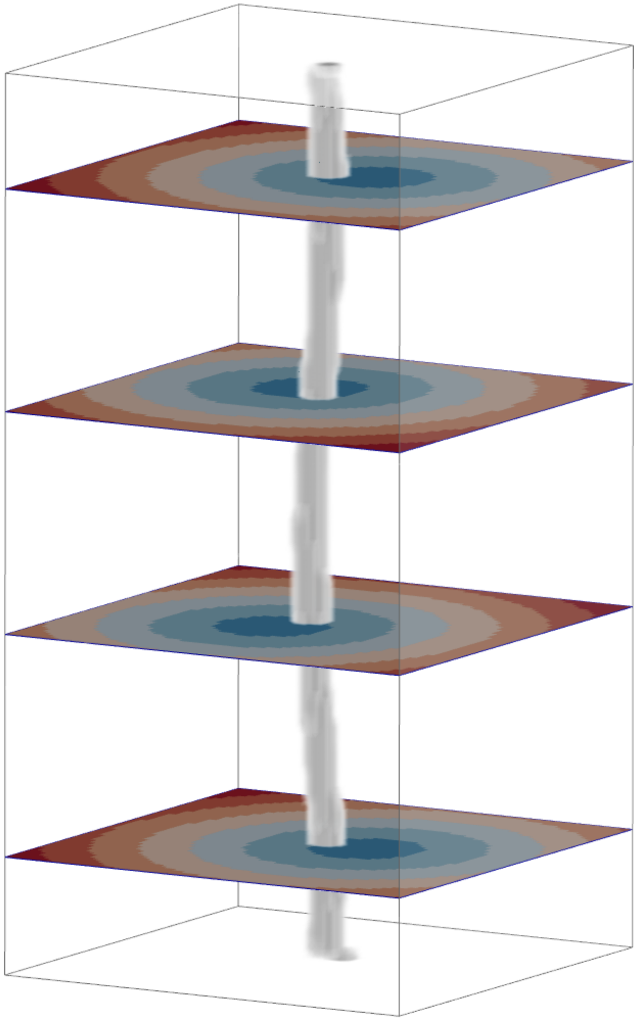
Find set by intersection of isosurfaces (e.g., marching lines), Newton iteration on cell faces, analytic solution (for triangular faces),…

Vortices – A Few General Remarks

- Steady vs. unsteady flow
  - Correction by consideration of spatial AND temporal velocity fluctuations (Material derivative)
  - (Usually) small effects, vortices are mostly instantaneous features
  - In unsteady flows vortices can be created, merge, split, braid, be dissipated,…
- Region vs. core line
  - Region: fails to detect rotation axis
  - Core line: no information on region of influence



Overview

1. Line integral convolution (LIC)
2. Vortex detectors
3. The parallel vectors operator
4. **Streamlines vs. Pathlines**
5. Limits of Vector Field Topology

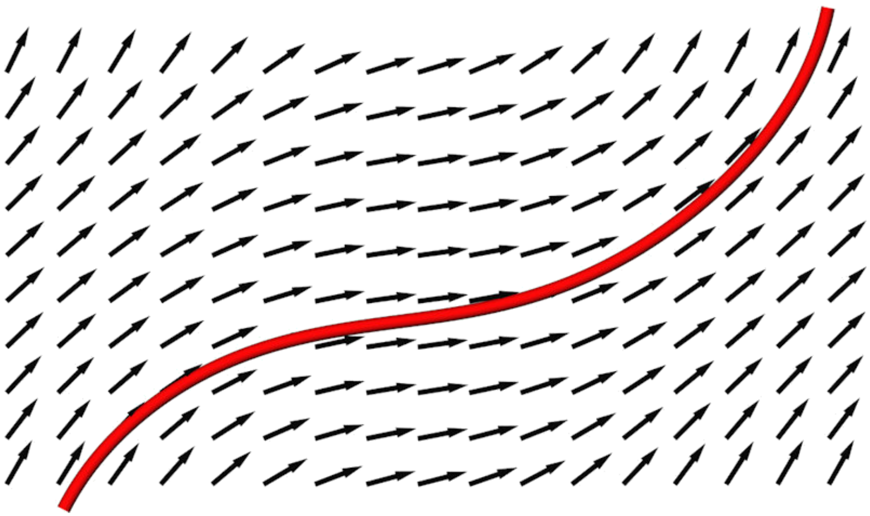
Streamlines vs. Pathlines

Characteristic Curves

- **tangent curves**:

$$\mathbf{s}(t) \longrightarrow \dot{\mathbf{s}}(t) = \mathbf{v}(\mathbf{s}(t))$$

  - solve initial value problem
  - describes path of a mass less particle

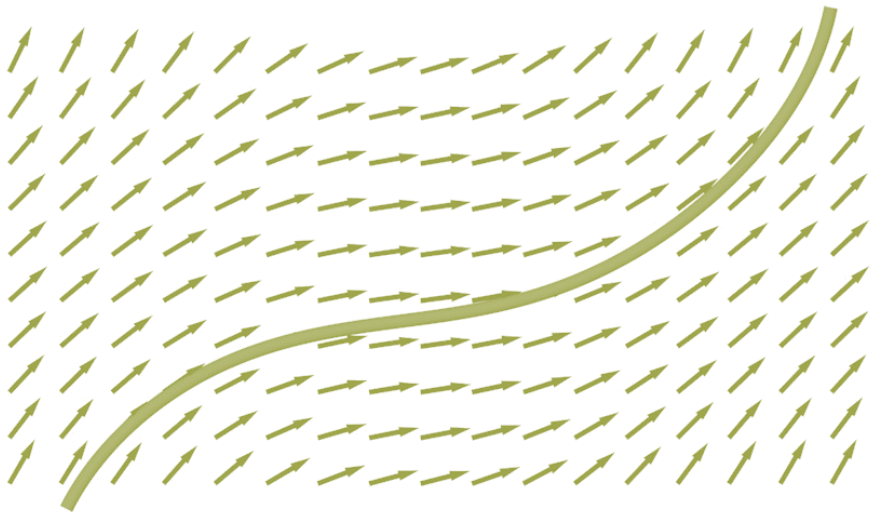


[Tino Weinkauf, MPI]

Streamlines vs. Pathlines

Characteristic Curves

- **tangent curves**:
  - tangent curves do not intersect
  - unique for any point in **v**
    - → exception: *critical points*
  - description of tangent curves:
    - → parametric description (only linear fields)
    - → numerical integration

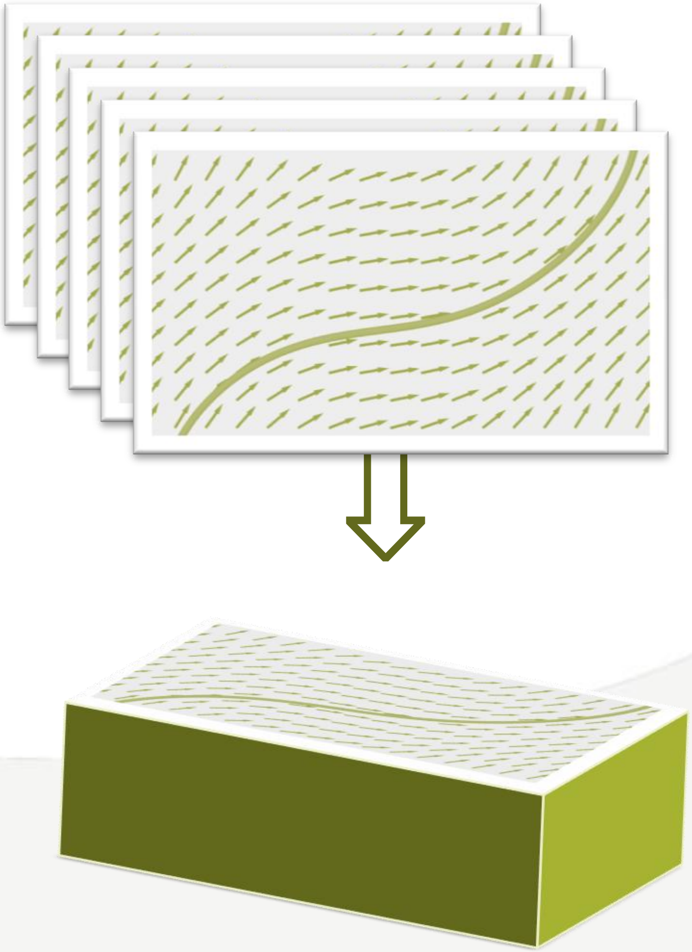


[Tino Weinkauf, MPI]

Streamlines vs. Pathlines

Space-time domain approach

- **concept to handle time-dependent data**
  - lift problem to higher dimension
  - time as additional spatial dimension
  - unsteady case ⇨ steady case
  - space and time can be handled in one set
  - extendable to arbitrary dimensions



Streamlines vs. Pathlines

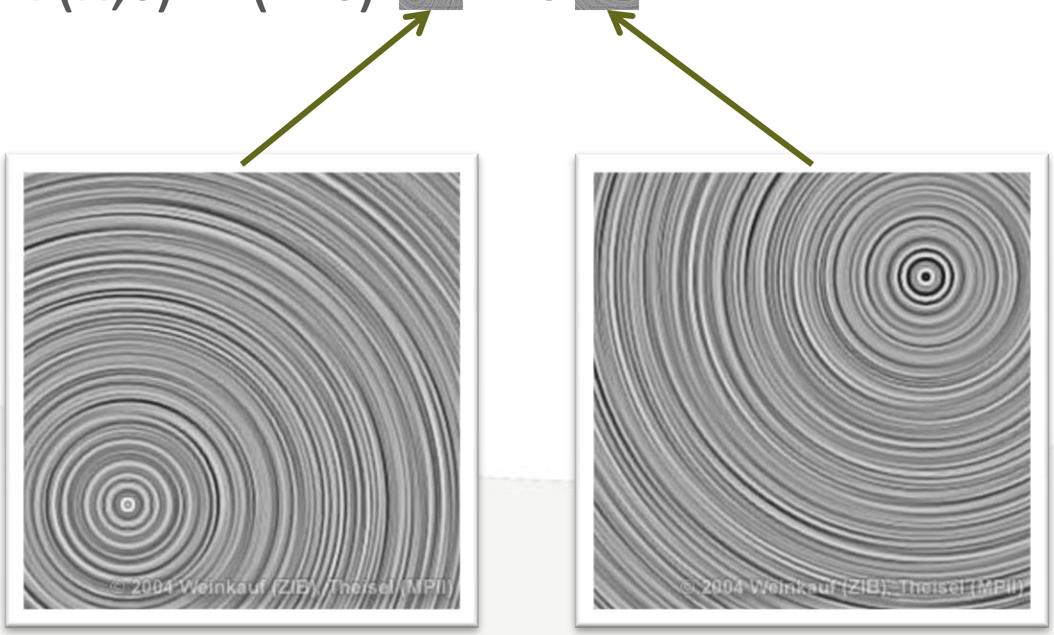
Space-time domain approach

- **vector field**: $$\mathbf{v}(x,y,t) = \begin{pmatrix} u(x,y,t) \\ v(x,y,t) \end{pmatrix}$$
- **steady case:** $$\mathbf{s}(x,y,t) = \begin{pmatrix} u(x,y,t) \\ v(x,y,t) \\ 0 \end{pmatrix}$$ ⇨ **streamlines**
- **unsteady case:** $$\mathbf{p}(x,y,t) = \begin{pmatrix} u(x,y,t) \\ v(x,y,t) \\ 1 \end{pmatrix}$$ ⇨ **pathlines**

Streamlines vs. Pathlines

Simple Example:

$$\mathbf{v}(\mathbf{x},t) = (1-t)\ \square + t\ \square$$

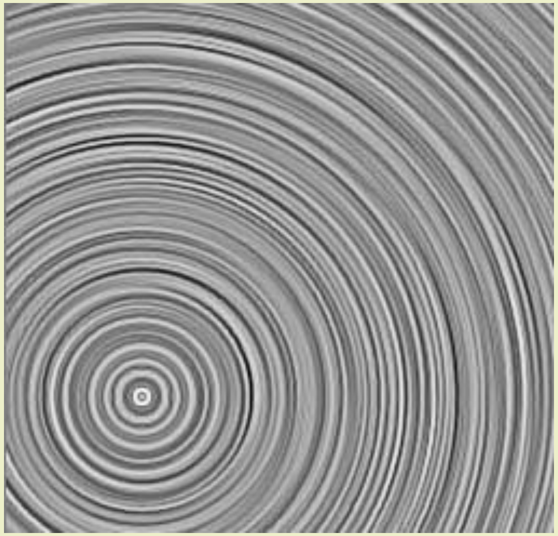


Steady Case 2D

$$\mathbf{v}(x, y)$$

$$\frac{d}{d\tau}\mathbf{x}(\tau) = \mathbf{v}(\mathbf{x}(\tau))$$
$$\text{with } \mathbf{x}(0) = \mathbf{x}_0$$

streamlines

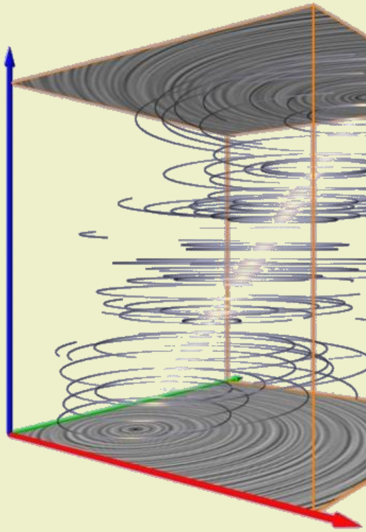


Unsteady Case 2D

$$\mathbf{v}(x, y, t)$$

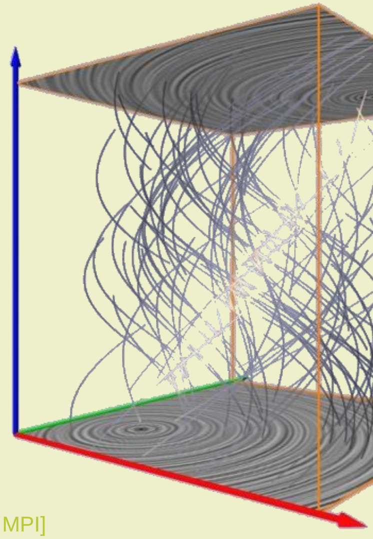
$$\frac{d}{d\tau}\mathbf{x}(\tau) = \mathbf{v}(\mathbf{x}(\tau), t_0)$$
$$\text{with } \mathbf{x}(0) = \mathbf{x}_0$$

streamlines



$$\frac{d}{dt}\mathbf{x}(t) = \mathbf{v}(\mathbf{x}(t), t)$$
$$\text{with } \mathbf{x}(t_0) = \mathbf{x}_0$$

pathlines

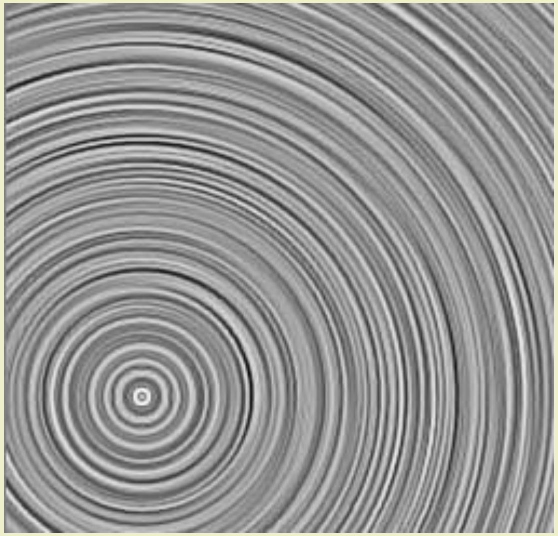


[Tino Weinkauf, MPI]

Steady Case 3D

$$\mathbf{v}(x,y,z) = \begin{pmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{pmatrix}$$

streamlines

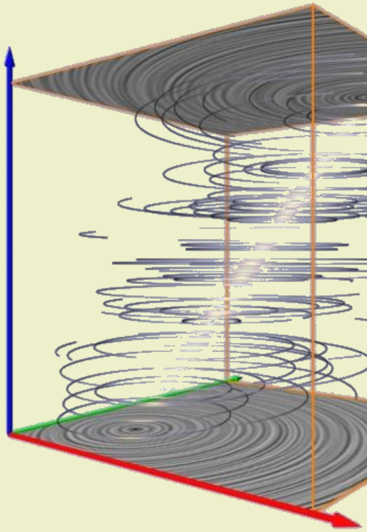


Unsteady Case 3D

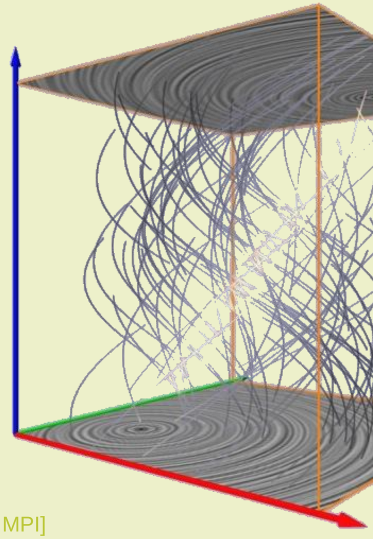
$$\mathbf{s}(x,y,z,t) = \begin{pmatrix} u(x,y,z,t) \\ v(x,y,z,t) \\ w(x,y,z,t) \\ 0 \end{pmatrix}$$

$$\mathbf{p}(x,y,z,t) = \begin{pmatrix} u(x,y,z,t) \\ v(x,y,z,t) \\ w(x,y,z,t) \\ 1 \end{pmatrix}$$

streamlines



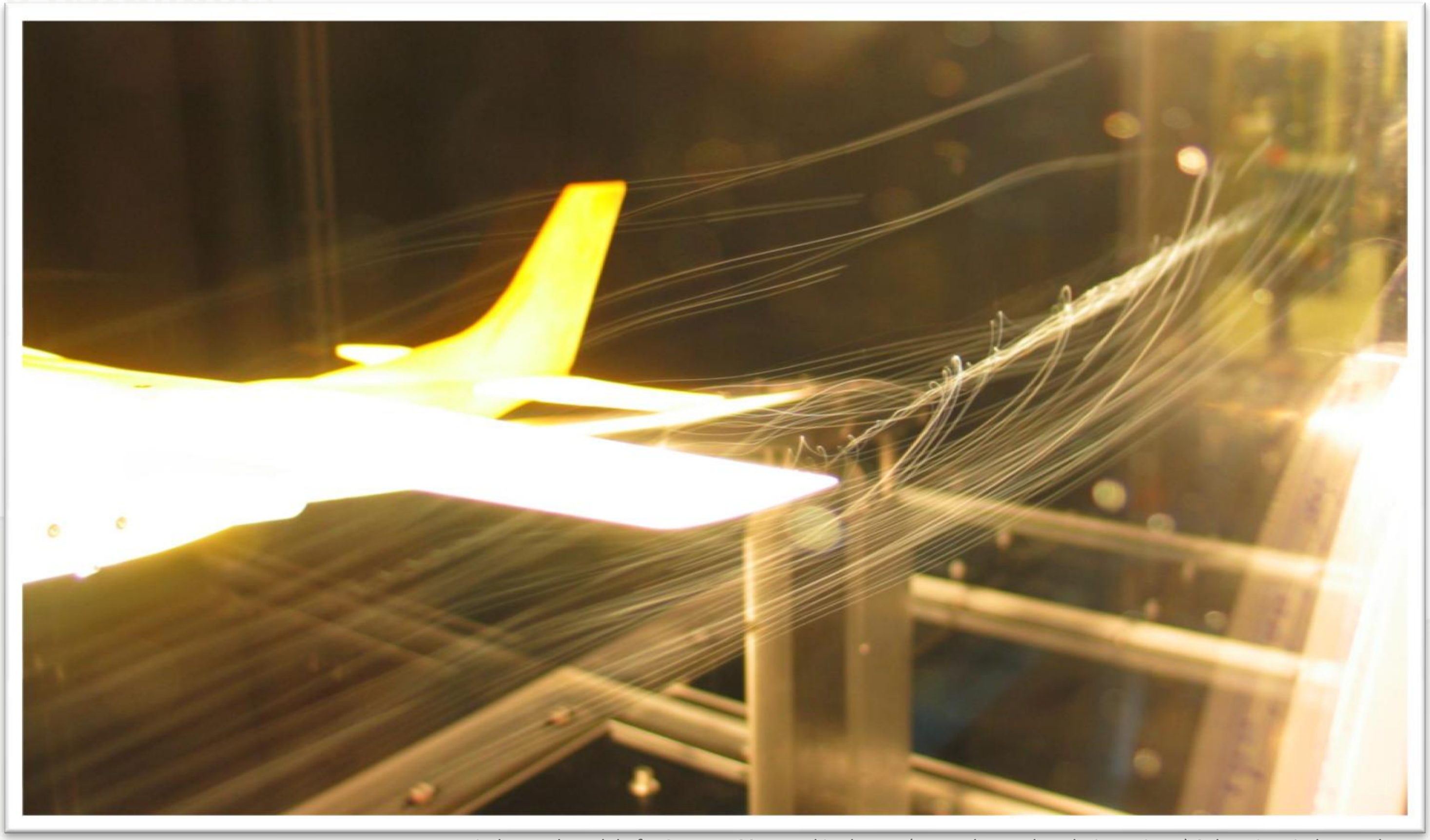
pathlines



[Tino Weinkauf, MPI]

Streamlines vs. Pathlines

Example Pathlines:

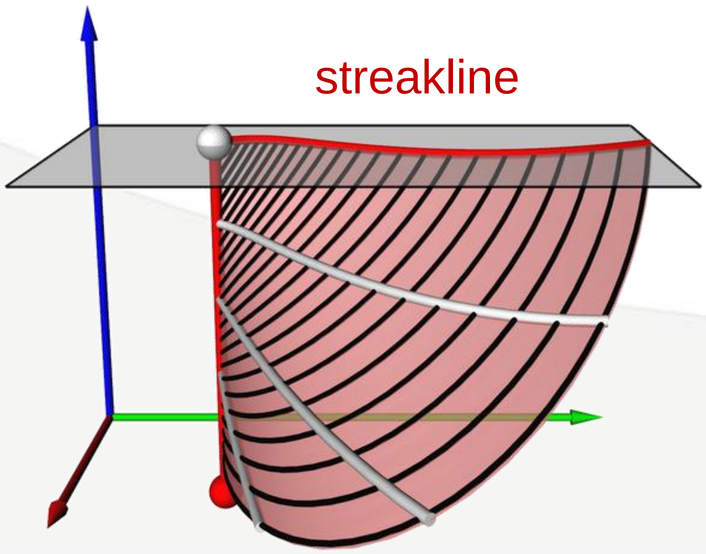


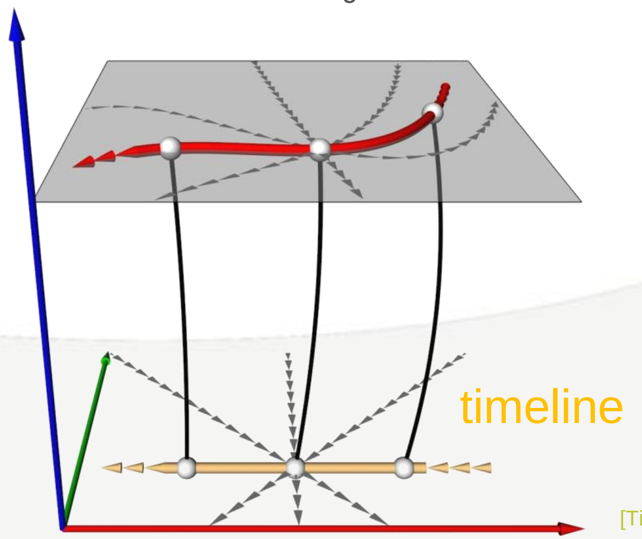
A wind tunnel model of a Cessna 182 Tested in the RPI (Rensselaer Polytechnic Institute) Subsonic Wind Tunnel.
By Ben FrantzDale (2007).

Streamlines vs. Pathlines

Characteristic Curves Overview:

- **Curve types:**
  - **streamlines:** parallel to **v**(**p**,t) in each point **p** for a fixed time
  - **pathlines:** motion of a particles over the time in an *unsteady* field **v**(**p**,t)
  - **streaklines:** location of all particles set out at a *fixed point* over time
  - **timelines:** evolution of a curve set out at a time t_0





[Tino Weinkauf, MPI]

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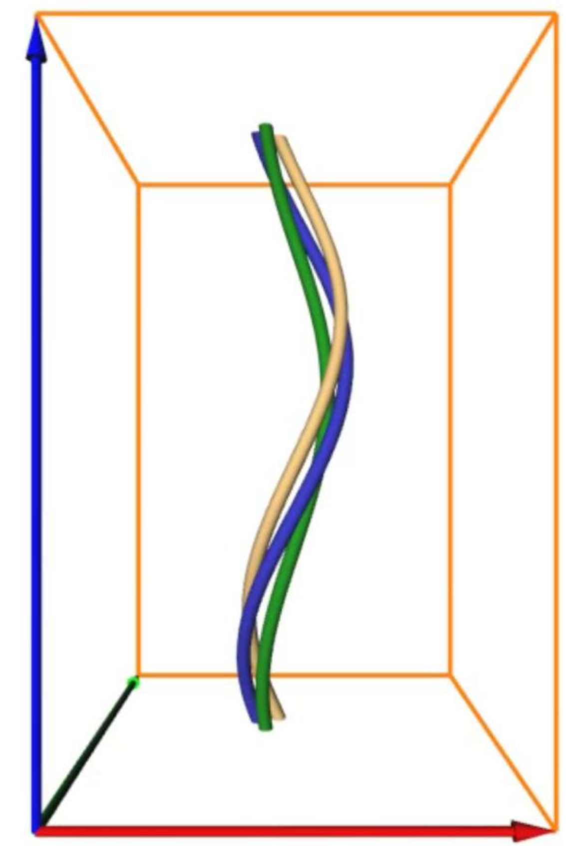
Limits of Vector Field Topology

Example 1: The Beads Problem [Wiebel et al., TopoInVis 2009]

- **Vector field:**
  - modeling aggregating cell behavior
  - simple rotating attractor

$$\mathbf{v}(x,y,t) = \begin{pmatrix} -(y - \frac{1}{3}\sin(t)) - (x - \frac{1}{3}\cos(t)) \\ (x - \frac{1}{3}\cos(t)) - (y - \frac{1}{3}\sin(t)) \end{pmatrix}$$

- applied methods:
  - critical points (green)
  - swirling pathline cores (blue)



[Tino Weinkauf, MPI]

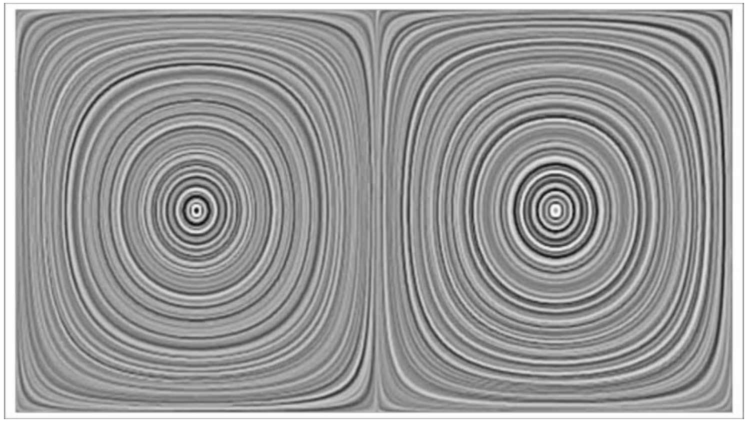
Limits of Vector Field Topology

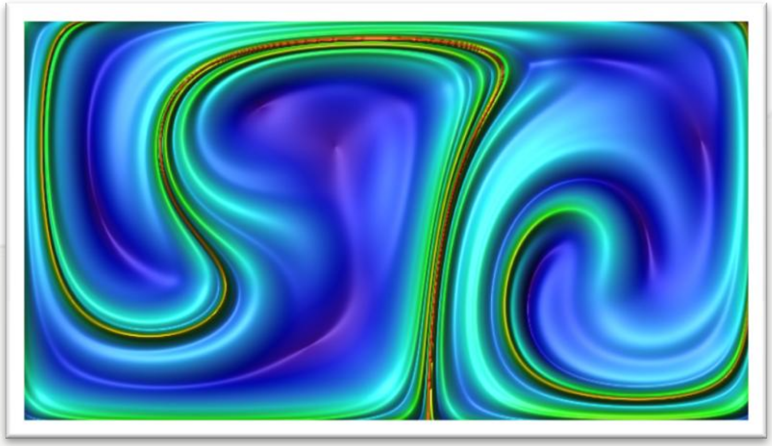
Example 2: Double Gyre [Shadden06]

- **static case:**
  - isolated critical features
    → two sliding orbits
  - separating structure

- **unsteady case:**
  - asymetric material structure
  - complex ridge evolvement
  - scalar field description

Example:

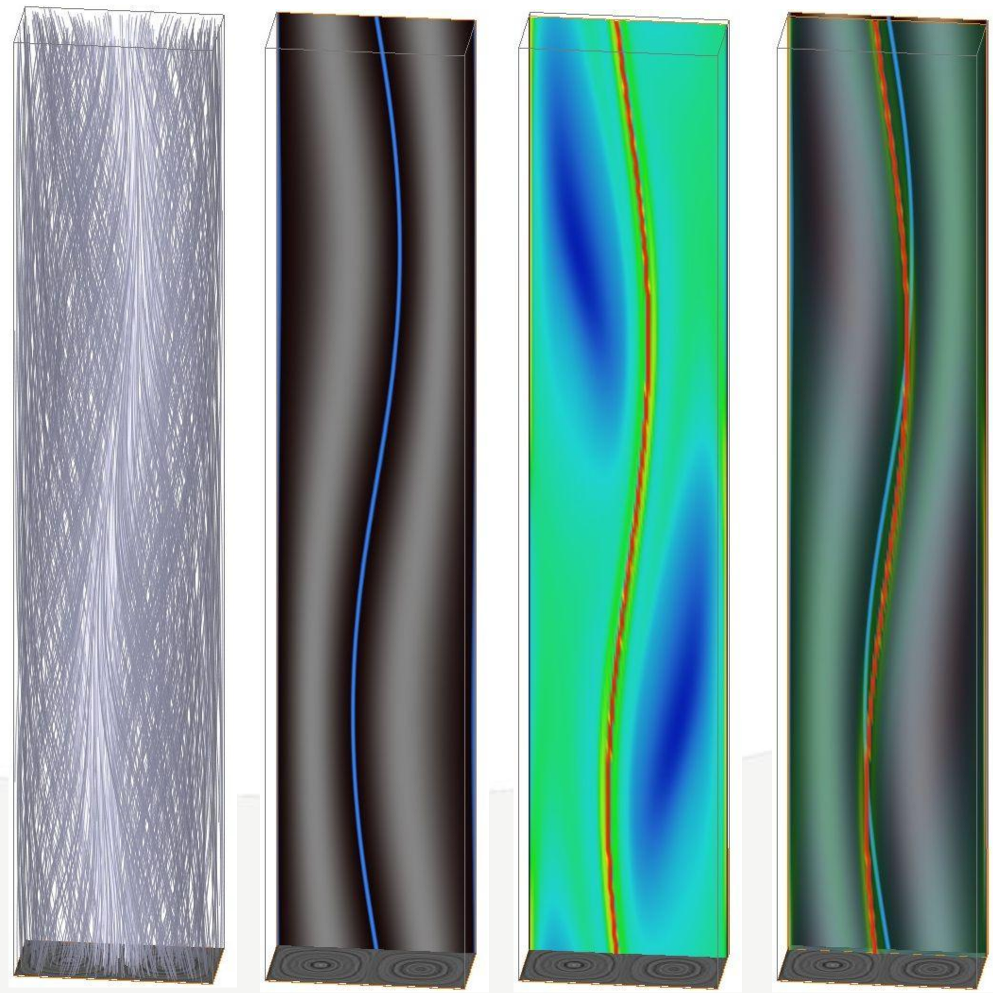




Limits of Vector Field Topology

Example 2: Double Gyre [Shadden06]

- **comparison:**
  a) pathlines
  b) vector length
  c) Lagrangian feature
  d) combined



Thank you for your attention!

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