Robust Bidirectional Search via Heuristic Improvement

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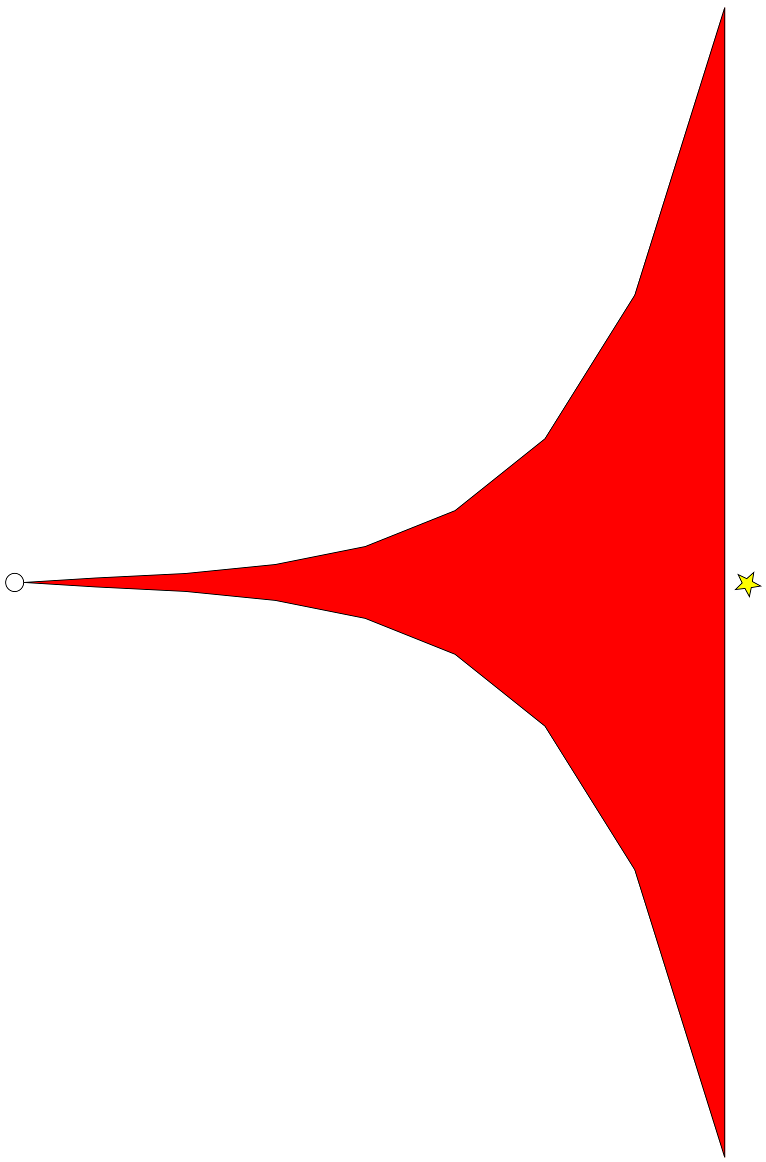
Bidirectional Search

Bidirectional?

- Worst case time complexity of a search is b^d
- Constructing two search trees with $b^{\frac{d}{2}}$ nodes each is much faster
- $b^d \ggg 2 \cdot b^{\frac{d}{2}}$

Bidirectional?



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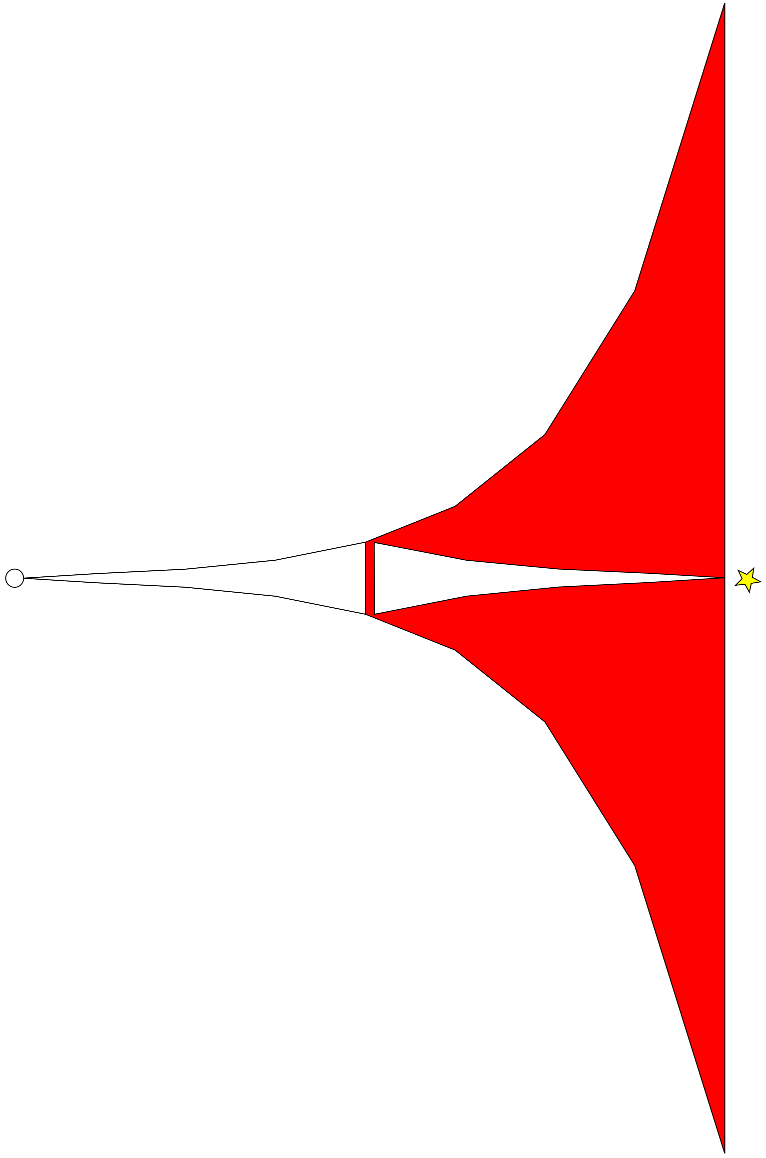

BHPA (Pohl 1971)

- Basic bidirectional heuristic search
- Searches from start to goal
- Searches from goal to start
- Terminates when incumbent solution is:

  1. Better than best forwards partial path
  2. Better than best backwards partial path

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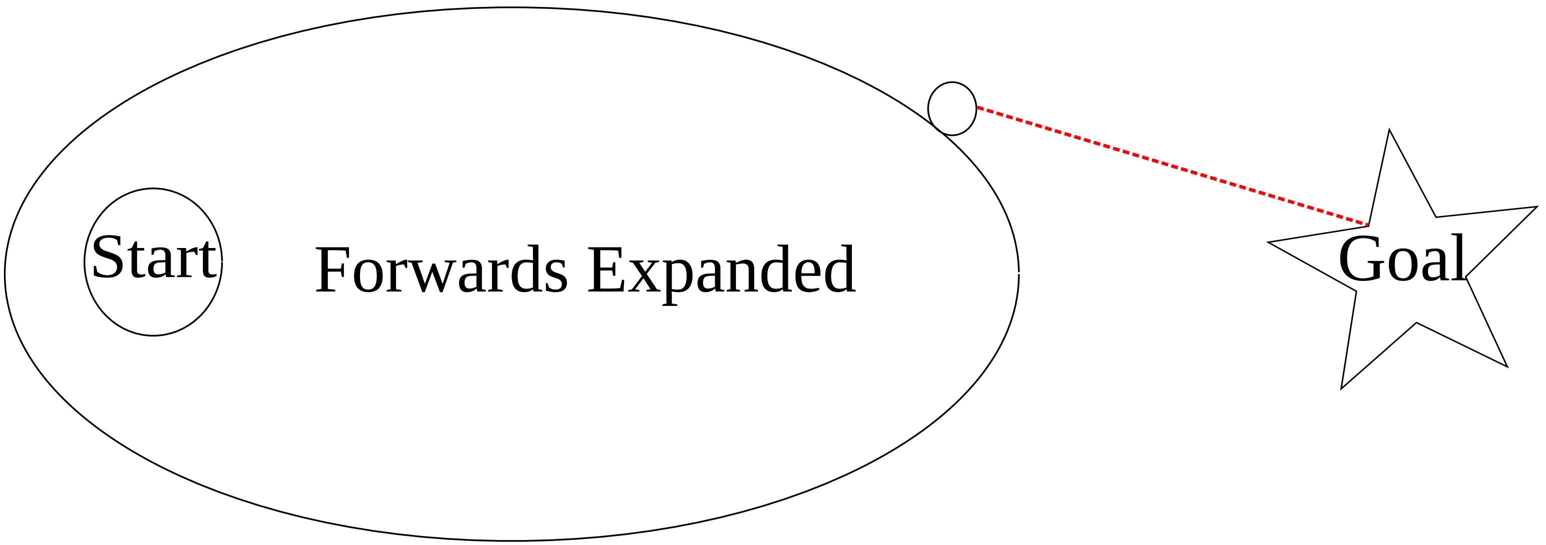
- This algorithm didn't live up to its $b^{\frac{d}{2}}$ promise
- Kainz and Kaindl (1997) showed:
  - BHPA found the optimal solution early
  - Spent the rest of the time proving optimality

Perimeter Search (Dillenburg 1994)

Perimeter Search Algorithm

1. Build a perimeter P by doing a limited backwards search
2. Do the forwards search with improved heuristic
   - $h(n) = min_{p \in P}(h(n,p) + g_{rev}(p))$



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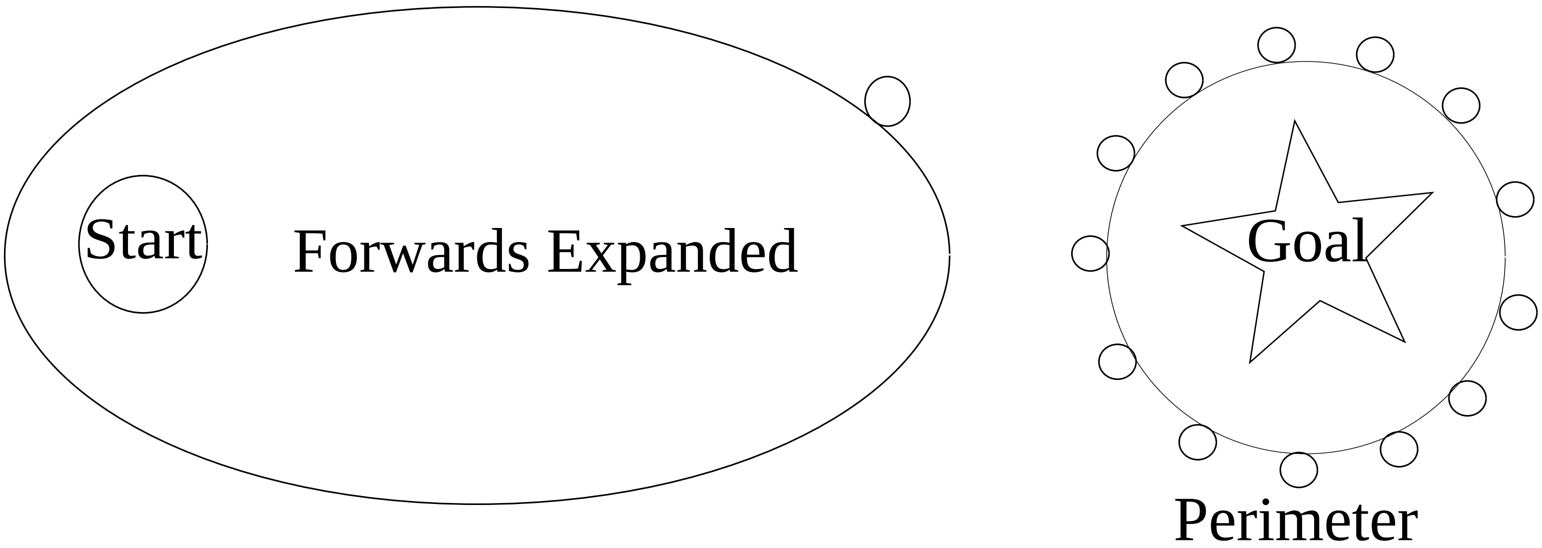

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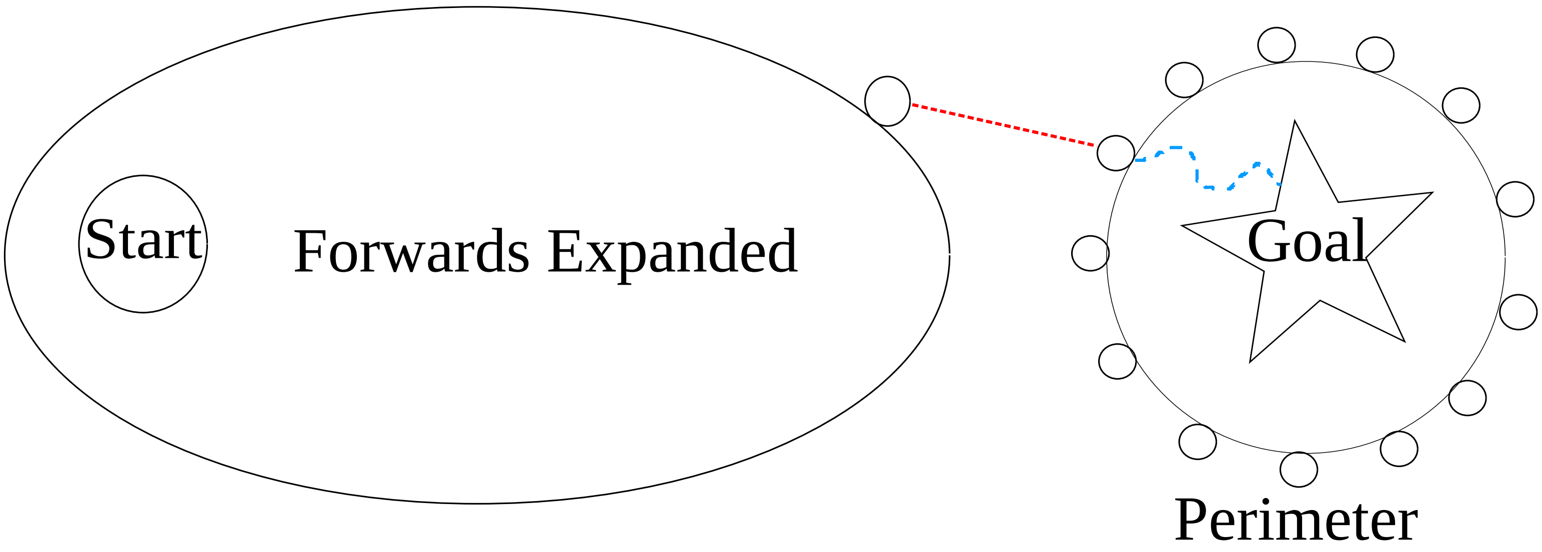

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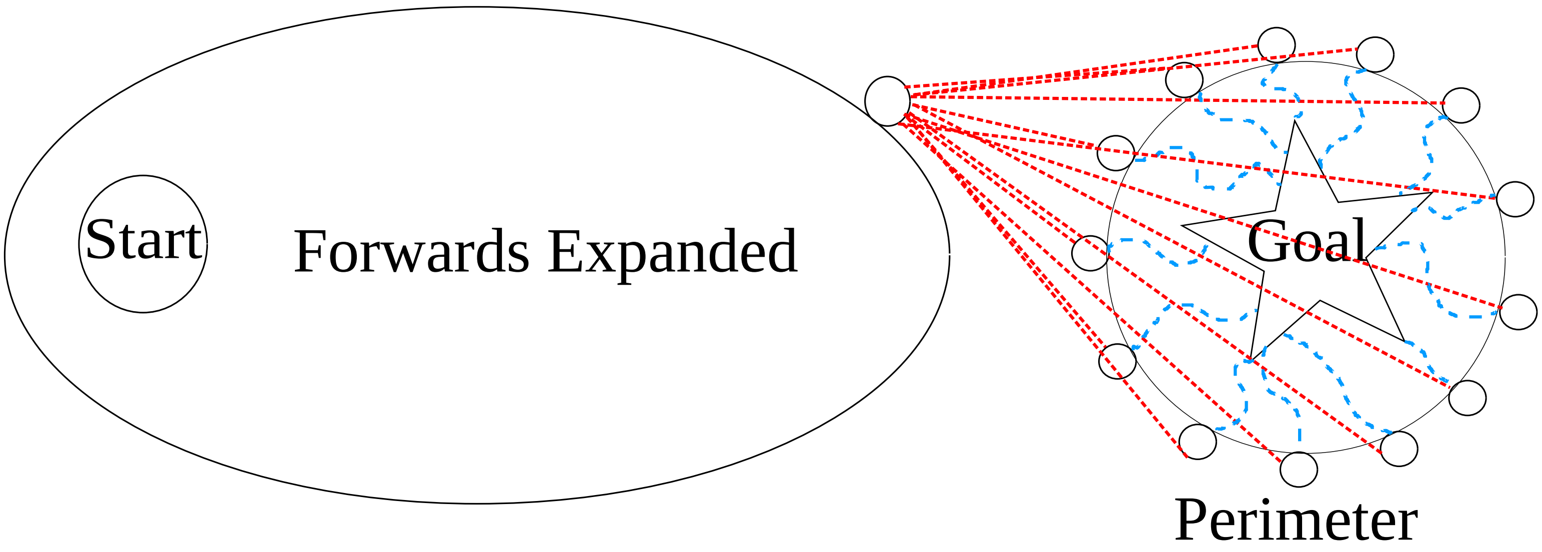

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The Bad:

- How much perimeter?
  - Analyzed by Linares-López (2002)
- Using the heuristic $|P|$ times can be slow

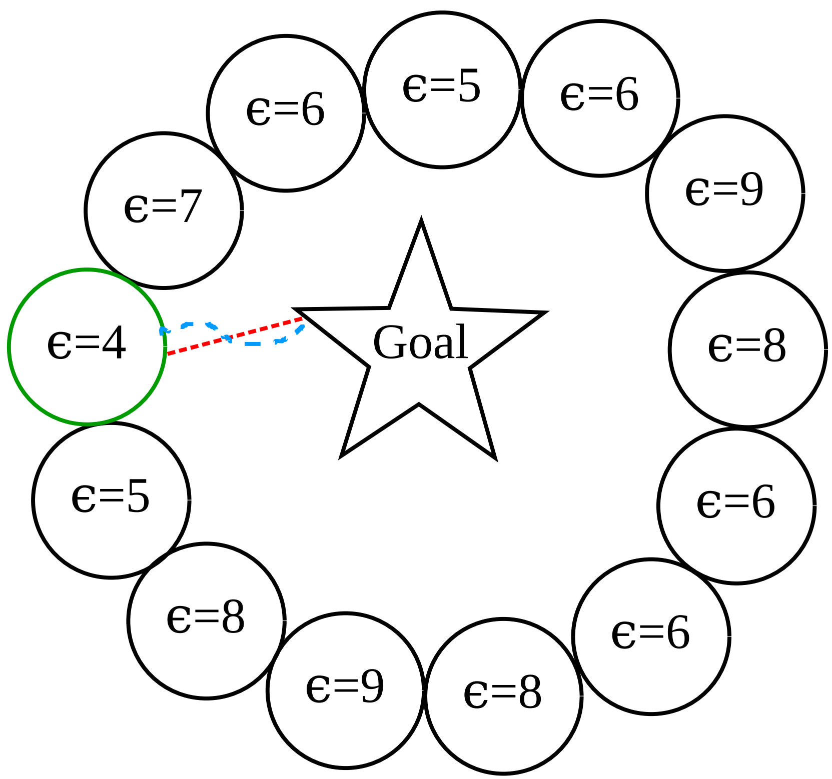
KKAdd (Kainz and Kaindl 1997)

Want to get benefit from perimeter without $|P|$ heuristic evaluations

$h(p)$ = Heuristic of p

$g_{rev}(p)$ = Optimal cost of getting from p to the goal

$\epsilon(p) = g_{rev}(p)$ - $h(p)$ = Heuristic Error at node p



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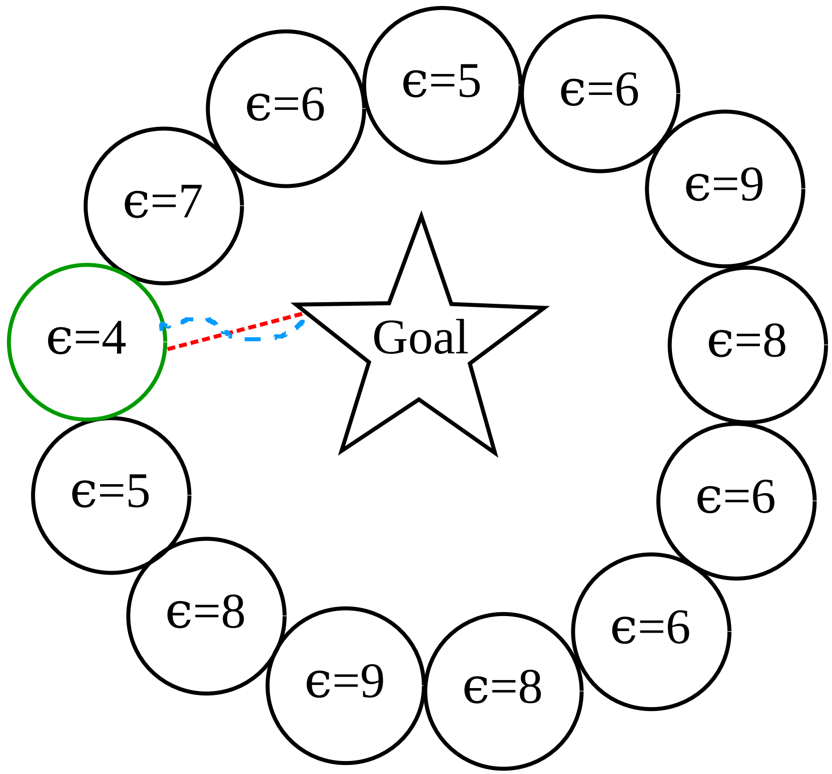

Theorem: If h is consistent, all nodes outside P have at least $min_{p \in P}(g_{rev}(p) - h(p))$ heuristic error.

A* with KKAdd Heuristic

1. Build a perimeter P by doing a limited backwards search
2. Do the forwards search with improved heuristic
   - $h_{KKAdd}(n) = h(n) + min_{p \in P}(g_{rev}(p) - h(p))$

$g_{rev}(p) =$ optimal cost of getting from the goal to p

$h(p) =$ heuristic at node p

$g_{rev}(p) - h(p) =$ error in the heuristic at node p

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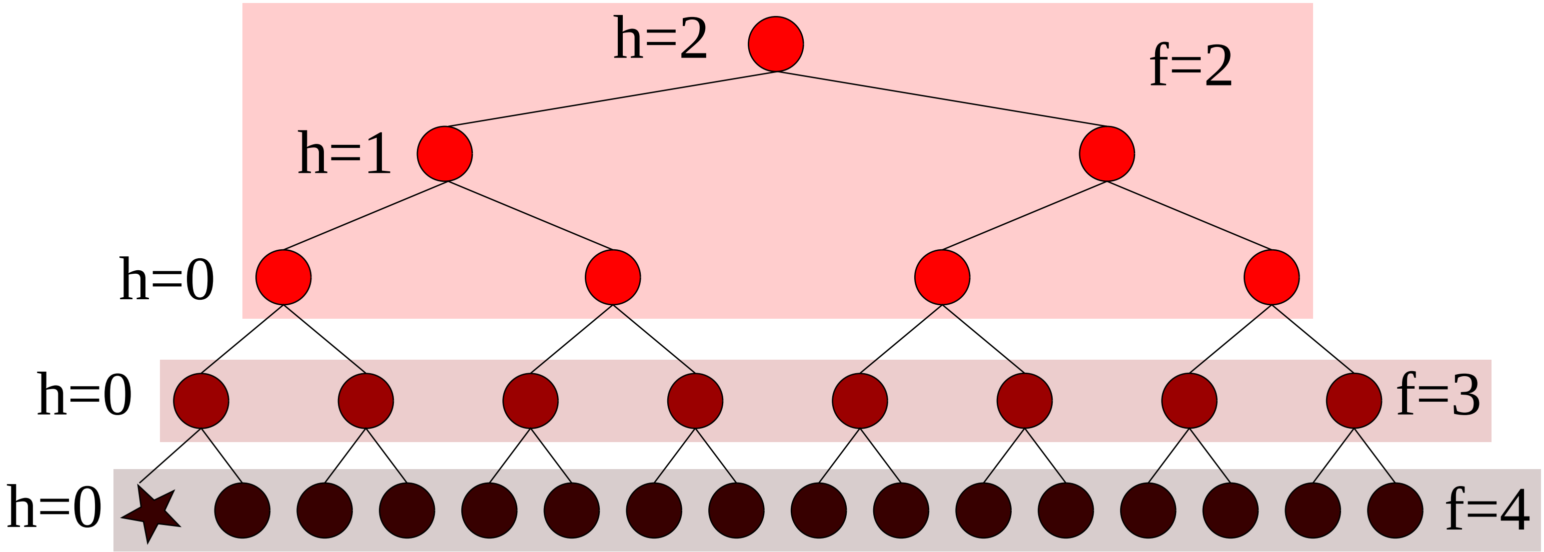
$min_{p \in P}(g_{rev}(p) - h(p)) = hAdd$

Called $hAdd$ because we **add** this quantity to the heuristic

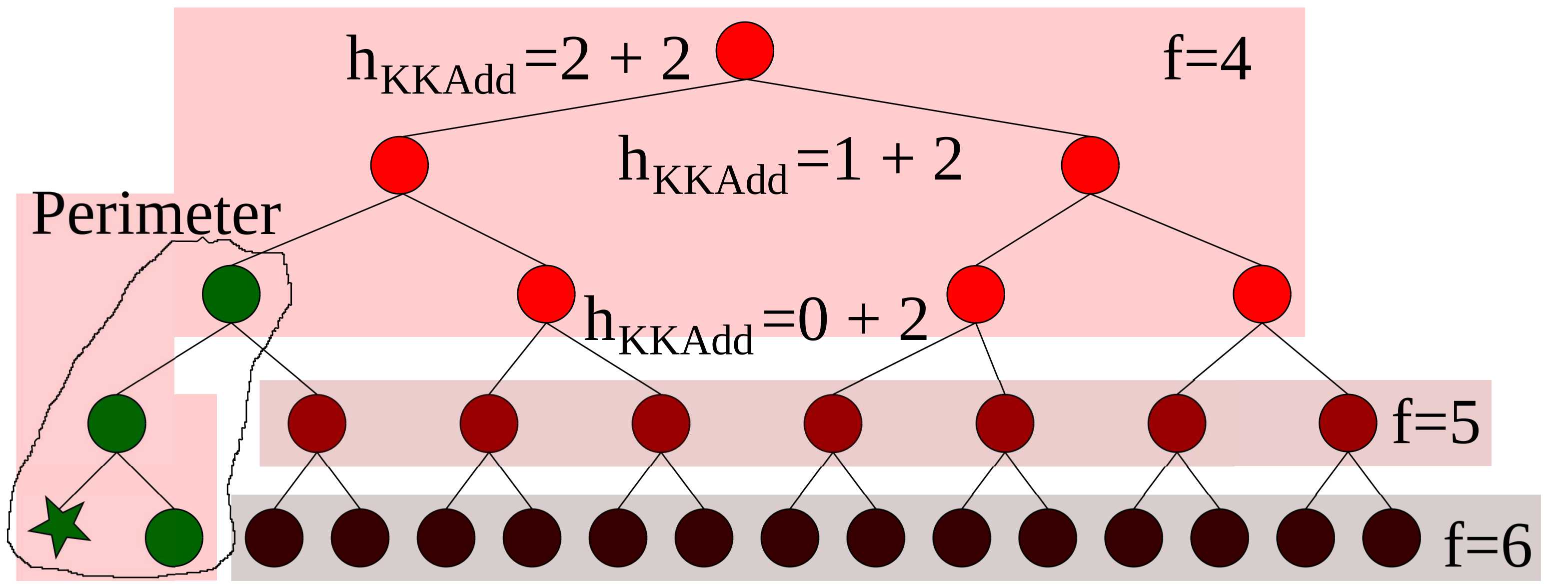
A* expands nodes in f layers



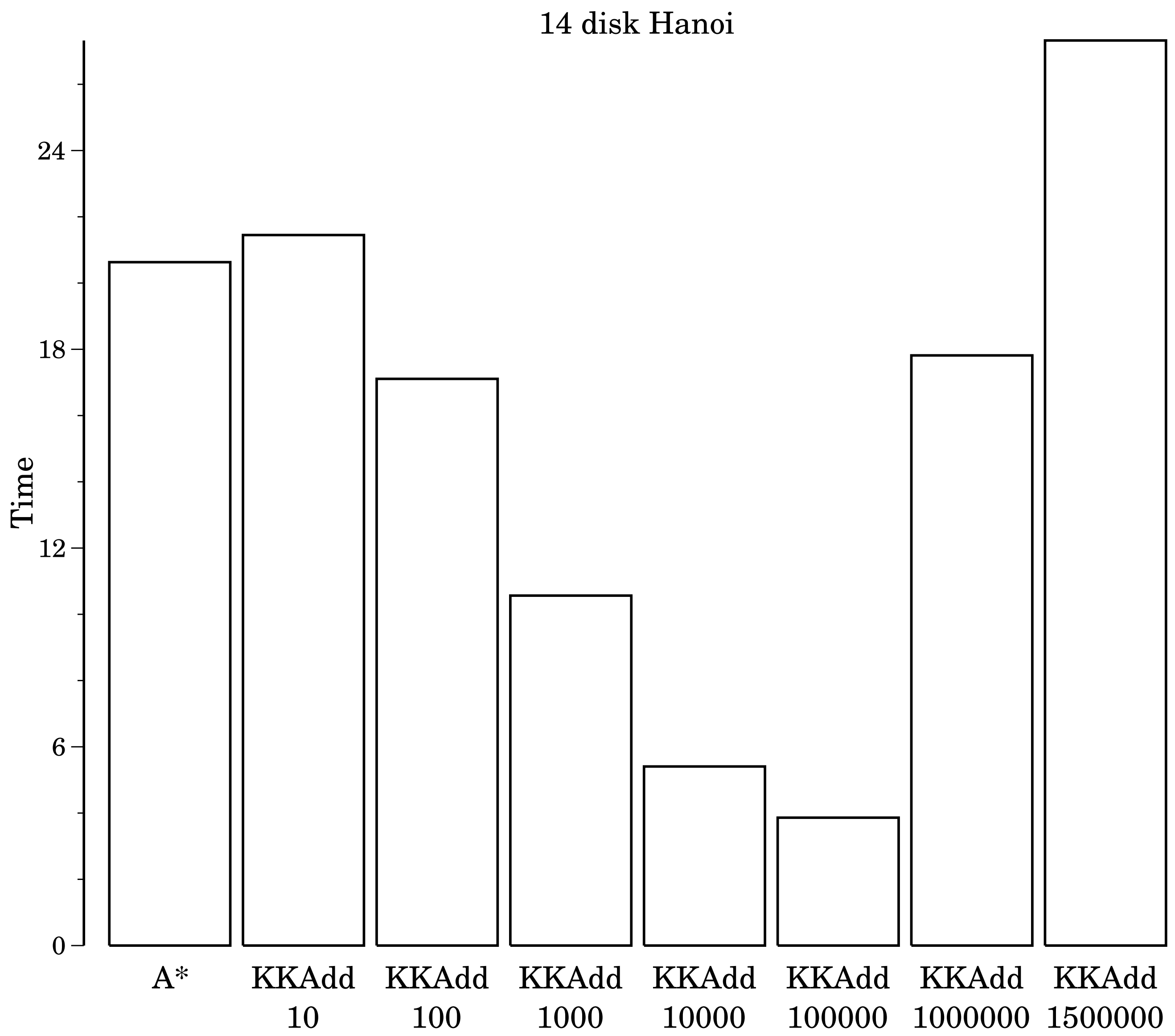
How does KKAdd reduce search effort?

The KKAdd heuristic can reduce the number of f layers

Now $h = h + hAdd$



KKAdd Results



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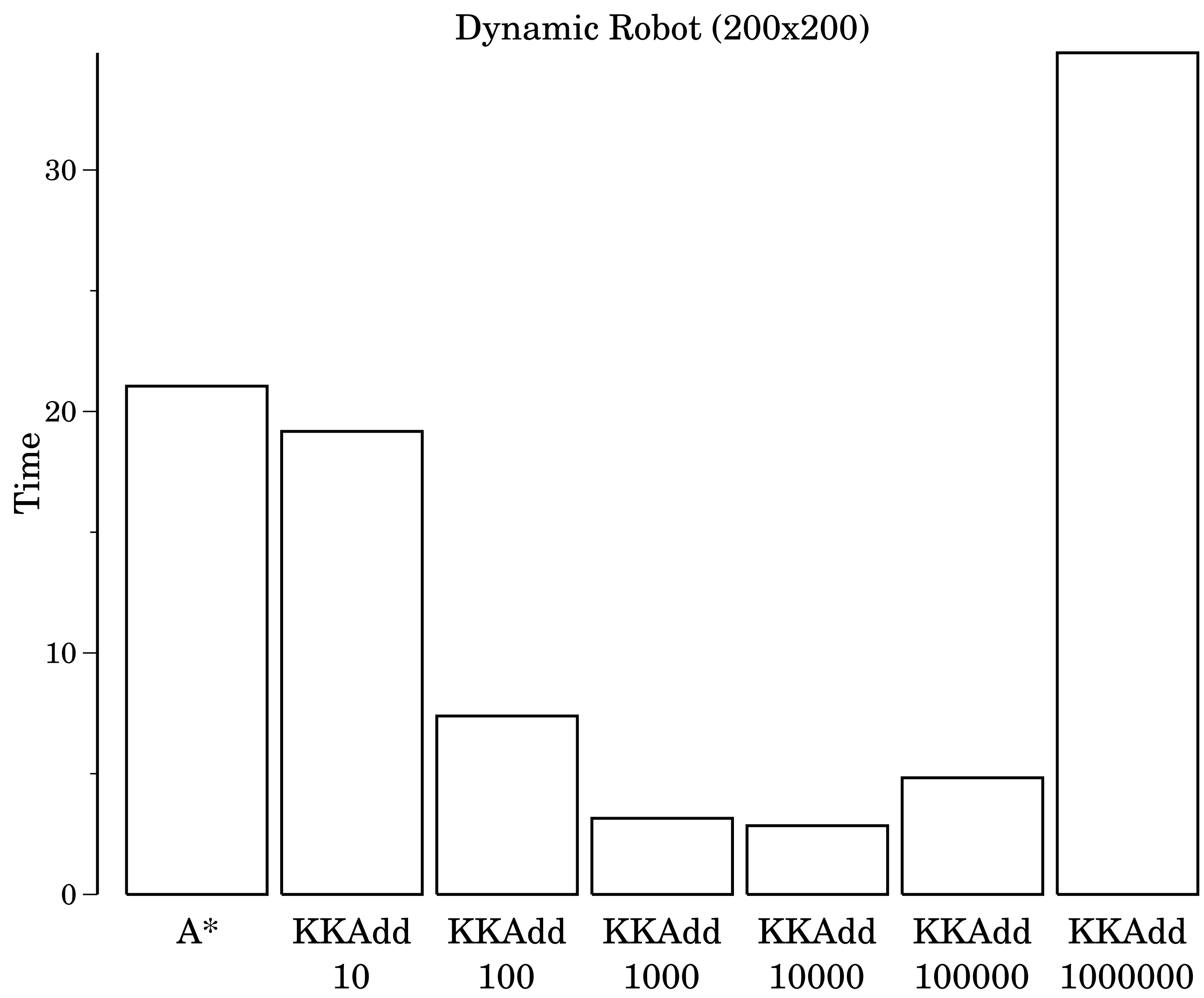

KKAdd Results

- KKAdd heuristic can speed up finding optimal solutions
- How much backwards search?
- Too little backwards search
  - No benefit
- Too much backwards search
  - Backwards search takes more time than plain A*

Incremental KKAdd

Changing Search Direction Online

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- Specify perimeter size as a **proportion** of all expansions

Proportion Advantages

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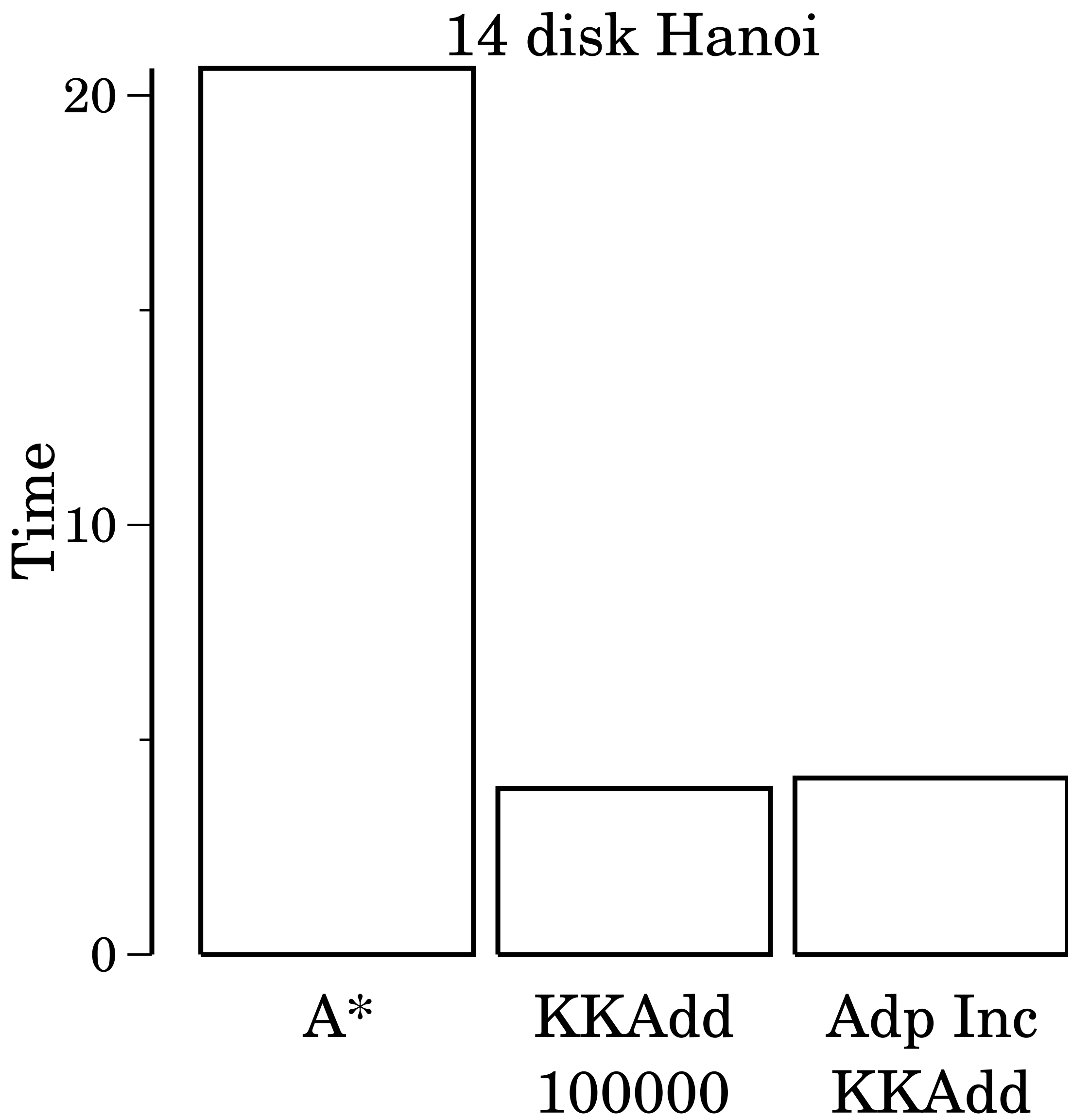
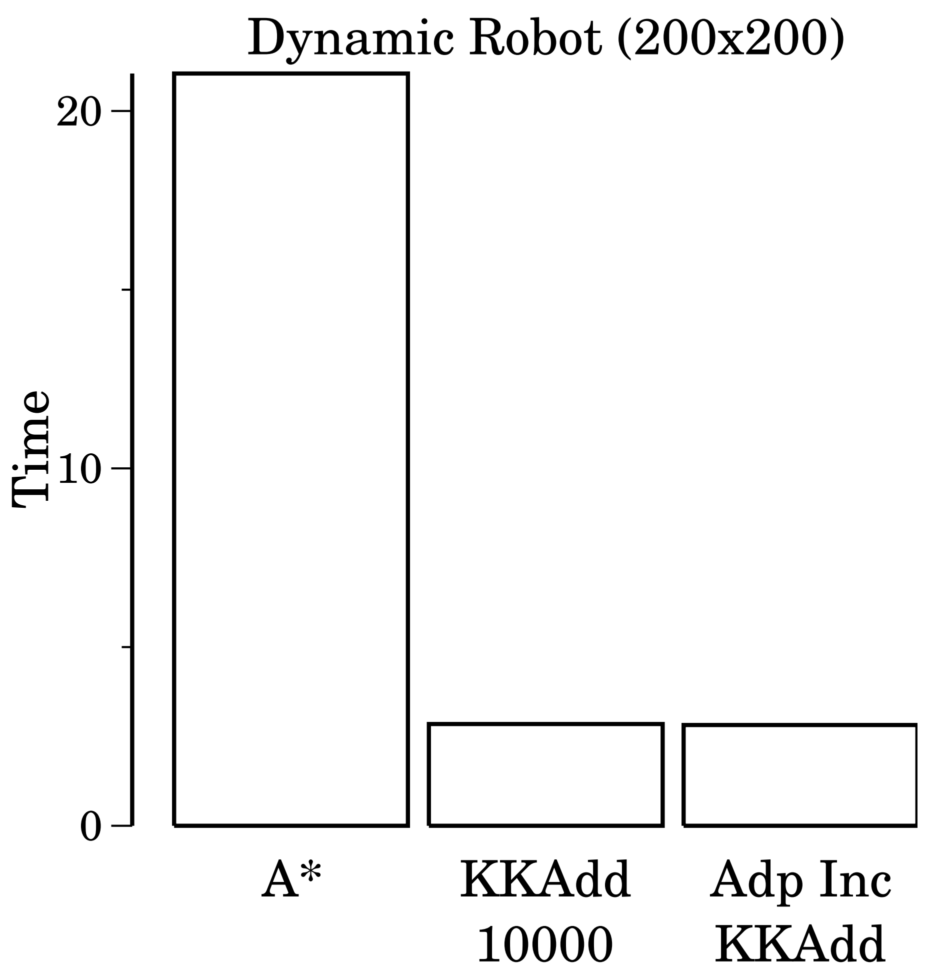
- Worst case bound:
  - $hAdd = 0$ so forwards search is the same as A*
  - Backwards work $< 2\cdot$ A* work $\cdot proportion$
  - e.g. guaranteed to be comparable to A*

Managing Backwards Expansions Online

- No proportion parameter
- Uses information at runtime to select direction to expand
- See paper for more details

Results





Adaptive KKAdd is able to capture the potential of KKAdd without the risk of doing too many or to few backwards expansions.

Robustness

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- KKAdd (Kainz and Kaindl 1997)
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- Incremental KKAdd
  - Worse case performance comparable to A*

Conclusion

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- **A framework for robust bidirectional search**